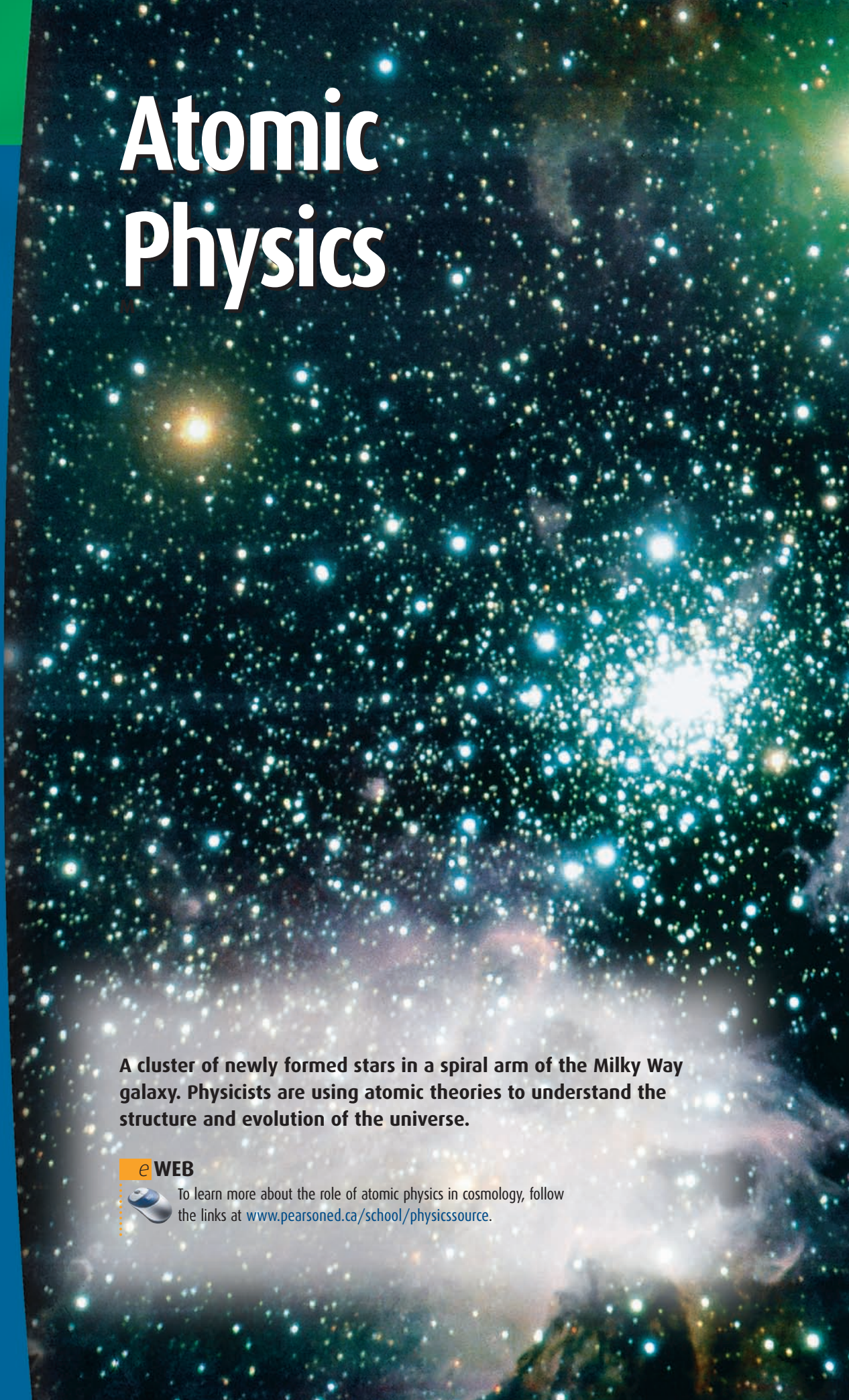
UNIT

VIII

Atomic Physics



A cluster of newly formed stars in a spiral arm of the Milky Way galaxy. Physicists are using atomic theories to understand the structure and evolution of the universe.

eWEB

To learn more about the role of atomic physics in cosmology, follow the links at www.pearsoned.ca/school/physicssource.

Unit at a Glance

CHAPTER 15 Electric force and energy quantization determine atomic structure.

CHAPTER 16 Nuclear reactions are among the most powerful energy sources in nature.

CHAPTER 17 The development of models of the structure of matter is ongoing.

Unit Themes and Emphases

- Energy and Matter

Focussing Questions

The study of atomic structure requires analyzing how matter and energy are related. As you study this unit, consider these questions:

- What is the structure of atoms?
- How can models of the atom be tested?
- How does knowledge of atomic structure lead to the development of technology?

Unit Project

How Atomic Physics Affects Science and Technology

- Unit VIII discusses radical changes in the understanding of matter and energy. In this project, you will investigate how advances in atomic physics influenced the development of technology and other branches of science.

CHAPTER

15

Electric force and energy quantization determine atomic structure.

Key Concepts

In this chapter, you will learn about:

- charge-to-mass ratio
- classical model of the atom
- continuous and line spectra
- energy levels
- quantum mechanical model

Learning Outcomes

When you have completed this chapter, you will be able to:

Knowledge

- describe matter as containing discrete positive and negative charges
- explain how the discovery of cathode rays helped develop atomic models
- explain the significance of J.J. Thomson's experiments
- explain Millikan's oil-drop experiment and charge quantization
- explain the significance of Rutherford's scattering experiment
- explain how electromagnetic theory invalidates the classical model of the atom
- describe how each element has a unique line spectrum
- explain continuous and line spectra
- explain how stationary states produce line spectra
- calculate the energy difference between states, and the characteristics of emitted photons

Science, Technology, and Society

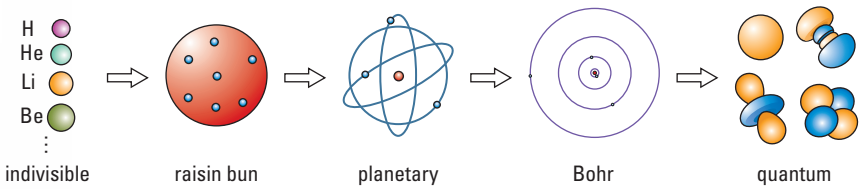
- explain how scientific knowledge and theories develop
- explain the link between scientific knowledge and new technologies

Max Planck began undergraduate studies at the University of Munich in 1874. During his first term, he took primarily mathematics courses, although he also had considerable musical talent and an interest in physics. Wondering which field to pursue, the teenaged Planck asked his physics professor, Philipp von Jolly, about the prospects for a career in physics. Jolly told Planck that there would be few opportunities since almost everything in physics had already been discovered, leaving only a few minor gaps to fill in.

Despite Jolly's discouraging advice, Planck went on to complete a doctorate in physics, and became a renowned professor at the University of Berlin. As described in Chapter 14, one of the "gaps" in theoretical physics was the puzzling distribution of energy among the wavelengths of radiation emitted by a heated body. Planck concentrated on this problem for months, and by the end of 1900 concluded that this distribution is possible only if energy is quantized.

At first, Planck and his contemporaries did not realize the huge significance of his findings. As you will learn in this chapter, Planck's discovery led to quantum theory, a concept that revolutionized atomic physics. You will also see that Planck was just the first of many researchers who demonstrated that there was still a great deal to be discovered in physics.

Figure 15.1 shows how radically our concept of the atom changed during the 20th century. Planck's idea of quantization of energy led Bohr to propose his model of the atom in 1913. In this chapter, you will study in detail how the model of the atom evolved.



▲ **Figure 15.1** The evolution of theories of atomic structure

15-1 QuickLab

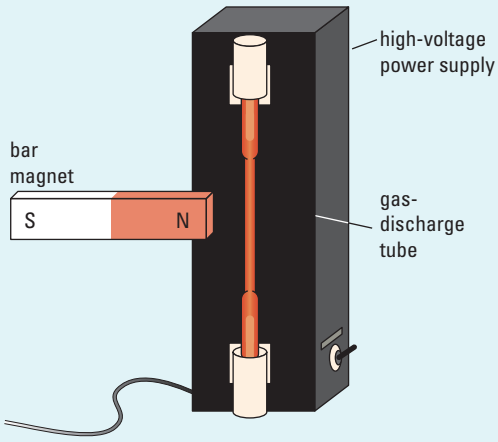
Cathode Rays and Magnetic Fields

Problem

What type of charge do cathode rays have?

Materials

gas-discharge tube (Figure 15.2)
high-voltage power supply
bar magnet or electromagnet



▲ **Figure 15.2** A gas-discharge tube

Procedure

1. Your teacher will give directions for setting up the particular tube and power supply that you will use. Follow these directions carefully. Note the location of the cathode in the discharge tube.
2. Turn on the power supply and note the beam that appears in the tube. To see the beam clearly, you may need to darken the room.
3. Bring the magnet close to the tube surface. Note the direction of the magnetic field and the direction in which the beam moves.
4. Repeat step 3 at various positions along the tube.

Questions

1. How did you determine the direction of the magnetic field?
2. What do you think causes the visible beam in the tube?
3. What evidence suggests that the magnet causes the beam to deflect?
4. How could you verify that the cathode rays originate from the negative terminal of the discharge tube?
5. What can you conclude about the charge of the cathode rays? Explain your reasoning.
6. Describe another way to determine the type of charge on cathode rays.

Think About It

1. What are atoms made of?
2. What holds atoms together?
3. How can physicists measure atomic structure when it is too small to be seen by even the most powerful microscope?

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes in your ideas.

15.1 The Discovery of the Electron

info **BIT**

Dalton's research included important findings about the aurora borealis, atmospheric gases, and the formation of dew. He also made the first scientific study of colour blindness, which he had himself.

Around 1803, the English chemist John Dalton (1766–1844) developed an atomic theory to explain the ratios in which elements combine to form compounds. Although later discoveries required modifications to Dalton's theory, it is a cornerstone for modern atomic theory. This theory could explain the observations made by Dalton and other chemists, but their experiments did not provide any direct evidence that atoms actually exist. At the end of the 19th century, there was still some doubt about whether all matter was made up of atoms. By 1900, experiments were providing more direct evidence. Current technology, such as scanning tunnelling microscopes, can produce images of individual atoms.

MINDS ON **Evidence for Atoms**

How do you know that atoms exist?

Work with a partner to list evidence that matter is composed of atoms.

Cathode-ray Experiments

cathode ray: free electrons emitted by a negative electrode

During the 1800s, scientists discovered that connecting a high voltage across the electrodes at opposite ends of an evacuated glass tube caused mysterious rays to flow from the negative electrode (the cathode) toward the positive electrode. These **cathode rays** caused the glass to glow when they struck the far side of the tube. The rays could be deflected by a magnetic field. In 1885, after several years of experiments with improved vacuum discharge tubes, William Crookes in England suggested that cathode rays must be streams of negatively charged particles. In 1895, Jean Baptiste Perrin in France showed that cathode rays entering a hollow metal cylinder built up a negative charge on the cylinder.

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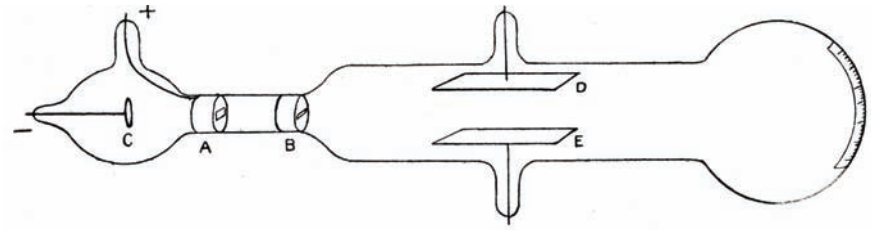
The Irish physicist G. Johnstone Stoney coined the term *electron* in 1891. Thomson called the particles in cathode rays "corpuscles."

In 1897, Joseph John Thomson (1856–1940) took these experiments a step further. First, he used an improved version of Perrin's apparatus to show even more clearly that cathode rays carry negative charge (Figure 15.3). Next, he tackled the problem of why no one had been able to deflect cathode rays with an electric field. Thomson's hypothesis was that the cathode rays ionized some of the air molecules remaining in the vacuum chamber and these ions then shielded the cathode rays from the electric field. By taking great care to get an extremely low pressure in his discharge tube, Thomson was able to demonstrate that cathode rays respond to electric fields just as negatively charged particles would. Thomson had discovered the electron.



The rays from the cathode C pass through a slit in the anode A, which is a metal plug fitting tightly into the tube and connected with the earth; after passing through a second slit in another earth-connected metal plug B, they travel between two parallel aluminium plates about 5 cm long by 2 broad and at a distance of 1.5 cm apart; they then fall on the end of the tube and produce a narrow well-defined phosphorescent patch. A scale pasted on the outside of the tube serves to measure the deflexion of this patch.

◀ **Figure 15.3** Reproduction of Thomson's diagram and part of his description of the apparatus he used for several of his experiments. To determine the charge-to-mass ratio for an electron, Thomson added an electromagnet beside the parallel plates in the middle of the tube.

MINDS ON **Are Electrons Positively or Negatively Charged?**

Outline two different methods for testing whether cathode rays consist of negatively or positively charged particles.

Charge-to-mass Ratio of the Electron

Thomson did not have a method for measuring either the mass of an electron or the charge that it carried. However, he did find a way to determine the ratio of charge to mass for the electron by using both an electric field and a magnetic field.

Recall from Chapter 11 that the electric force acting on a charged particle is

$$\vec{F}_e = q\vec{E}$$

where $\vec{F}_e$ is the electric force, q is the magnitude of the charge on the particle, and $\vec{E}$ is the electric field.

Section 12.2 describes how the left-hand rule gives the direction of the magnetic force acting on a negative charge moving through a magnetic field. The magnitude of this force is

$$|\vec{F}_m| = qv_\perp|\vec{B}|$$

where q is the magnitude of the charge on the particle, $v_\perp$ is the component of the particle's velocity perpendicular to the magnetic field, and B is the magnitude of the magnetic field.

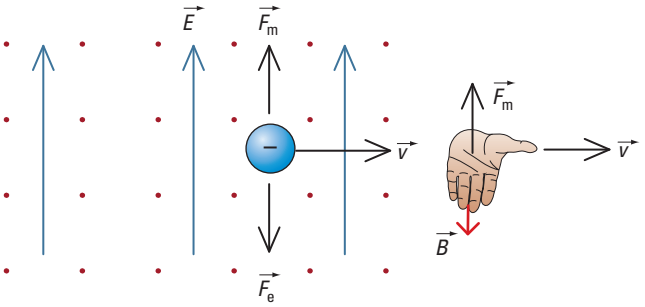
For a particle moving perpendicular to a magnetic field, $v_\perp = v$ and

$$|\vec{F}_m| = qv|\vec{B}|$$

Consider the perpendicular electric and magnetic fields shown in Figure 15.4. The electric field exerts a downward force on the negative charge while the magnetic field exerts an upward force. The gravitational force acting on the particle is negligible. If the net force on the charged particle is zero, the electric and magnetic forces must be equal in magnitude but opposite in direction:

PHYSICS INSIGHT

Dots and ×s are a common way to show vectors directed out of or into the page. A dot represents the tip of a vector arrow coming toward you, and × represents the tail of an arrow moving away from you.



▲ **Figure 15.4** Perpendicular electric and magnetic fields act on a moving negative charge. The red dots represent a magnetic field directed out of the page. Using the left-hand rule for magnetic force, your thumb points in the direction of electron flow (right), fingers point in the direction of the magnetic field (out of the page), and the palm indicates the direction of the magnetic force (toward the top of the page).

$$\vec{F}_{net} = \vec{F}_e + \vec{F}_m$$
$$\vec{F}_{net} = 0, \text{ so}$$
$$F_e = F_m$$
$$|\vec{E}|q = |\vec{B}|qv$$
$$|\vec{E}| = |\vec{B}|v$$

The speed of the particle is, therefore,

$$v = \frac{|\vec{E}|}{|\vec{B}|}$$

Example 15.1

A beam of electrons passes undeflected through a 0.50-T magnetic field combined with a 50-kN/C electric field. The electric field, the magnetic field, and the velocity of the electrons are all perpendicular to each other. How fast are the electrons travelling?

Given

$|\vec{E}| = 50 \text{ kN/C} = 5.0 \times 10^4 \text{ N/C}$ $\qquad$ $|\vec{B}| = 0.50 \text{ T}$

Required

speed (v)

Analysis and Solution

Since the electrons are not deflected, $\vec{F}_{net} = 0$, so the magnitude of $\vec{F}_e$ equals the magnitude of $\vec{F}_m$.

$$v = \frac{|\vec{E}|}{|\vec{B}|}$$
$$= \frac{5.0 \times 10^4 \text{ N/C}}{0.50 \text{ T}}$$
$$= 1.0 \times 10^5 \text{ m/s}$$

Paraphrase

The electrons are travelling at a speed of 1.0×10^5 m/s.

Practice Problems

1. A beam of electrons passes undeflected through a 2.50-T magnetic field at right angles to a 60-kN/C electric field. How fast are the electrons travelling?
2. What magnitude of electric field will keep protons from being deflected while they move at a speed of 1.0×10^5 m/s through a 0.05-T magnetic field?
3. What magnitude of magnetic field will stop ions from being deflected while they move at a speed of 75 km/s through an electric field with a magnitude of 150 N/C?

Answers

1. 2.4×10^4 m/s
2. 5×10^3 N/C
3. 2.0×10^{-3} T

Concept Check

What would happen to the beam of electrons in Example 15.1 if their speed were greatly decreased?

Thomson used mutually perpendicular electric and magnetic fields to determine the speed of the cathode rays. He then measured the deflection of the rays when just one of the fields was switched on. These deflections depended on the magnitude of the field, the length of the path in the

field, and the speed, charge, and mass of the cathode-ray particles. Thomson could determine only the first three quantities, but he could use his measurements to calculate the ratio of the two unknowns, the charge and mass of the particles. Thomson made measurements with a series of cathode-ray tubes that each had a different metal for the electrode that emitted the rays. Since he found reasonably consistent values for the charge-to-mass ratio, Thomson concluded that all cathode rays consist of identical particles with exactly the same negative charge.

Thomson's experiments showed that $\frac{q}{m}$ for an electron is roughly 10^{11} C/kg. This ratio is over a thousand times larger than the ratio for a hydrogen ion. Other physicists had shown that cathode rays can pass through thin metal foils and travel much farther in air than atoms do. Therefore, Thomson reasoned that electrons are much smaller than atoms. In his Nobel Prize lecture in 1906, he stated that "we are driven to the conclusion that the mass of the corpuscle is only about 1/1700 of that of the hydrogen atom." This value is within a few percent of the mass determined by the latest high-precision measurements.

Thomson put forward the daring theory that atoms were divisible, and the tiny particles in cathode rays were "the substance from which all the chemical elements are built up." Although he was incorrect about electrons being the *only* constituents of atoms, recognizing that electrons are subatomic particles was a major advance in atomic physics.

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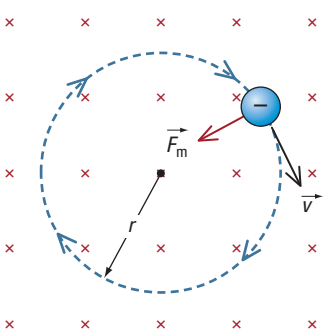
Around 1900, the idea that atoms could be subdivided was so radical that some physicists thought Thomson was joking when he presented his findings in a lecture to the Royal Institute in England.

Determining Charge-to-mass Ratios

Thomson measured the deflection of cathode rays to determine the charge-to-mass ratio of the electron. You can also determine mass-to-charge ratios by measuring the path of charged particles in a uniform magnetic field.

When a charged particle moves perpendicular to a magnetic field, the magnetic force is perpendicular to the particle's velocity, so the particle's direction changes, but its speed is constant. In a uniform magnetic field, $|\vec{B}|$ is constant, so the magnitude of the magnetic force, $|\vec{B}|qv$, is also constant. As described in Chapter 6, a force of constant magnitude perpendicular to an object's velocity can cause uniform circular motion. A charged particle moving perpendicular to a uniform magnetic field follows a circular path, with the magnetic force acting as the centripetal force (Figure 15.5).



$$|\vec{F}_{net}| = |\vec{F}_m|$$
$$|\vec{F}_c| = |\vec{F}_m|$$
$$\frac{mv^2}{r} = |\vec{B}|qv$$
$$\frac{q}{m} = \frac{v}{|\vec{B}|r}$$

▲ **Figure 15.5** The path of an electron travelling perpendicular to a uniform magnetic field. The red ×s represent a magnetic field directed into the page.

Concept Check

How do you know that the magnetic force on the particle is directed toward the centre of a circular path in Figure 15.5?

Example 15.2

When a beam of electrons, accelerated to a speed of 5.93×10^5 m/s, is directed perpendicular to a uniform 100-μT magnetic field, they travel in a circular path with a radius of 3.37 cm (Figure 15.6). Determine the charge-to-mass ratio for an electron.

Practice Problems

1. Find the charge-to-mass ratio for an ion that travels in an arc of radius 1.00 cm when moving at 1.0×10^6 m/s perpendicular to a 1.0-T magnetic field.
2. Find the speed of an electron moving in an arc of radius 0.10 m perpendicular to a magnetic field with a magnitude of 1.0×10^{-4} T.
3. A carbon-12 ion has a charge-to-mass ratio of 8.04×10^6 C/kg. Calculate the radius of the ion's path when the ion travels at 150 km/s perpendicular to a 0.50-T magnetic field.

Answers

1. 1.0×10^8 C/kg
2. 1.8×10^6 m/s
3. 0.037 m

Given

$v_e = 5.93 \times 10^5$ m/s

$|\vec{B}| = 100\ \mu\text{T} = 1.00 \times 10^{-4}$ T

$r = 3.37$ cm $= 3.37 \times 10^{-2}$ m

Required

charge-to-mass ratio $\left(\frac{q}{m}\right)$

Analysis and Solution

Since the magnetic force acts as the centripetal force,

$$|\vec{F}_m| = |\vec{F}_c|$$

$$|\vec{B}|qv = \frac{mv^2}{r}$$

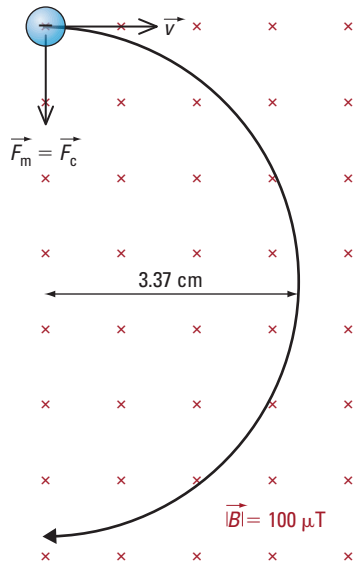
$$|\vec{B}|q = \frac{mv}{r}$$

$$\frac{q}{m} = \frac{v}{|\vec{B}|r}$$

Substituting the known values for the beam of electrons gives

$$\frac{q}{m} = \frac{5.93 \times 10^5\ \text{m/s}}{(1.00 \times 10^{-4}\ \text{T})(3.37 \times 10^{-2}\ \text{m})}$$

$$= 1.76 \times 10^{11}\ \text{C/kg}$$



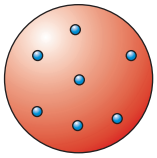
▲ **Figure 15.6**

Paraphrase

The charge-to-mass ratio for an electron is about 1.76×10^{11} C/kg.

Thomson's Raisin-bun Model

Most atoms are electrically neutral. If electrons are constituents of atoms, atoms must also contain some form of positive charge. Since no positively charged subatomic particles had yet been discovered, Thomson suggested that atoms might consist of electrons embedded in a blob of massless positive charge, somewhat like the way raisins are embedded in the dough of a raisin bun. Figure 15.7 shows Thomson's model of the atom.



▲ **Figure 15.7** Thomson's model of the atom: negative electrons embedded in a positive body

Concept Check

What characteristics should a scientific model have? Does Thomson's raisin-bun model of the atom have these characteristics?

THEN, NOW, AND FUTURE — The Mass Spectrometer

Thomson used electric and magnetic fields to determine the charge-to-mass ratio of electrons. In later experiments, he made similar measurements for positive ions. These experiments led to the development of the mass spectrometer, an instrument that can detect compounds, measure isotope masses, and determine molecular structures (Figure 15.8).

Many mass spectrometers use a four-stage process enclosed in a vacuum chamber:

Ionization: If the sample is not already a gas, the ion source vaporizes it, usually by heating. Heating may also break complex compounds into smaller fragments that are easier to identify. Next, the neutral compounds in the sample are ionized so that they will respond to electric and magnetic fields. Usually, the ion source knocks one or two electrons off the compound to produce a positive ion.

Acceleration: High-voltage plates then accelerate a beam of these ions into the velocity selector.

Velocity Selection: The velocity selector has crossed electric and magnetic fields arranged such that only the ions that have a speed of $v = \frac{|\vec{E}|}{|\vec{B}|}$ pass straight through. Ions with slower or faster speeds are deflected away from the entrance to the detection chamber. Thus, all the ions entering the next stage of the mass spectrometer have the same known speed.

Detection: A uniform magnetic field in the detection chamber makes the ions travel in circular paths. The radius of each path depends on the charge and mass of the ion. Ions arriving at the detector produce an electrical current that is proportional to the number of ions. The spectrometer can produce a graph of the charge-to-mass ratios for a sample by moving the detector or by varying the electric or magnetic fields.

Mass spectrometers can detect compounds in concentrations as small as a few parts per billion. These versatile machines have a huge range of applications in science, medicine, and industry.

Questions

1. Write an expression for the radius of the path of ions in a mass spectrometer.
2. How does the mass of an ion affect the radius of its path in the detection chamber?
3. Describe how you could use a mass spectrometer to detect an athlete's use of a banned performance-enhancing drug.

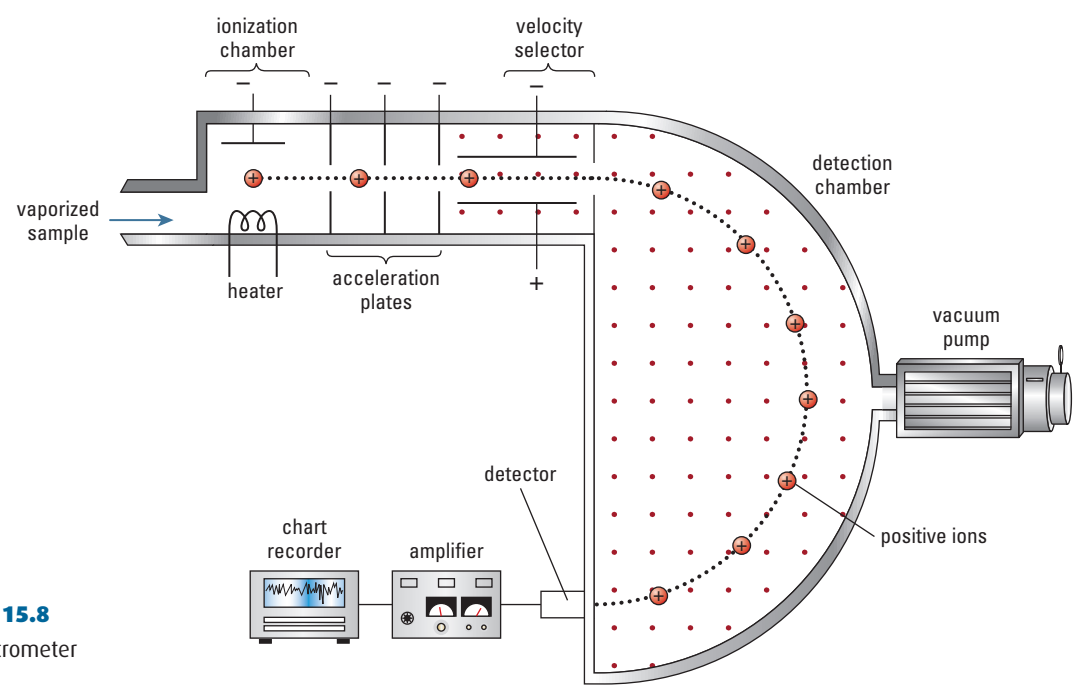


Figure 15.8
Mass spectrometer

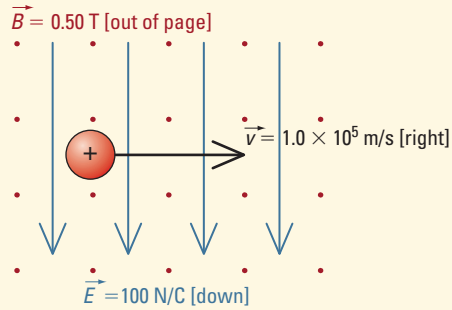
15.1 Check and Reflect

Knowledge

1. An electron moving at 5.0×10^5 m/s enters a magnetic field of magnitude 100 mT.
   (a) What is the maximum force that the magnetic field can exert on the electron? When will the magnetic field exert this maximum force?
   (b) What is the minimum force that the magnetic field can exert on the electron? When will the magnetic field exert this minimum force?
2. Explain why improved vacuum pumps were a key to the success of Thomson's experiments.
3. What experimental results led Thomson to conclude that all cathode rays consist of identical particles?

Applications

4. A beam of electrons enters a vacuum chamber that has a 100-kN/C electric field and a 0.250-T magnetic field.
   (a) Sketch an orientation of electric and magnetic fields that will let the electrons pass undeflected through the chamber.
   (b) At what speed would electrons pass undeflected through the fields in part (a)?
5. Electrons are observed to travel in a circular path of radius 0.040 m when placed in a magnetic field of strength 0.0025 T. How fast are the electrons moving?
6. How large a magnetic field is needed to deflect a beam of protons moving at 1.50×10^5 m/s in a path of radius 1.00 m?
7. Use the appropriate hand rule to determine the direction of the magnetic field in question 6 if the protons rotate counterclockwise in the same plane as this page.
8. A mass spectrometer has the electric field in its velocity selector set to 8.00×10^2 N/C and the magnetic field set to 10.0 mT. Find the speed of the ions that travel straight through these fields.
9. This diagram shows a proton moving at 1.0×10^5 m/s through perpendicular electric and magnetic fields.



   (a) Calculate the net force acting on the particle.
   (b) Will the net force change over time? Explain your reasoning.

Extensions

10. Suppose that a passenger accumulates 5 μC of negative charge while walking from left to right across the carpeted floor to the security gate at an airport.
    (a) If the metal detector at the security gate exerts an upward force on this charge, what is the direction of the magnetic field within the detector?
    (b) If the metal detector uses a 0.05-T magnetic field, roughly how fast would the passenger have to run through the detector in order to feel weightless?
    (c) Explain whether it would be practical to use an airport metal detector as a levitation machine.

e TEST

To check your understanding of Thomson's experiments, follow the eTEST links at www.pearsoned.ca/school/physicssource.

15.2 Quantization of Charge

As you learned in Chapter 14, Planck and Einstein introduced the concept of quantization in physics in the early 20th century. The discovery of the electron raised the intriguing idea that electric charge might also be quantized.

15-2 QuickLab

Determining a Fundamental Quantity

Problem

Does a set of containers contain only identical items?

Materials

5 sealed containers
laboratory balance

Procedure

1. Measure the mass of each container to a precision of 0.1 g.
2. Discuss with your partner how to record and present your data.
3. Pool your data with the rest of the class.

Questions

1. Look for patterns in the pooled data. Discuss as a class the best way to tabulate and graph all of the data. How can you arrange the data to make it easier to analyze?
2. Consider the differences in mass between pairs of containers. Explain whether these differences indicate that the masses vary only by multiples of a basic unit. If so, calculate a value for this basic unit.
3. Can you be sure that the basic unit is not smaller than the value you calculated? Explain why or why not.
4. What further information do you need in order to calculate the number of items in each container?

The American physicist Robert Andrews Millikan (1868–1953) and his graduate student Harvey Fletcher made the next breakthrough in the study of the properties of the electron. In 1909, Millikan reported the results from a beautiful experiment that determined the charge on the electron and showed that it was a fundamental unit of electrical charge.

Millikan's Oil-drop Experiment

Millikan and Fletcher used an atomizer to spray tiny drops of oil into the top of a closed vessel containing two parallel metal plates (Figure 15.9). Some of the oil drops fell into the lower part of the vessel through a small hole in the upper plate. Friction during the spraying process gave some of the oil drops a small electric charge. Millikan also used X rays to change the charge on the oil drops.

Since these oil drops were usually spherical, Millikan could calculate the mass of each drop from its diameter and the density of the oil. He connected a high-voltage battery to the plates, then observed the motion of the oil drops in the uniform electric field between the plates. By analyzing this motion and allowing for air resistance, Millikan calculated the electric force acting on each drop, and thus determined the charge on the drop.

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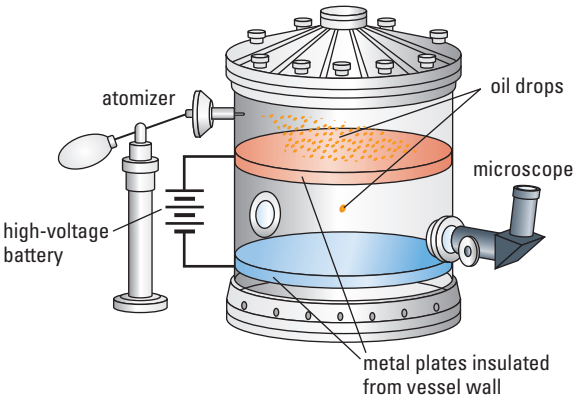
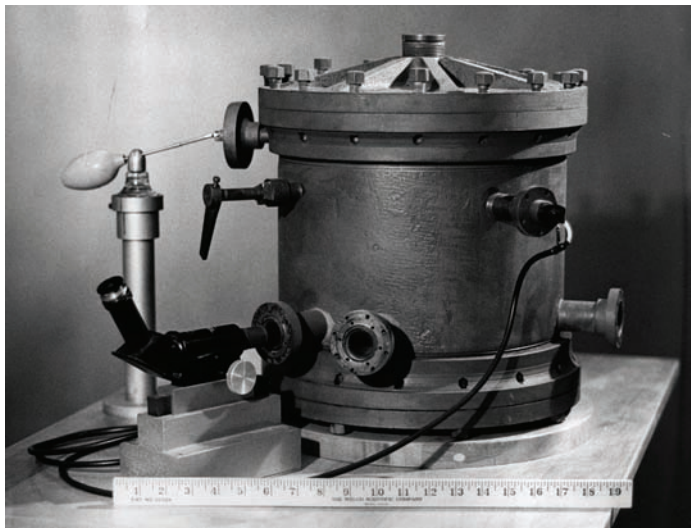
Millikan won the Nobel Prize in physics in 1923. In his Nobel lecture on the oil-drop experiment, he did not mention Fletcher's work at all.





▲ **Figure 15.9** Millikan's oil-drop apparatus

eSIM

To see a simulation of Millikan's oil-drop experiment, follow the links at www.pearsoned.ca/school/physicssource.

Millikan made numerous measurements with oil drops of various sizes. He found that the charged oil drops had either 1.6×10^{-19} C of charge or an integer multiple of this value. Millikan reasoned that the smallest possible charge that a drop can have is the charge acquired either by gaining or losing an electron. Hence, the charge on the electron must be -1.6×10^{-19} C.

Millikan showed that charge is not a continuous quantity; it exists only in discrete amounts. This finding parallels Planck's discovery in 1900 that energy is quantized (see section 14.1).

elementary unit of charge, e: the charge on a proton

Recent measurements have determined that the **elementary unit of charge, e**, has a value of $1.602\ 176\ 462 \pm 0.000\ 000\ 063 \times 10^{-19}$ C. A value of 1.60×10^{-19} C is accurate enough for the calculations in this textbook. Note that a proton has a charge of +1e and an electron has a charge of −1e.

info BIT

Thomson and others had tried to measure the charge on an electron using tiny droplets of water. However, evaporation and other problems made these measurements inaccurate.

Since Thomson and others had already determined the charge-to-mass ratio for electrons, Millikan could now calculate a reasonably accurate value for the mass of the electron. This calculation showed that the mass of the electron is roughly 1800 times less than the mass of the lightest atom, hydrogen.

Millikan and Controversy

In the mid 1970s, historians of science made a disturbing discovery: Millikan had, on several occasions, stated that he used all of his data in coming to the conclusion that the charge of the electron was quantized. In fact, his notebooks contain 175 measurements of which he only reported 58! When all of Millikan's data are used, his evidence for the quantization of charge is far less conclusive! Was Millikan guilty of scientific fraud, or was it a deeper, intuitive insight that led him to select only the data that clearly supported his claim that the charge on the electron is quantized? Historians will debate this question for many years to come, but we still acknowledge Millikan as the first person to measure the charge of the electron and to establish the quantization of charge.

eMATH

For a simple exercise to determine the fundamental unit of charge using a method similar to Millikan's, visit www.pearsoned.ca/school/physicssource.

Concept Check

Why is it almost impossible to determine the mass of an electron without determining its charge first?

Example 15.3

An oil drop of mass 8.2×10^{-15} kg is suspended in an electric field of 1.0×10^5 N/C [down]. How many electrons has the oil drop gained or lost?

Given

$\vec{E} = 1.0 \times 10^5$ N/C [down]

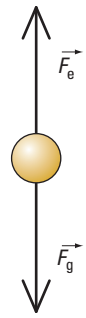
$m = 8.2 \times 10^{-15}$ kg

Required

number of electrons gained or lost (n)

Analysis and Solution

To balance the gravitational force, the electric force must be directed upward (Figure 15.10). Since the electric force is in the opposite direction to the electric field, $\vec{E}$, the charge on the oil drop must be negative. The oil drop must have gained electrons. For the electric and gravitational forces to balance, their magnitudes must be equal. Since $\vec{F}_{net} = \vec{F}_e + \vec{F}_g$ and $\vec{F}_{net} = 0$, then $0 = |\vec{F}_e| - |\vec{F}_g|$.



▲ **Figure 15.10**

$$|\vec{F}_e| = |\vec{F}_g|$$

$$q|\vec{E}| = mg$$

where q is the magnitude of the charge on the drop and g is the magnitude of the gravitational field.

Solving for q gives

$$q = \frac{mg}{|\vec{E}|}$$

$$= \frac{(8.2 \times 10^{-15}\text{ kg})(9.81\text{ m/s}^2)}{1.0 \times 10^5\text{ N/C}}$$

$$= 8.04 \times 10^{-19}\text{ C}$$

The net charge on the oil drop equals the number of electrons gained times the charge on each electron. Thus,

$$q = ne \text{ and } n = \frac{q}{e}$$

Note that n must be a whole number.

$$n = \frac{q}{e}$$

$$= \frac{8.04 \times 10^{-19}\text{ C}}{1.60 \times 10^{-19}\text{ C}}$$

$$= 5$$

Paraphrase

The oil drop has gained five electrons.

Practice Problems

1. How many electrons are gained or lost by a plastic sphere of mass 2.4×10^{-14} kg that is suspended by an electric field of 5.0×10^5 N/C [up]?
2. What electric field will suspend an oil drop with a mass of 3.2×10^{-14} kg and a charge of $+2e$?

Answers

1. The sphere has lost three electrons.
2. 9.8×10^5 N/C [up]

In Example 15.3, you studied what happens when the electric force on a charged droplet exactly balances the gravitational force on the same droplet. What happens if these two forces do not balance each other?

Example 15.4

A tiny plastic sphere of mass 8.2×10^{-15} kg is placed in an electric field of 1.0×10^5 N/C [down] within a vacuum chamber. The sphere has 10 excess electrons. Determine whether the sphere will accelerate, and if so, in which direction.

Given

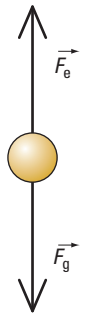
Choose up to be positive.

$\vec{E} = 1.0 \times 10^5$ N/C [down] $= -1.0 \times 10^5$ N/C

$m = 8.2 \times 10^{-15}$ kg

$q = -10e$

$\vec{g} = 9.81$ m/s^2 [down] $= -9.81$ m/s^2



▲ **Figure 15.11 (a)**

Required

acceleration of the plastic sphere ($\vec{a}$)

Analysis and Solution

Express the charge on the sphere in coulombs:

$$q = -10e = -10\,(1.60 \times 10^{-19}\text{ C}) = -1.60 \times 10^{-18}\text{ C}$$

Draw a free-body diagram of the forces acting on the sphere (Figure 15.11 (a)). Because the charge is negative, the electric force is in the opposite direction to the electric field, $\vec{E}$.

Calculate the magnitude of both the electric and gravitational forces acting on the sphere.

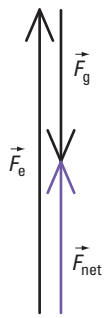
$$\begin{aligned}\vec{F}_g &= m\vec{g}\\ &= (8.2 \times 10^{-15}\text{ kg})(-9.81\text{ m/s}^2)\\ &= -8.04 \times 10^{-14}\text{ N}\\ \vec{F}_e &= q\vec{E}\\ &= (-1.60 \times 10^{-18}\text{ C})(-1.0 \times 10^5\text{ N/C})\\ &= +1.60 \times 10^{-13}\text{ N}\end{aligned}$$

Determine the net force on the sphere. From Figure 15.11 (b),

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_g + \vec{F}_e\\ &= -8.04 \times 10^{-14}\text{ N} + (+1.60 \times 10^{-13}\text{ N})\\ &= +7.96 \times 10^{-14}\text{ N}\end{aligned}$$

Now find the acceleration of the sphere:

$$\begin{aligned}\vec{F}_{net} &= m\vec{a}\\ \vec{a} &= \frac{\vec{F}_{net}}{m}\\ &= \frac{+7.96 \times 10^{-14}\text{ N}}{8.2 \times 10^{-15}\text{ kg}}\\ &= +9.7\text{ m/s}^2\end{aligned}$$



▲ **Figure 15.11 (b)**

Paraphrase

The sphere will accelerate upward at a rate of 9.7 m/s^2.

Practice Problems

1. Calculate the net force on a sphere of charge $-5e$ and mass 2.0×10^{-14} kg when placed in an electric field of strength 1.0×10^5 N/C [up] in a vacuum chamber.
2. Calculate the acceleration of the sphere if the direction of the electric field in question 1 is reversed.

Answers

1. 1.2×10^{-13} N [down]
2. 14 m/s^2 [down]

15.2 Check and Reflect

Knowledge

1. Explain the term *quantization of charge.*
2. Which two properties of the electron was Millikan able to determine with the results from his oil-drop experiment?
3. Calculate the charge, in coulombs, on an oil drop that has gained four electrons.
4. Determine the electric force acting on an oil drop with a charge of $-5e$ in a uniform electric field of 100 N/C [down].

Applications

5. (a) What is the net force acting on a charged oil drop falling at a constant velocity in the absence of an electric field? Explain your reasoning, using a free-body diagram to show the forces acting on the drop.
   (b) Describe the motion of the oil drop in an electric field that exerts an upward force greater than the gravitational force on the drop. Draw a diagram to show the forces acting on the drop.
6. An oil droplet with a mass of 6.9×10^{-15} kg is suspended motionless in a uniform electric field of 4.23×10^{4} N/C [down].
   (a) Find the charge on this droplet.
   (b) How many electrons has the droplet either gained or lost?
   (c) In what direction will the droplet move if the direction of the electric field is suddenly reversed?

Extensions

7. A student attempting to duplicate Millikan's experiment obtained these results. Explain why you might suspect that there was a systematic error in the student's measurements.

Droplet #	Charge ($\times 10^{-19}$ C)
1	19.0
2	17.2
3	10.0
4	20.8
5	26.2
6	24.4
7	20.8
8	22.6
9	15.4
10	24.4

8. Some critics of Millikan have noted that he used only a third of his measurements when reporting the results of his oil-drop experiment. Use a library or the Internet to learn more about this issue. Explain whether you feel that Millikan was justified in presenting only his "best" data.

eTEST

To check your understanding of charge quantization, follow the eTEST links at www.pearsoned.ca/school/physicssource.

15.3 The Discovery of the Nucleus

By the beginning of the 20th century, the work of Thomson, Perrin, and others had provided strong evidence that atoms were not the smallest form of matter. Physicists then started developing models to describe the structure of atoms and devising experiments to test these models.

15-3 QuickLab

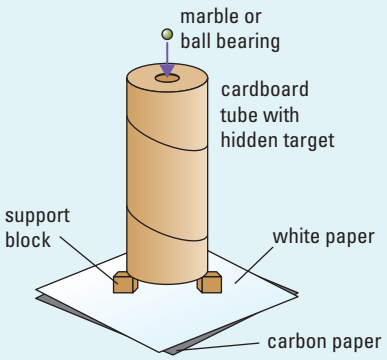
Using Scattering to Measure a Hidden Shape

Problem

What can scattering reveal about the shape of an unseen target?

Materials

cardboard tube 10–15 cm in diameter
small marbles or ball bearings
carbon paper
white letter-size paper



▲ **Figure 15.12**

Procedure

1. Your teacher will prepare several "beam tubes" containing a hidden target. A cover blocks the top of each tube, except for a small opening (Figure 15.12). No peeking!
2. Work with a small group. Place a sheet of carbon paper, carbon side up, on the desk, and put a piece of white paper on top of the carbon paper. Then carefully set the cardboard tube on blocks on top of the paper.
3. Drop a marble or ball bearing through the opening at the top of the cardboard tube. Retrieve the marble or bearing, then drop it through the tube again, for a total of 50 times.
4. Remove the piece of paper, and look for a pattern in the marks left on it.

Questions

1. Discuss with your group what the scattering pattern reveals about the shape and size of the target hidden in the cardboard tube. Sketch the likely shape of the target.
2. Explain how you can use your scattering results to estimate the dimensions of the target.
3. Compare your predictions to those made by groups using the other prepared tubes.
4. What is the smallest dimension that you could measure with the equipment in this experiment?

Rutherford's Scattering Experiment

Ernest Rutherford (1871–1937), a brilliant experimenter from New Zealand, was fascinated by radioactivity. By 1909, he had shown that some radioactive elements, such as radium and thorium, emitted positively charged helium ions, which are often called alpha (α) particles. Rutherford had also observed that a beam of these particles spread out somewhat when passing through a thin sheet of mica. So, he had two assistants, Hans Geiger and Ernest Marsden, measure the proportion of alpha particles scattered at different angles from various materials.

Figure 15.13 shows the technique Rutherford devised for these measurements. A few milligrams of radium in a lead block with a small opening produced a pencil-shaped beam of alpha particles. The experimenters positioned a sheet of thin gold foil at right angles to the beam of alpha particles and used a screen coated with zinc sulfide to detect particles scattered by the foil. When an alpha particle struck the screen, the zinc sulfide gave off a faint flash of light, just enough to be visible through the microscope. By rotating the screen and microscope around the gold film, Geiger and Marsden measured the rates at which alpha particles appeared at various angles.

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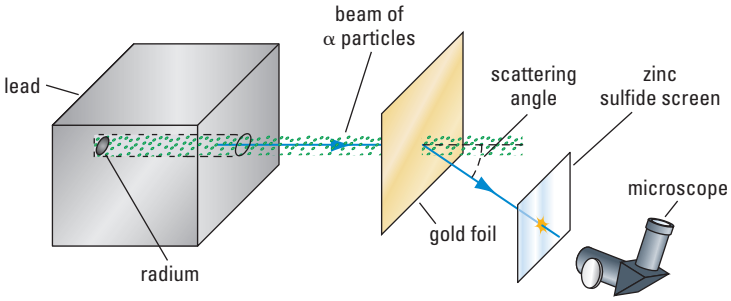
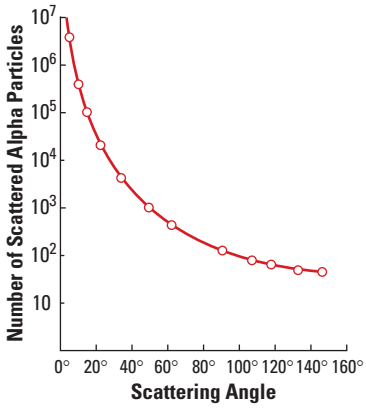
Rutherford was a professor at McGill University in Montreal from 1898 to 1907. During that time, he discovered an isotope of radon, showed that radioactive elements can decay into lighter elements, and developed a method for dating minerals and estimating the age of Earth. Although he considered himself a physicist, he cheerfully accepted the Nobel Prize for chemistry in 1908.



◀ **Figure 15.13** Rutherford's scattering experiment

Most of the alpha particles travelled through the foil with a deflection of a degree or less. The number of alpha particles detected dropped off drastically as the scattering angle increased. However, a few alpha particles were scattered at angles greater than 140°, and once in a while an alpha particle would bounce almost straight back. Figure 15.14 shows the relationship between the number of scattered alpha particles and the angle at which they scattered.



▲ **Figure 15.14** The scattering pattern observed by Geiger and Marsden

Rutherford was startled by these results. He knew that the deflections caused by attraction to the electrons in the gold atoms would be tiny because the alpha particles were fast-moving and roughly 8000 times heavier than electrons. He also calculated that deflection caused by repulsion of the alpha particles by the positive charge in the gold atoms would be no more than a degree if this charge were distributed evenly throughout each atom in accordance with Thomson's model. So, Rutherford did not expect *any* alpha particles to be scattered at large angles. In a lecture describing the experiment, he said that seeing this scattering "was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you!"

Rutherford spent several weeks analyzing the scattering data. He concluded that the positive charge in a gold atom must be concentrated in an incredibly tiny volume, so most of gold foil was actually empty space!

Rutherford's team then tried scattering alpha particles from other metals. Using data from scattering experiments with aluminium foil, Rutherford showed that the positive charge and most of the mass of an atom are contained in a radius of less than 10^{-14} m. Rutherford had discovered the nucleus—and disproved the raisin-bun model.

Concept Check

On average, atoms have a radius of roughly 10^{-10} m. Use Rutherford's estimate for the size of the nucleus to calculate the proportion of the human body that is just empty space.

The Planetary Model

planetary model: atomic model that has electrons orbiting a nucleus

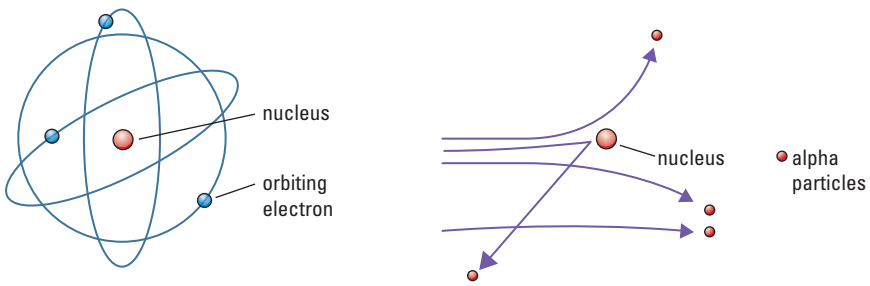
Rutherford's discovery of the nucleus quickly led to the **planetary model** of the atom (Figure 15.15). In this model, the electrons orbit the nucleus much like planets orbiting the Sun. The electrostatic attraction between the positive nucleus and the negative electrons provides the centripetal force that keeps the electrons in their orbits. This model is also known as the solar-system, nuclear, or Rutherford model.

To calculate the size of the nucleus, Rutherford applied the law of conservation of energy and an equation for electric potential energy. He knew that the electric potential energy that a charge q_1 gains from the field around charge q_2 is

$$E_p = \frac{kq_1q_2}{d}$$

where k is Coulomb's constant and d is the distance between the charges.

This equation can be derived from Coulomb's law using basic calculus.



▲ **Figure 15.15** The planetary model of the atom can explain the results of Rutherford's scattering experiments. The extreme scattering of some alpha particles could only be explained by having most of the mass and positive charge of the atom concentrated in a very small nucleus.

Example 15.5

In a scattering experiment, some alpha particles are scattered almost straight back from a sheet of gold foil. Each of these particles had an initial kinetic energy of 1.6×10^{-12} J. The charge on an alpha particle is +2e, and the charge on a gold nucleus is +79e. Estimate the maximum possible size of a gold nucleus, given that the alpha particles do not hit the nucleus.

Given

$E_k = 1.6 \times 10^{-12}$ J $\quad q_1 = q_\alpha = +2e \quad q_2 = q_{gold} = +79e$

Required

radius of the nucleus of a gold atom (r)

Analysis and Solution

Since energy is conserved, the total kinetic and potential energy of an alpha particle does not change during scattering.

As an alpha particle approaches a nucleus, the force of repulsion between the positive charges causes the alpha particle to slow down. This process converts the particle's kinetic energy to electric potential energy until the kinetic energy is zero. Then the particle starts moving away from the nucleus and regains its kinetic energy. At the point where the alpha particle is closest to the nucleus, all of the particle's kinetic energy has been converted to electric potential energy.

The electric potential energy of the alpha particle is $E_p = \frac{kq_1q_2}{d}$, where k is Coulomb's constant and d is the distance between the alpha particle and the nucleus.

The initial distance between the alpha particle and the nucleus is vastly larger than the distance between them when they are closest together. Therefore, the initial electric potential energy of the alpha particle is negligible for this calculation.

Initially, kinetic energy + electric potential energy $= 1.6 \times 10^{-12}$ J + 0

When the alpha particle is closest to the nucleus,

kinetic energy + electric potential energy $= 0 + \frac{kq_1q_2}{d}$

Since the total energy is conserved, $\frac{kq_1q_2}{d} = 1.6 \times 10^{-12}$ J

Solving for d gives

$$d = \frac{kq_1q_2}{1.6 \times 10^{-12}\ \text{J}}$$

$$= \frac{\left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\cancel{\text{C}^2}}\right)(2 \times 1.60 \times 10^{-19}\ \cancel{\text{C}})(79 \times 1.60 \times 10^{-19}\ \cancel{\text{C}})}{1.6 \times 10^{-12}\ \text{J}}$$

$$= 2.3 \times 10^{-14}\ \text{m}$$

At its closest approach, the alpha particle is about 2.3×10^{-14} m from the centre of the nucleus.

Paraphrase

The radius of a gold nucleus cannot be larger than 2.3×10^{-14} m.

Practice Problems

1. The charge on a tin nucleus is +50e. How close can an alpha particle with an initial kinetic energy of 1.6×10^{-12} J approach the nucleus of a tin atom?
2. An iron nucleus has 56 protons. What is the electric potential energy of a proton located 5.6×10^{-13} m from the centre of an iron nucleus?

Answers

1. 1.4×10^{-14} m
2. 2.3×10^{-14} J

15.3 Check and Reflect

Knowledge

1. Explain how the results from Rutherford's gold-foil experiment disproved Thomson's model of the atom.
2. Briefly describe the planetary model of the atom.
3. Find the potential energy of an alpha particle that is
   (a) 1.0×10^{-10} m from the centre of a gold nucleus
   (b) 1.0×10^{-14} m from the centre of a gold nucleus
4. Why does the scattering angle increase as alpha particles pass closer to the nucleus?

Applications

5. Why did Rutherford conclude that it was just the nucleus that must be extremely tiny in an atom and not the entire atom?
6. (a) By 1900, physicists knew that 1 m^3 of gold contains approximately 6×10^{28} atoms. Use this information to estimate the radius of a gold atom. List any assumptions you make.
   (b) Compare this estimate to the estimate of the size of a gold nucleus in Example 15.5.
7. In scattering experiments with aluminium foil, Rutherford found that the alpha particles observed at angles close to 180° did not behave like they had been scattered only by electrostatic repulsion. Rutherford thought these alpha particles might have actually hit an aluminium nucleus.
   (a) Why did Rutherford see only electrostatic scattering when he used the same source of alpha particles with gold foil?
   (b) Rutherford used alpha particles with an energy of about 1.2×10^{-12} J. Estimate the radius of an aluminium nucleus.

Extensions

8. According to Rutherford's calculations, the positive charges in an atom are packed tightly together in the nucleus. Why would physicists in 1900 expect such an arrangement to be highly unstable? What did Rutherford's results suggest about forces in the nucleus?

eTEST

To check your understanding of atomic models, follow the eTEST links at www.pearsoned.ca/school/physicssource.

15.4 The Bohr Model of the Atom

In 1912, Niels Bohr (1885–1962), a Danish physicist, studied for a few months at Rutherford's laboratory. Both Bohr and Rutherford recognized a critical flaw in the planetary model of the atom. Experiments had shown that an accelerating charge emits electromagnetic waves, as predicted by the mathematical model for electromagnetism developed by the Scottish physicist James Clerk Maxwell (1831–1879). Electrons orbiting a nucleus are constantly accelerating, so they should emit electromagnetic waves. These waves would take energy from the orbiting electrons. As a result, the electrons in an atom should spiral into the nucleus in a few microseconds (Figure 15.16). But empirical evidence indicates that electrons do not spiral into their atomic nuclei. If they did, stable matter would not exist.

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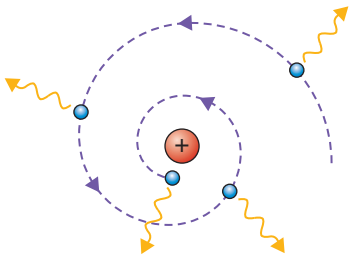
Rutherford and Bohr shared an interest in soccer: Rutherford was a fan, and Bohr was an excellent player.



◀ **Figure 15.16** According to Maxwell's laws of electromagnetism, the orbiting electron should continuously radiate energy and spiral into the nucleus, which it does not do.

Bohr thought Planck's concept of quantized energy might provide a solution, and puzzled for months over how to fit this concept into a model of the atom. Then, a casual conversation with a colleague, his former classmate Hans Marius Hansen, gave Bohr the key to the answer. Hansen had recently returned from studying in Germany with an expert in **spectroscopy**. Hansen told Bohr that the wavelengths of the light in the spectrum of hydrogen have a mathematical pattern. No one had yet explained why this pattern occurs. Bohr found the explanation, and provided the first theoretical basis for spectroscopy.

spectroscopy: the study of the light emitted and absorbed by different materials

Spectroscopy

A prism or diffraction grating can spread light out into a spectrum with colours distributed according to their wavelengths. In 1814, Josef von Fraunhofer (1787–1826) noticed a number of gaps or dark lines in the spectrum of the Sun. By 1859, another German physicist, Gustav Kirchhoff (1824–1887), had established that gases of elements or compounds under low pressure each have a unique spectrum. Kirchhoff and others used spectra to identify a number of previously unknown elements. Kirchhoff's laws for spectra explain how temperature and pressure affect the light produced or absorbed by a material:

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The element names *cesium* and *rubidium* come from the Latin words for the colours of the spectral lines used to discover these elements. Cesium comes from *cesius*, which is Latin for sky blue. Rubidium comes from *rubidus* — Latin for dark red.

emission line spectrum: a pattern of bright lines produced by a hot gas at low pressure

absorption line spectrum: a pattern of dark lines produced when light passes through a gas at low pressure

- A hot, dense material emits a continuous spectrum, without any dark or bright lines.
- A hot gas at low pressure has an **emission line spectrum** with bright lines at distinct characteristic wavelengths.
- A gas at low pressure absorbs light at the same wavelengths as the light it emits when heated. Shining white light through the gas produces an **absorption line spectrum** with dark lines that match the bright lines in the emission spectrum for the gas.

Figure 15.17 illustrates these three types of spectra.

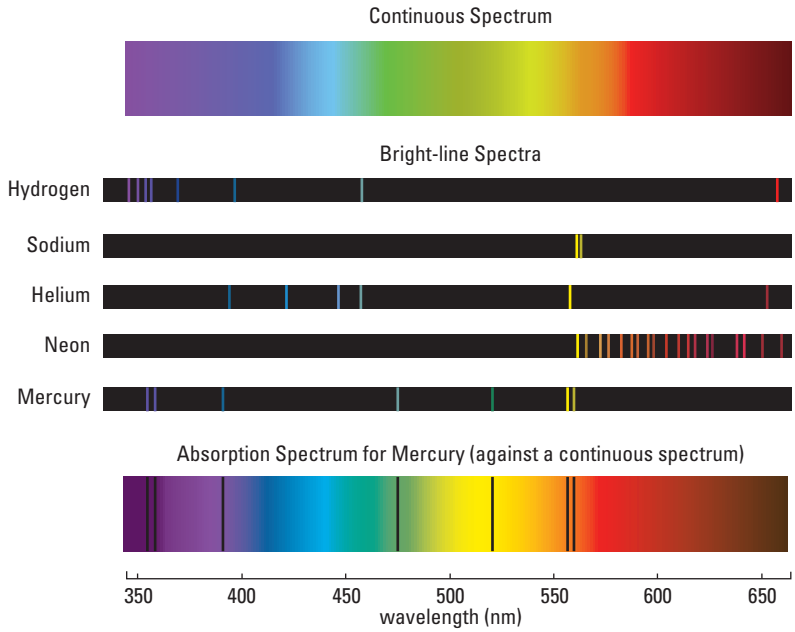


Figure 15.17
Continuous, emission line, and absorption line spectra

15-4 Design a Lab

Emission Spectra of Elements

The Question

In what ways are emission line spectra characteristic of the elements that produce them?

Design and Conduct Your Investigation

Investigate the emission spectra produced by various elements. Here are some ways you can heat different elements enough to produce visible light:

- Use a Bunsen burner to vaporize a small amount of an element.
- Use commercially available discharge tubes containing gaseous elements.
- Observe forms of lighting that use a vaporized element, such as sodium or mercury arc lamps.

Often, a diffraction grating is the simplest method for observing a spectrum. Check with your teacher if you need directions for using a diffraction grating. Note the overall colour of the light from each element, and sketch or photograph its emission spectrum.

Extending

Investigate absorption lines. You could try comparing the spectra of direct sunlight and sunlight that passes through clouds. Alternatively, you could shine white light at coloured solutions and diffract the light that passes through. Why are distinct absorption lines generally more difficult to produce than emission lines?

Improvements in the resolution of **spectrometers** made them a powerful analytic tool. For example, scientists have painstakingly matched the dark **Fraunhofer lines** in the solar spectrum (Figure 15.18) to the spectral patterns of dozens of elements, thus proving that these elements are present in the Sun's atmosphere. However, at the start of the 20th century, the reasons why elements produce spectral lines were still a mystery.

spectrometer: a device for measuring the wavelengths of light in a spectrum

Fraunhofer line: a dark line in the spectrum of the Sun

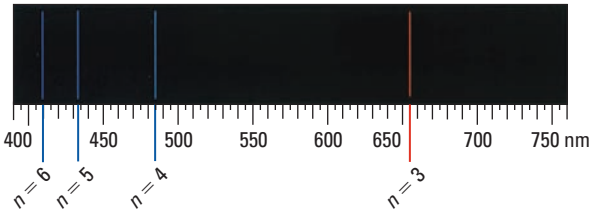
▲ **Figure 15.18** Fraunhofer lines are the dark lines in the visible part of the solar spectrum.

The first hint that spectral lines were not just randomly spaced came in 1885. Johann Jacob Balmer, a Swiss mathematics teacher with an interest in numerology, found a formula for the wavelengths of the lines in the hydrogen spectrum. In 1890, Johannes Robert Rydberg generalized Balmer's formula and applied it with varied success to other elements. Here is the formula for hydrogen:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where R_H is Rydberg's constant for hydrogen, $1.097 \times 10^7\ m^{-1}$, and n has the whole number values 3, 4, 5, ...

The emission line spectrum of hydrogen is given in Figure 15.19.



▲ **Figure 15.19** Some of the bright lines in the spectrum of hydrogen

Hansen told Bohr about this formula. Bohr later remarked, "As soon as I saw Balmer's formula, the whole thing was immediately clear to me." Bohr had realized that the spectral lines corresponded to *differences* between quantized energy levels in the hydrogen atom. This concept was the foundation of a powerful new model of the atom.

eWEB

To learn more about spectra, follow the links at www.pearsoned.ca/school/physicssource.

The Bohr Model of the Atom

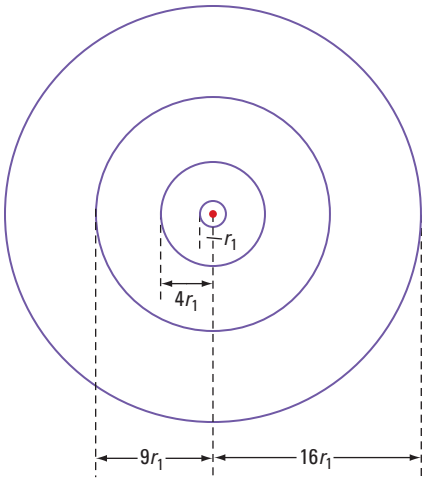
In 1913, Bohr published a paper suggesting a radical change to the planetary model of the atom. Here are the basic principles of Bohr's model:

- Electrons can orbit the nucleus only at certain specific distances from the nucleus. These distances are particular multiples of the radius of the smallest permitted orbit (Figure 15.20). Thus, the orbits in an atom are quantized.
- Both the kinetic energy and the electric potential energy of an electron in orbit around a nucleus depend on the electron's distance from the nucleus. So, the energy of an electron in an atom is also quantized. Each orbit corresponds to a particular **energy level** for the electron.
- An electron can move from one energy level to another only by either emitting or absorbing energy equal to the difference between the two energy levels. An electron that stays in a particular orbit does not radiate any energy. Since the size and shape of the orbit remain constant along with the energy of the electron, the orbits are often called **stationary states**.

energy level: a discrete and quantized amount of energy

stationary state: a stable state with a fixed energy level

principal quantum number: the quantum number that determines the size and energy of an orbit



▶ **Figure 15.20** In Bohr's model of the atom, electrons can orbit only at specific distances from the nucleus.

Bohr reasoned that Balmer's formula shows that the energy of an orbiting electron depends inversely on the square of a quantum number n, now known as the **principal quantum number**. Bohr then used the equations for uniform circular motion, Coulomb's law, and electric potential energy to derive expressions for the size of the hydrogen atom and the energy of the electron in the atom.

Orbit Sizes

Bohr's model of the hydrogen atom states that electrons can orbit the nucleus only at specific locations given by the expression:

$$r_n = \frac{h^2}{4\pi^2 mke^2} n^2$$

where r_n is the radius of the nth possible orbit for an electron and n is the principal quantum number, which can have the values 1, 2, 3, ... The other symbols in the equation all represent constants: k is Coulomb's constant, h is Planck's constant, e is the elementary charge, and m is the mass of the electron.

By combining all the constants, this equation can be simplified to

$$r_n = r_1 n^2, \quad \text{where } r_1 = \frac{h^2}{4\pi^2 mke^2} = 5.29 \times 10^{-11}\ \text{m}$$

Bohr radius: radius of the smallest orbit in a hydrogen atom

ground state: the lowest possible energy level

The quantity r_1 is known as the **Bohr radius**. It is the radius of the lowest possible energy level or **ground state** of the hydrogen atom.

Concept Check

Sketch the first three orbits for a hydrogen atom to scale. Describe how the size of this atom changes as n increases.

Energy Levels

An orbiting electron has both kinetic energy and electric potential energy. By combining equations for kinetic and electric potential energies, Bohr derived this expression for E_n, the total energy of the electron in an energy level:

$$E_n = -\frac{1}{n^2}\left(\frac{2\pi^2 mk^2e^4}{h^2}\right)$$

As with the expression for the orbit radii, the constants can be combined to simplify the equation:

$$E_n = -\frac{E_1}{n^2}, \text{ where } E_1 = \frac{2\pi^2 mk^2e^4}{h^2} = 2.18 \times 10^{-18}\ \text{J or } 13.6\ \text{eV}$$

E_1 is the ground-state energy of the hydrogen atom. When $n > 1$, the electron has greater energy and the atom is in an **excited state.** When $n = \infty$, the electron is no longer bound to the nucleus because $E_\infty = 0$. Thus, Bohr's model predicts that the **ionization energy** for hydrogen is 2.18×10^{-18} J or 13.6 eV, corresponding to E_1.

excited state: any energy level higher than the ground state

ionization energy: energy required to remove an electron from an atom

Concept Check

Does it make sense that the energy in the equation $E_n = -\frac{1}{n^2}\left(\frac{2\pi^2 mk^2e^4}{h^2}\right)$ is negative? Consider what you have to do to remove an electron from an atom.

Example 15.6

How much energy must a hydrogen atom absorb in order for its electron to move from the ground state to the $n = 3$ energy level?

Given

$n_{\text{initial}} = 1$ $\qquad$ $n_{\text{final}} = 3$

Required

energy absorbed by atom (ΔE)

Analysis and Solution

The energy the atom must absorb is equal to the difference between the two energy levels.
The energy for each energy level is

$$E_n = -\frac{E_1}{n^2}$$

$$= -\frac{2.18 \times 10^{-18}\ \text{J}}{n^2}$$

$$\Delta E = E_3 - E_1$$

$$= -\frac{2.18 \times 10^{-18}\ \text{J}}{3^2} - \left(-\frac{2.18 \times 10^{-18}\ \text{J}}{1^2}\right)$$

$$= (2.18 \times 10^{-18}\ \text{J})\left(-\frac{1}{9} + 1\right)$$

$$= 1.94 \times 10^{-18}\ \text{J}$$

Paraphrase

The electron in a hydrogen atom requires 1.94×10^{-18} J of energy to move from the ground state to the $n = 3$ level.

Practice Problems

1. How much energy does it take to move the electron in a hydrogen atom from the ground state to the $n = 4$ energy level?
2. How much energy does the electron in a hydrogen atom lose when dropping from the $n = 5$ energy level to the $n = 2$ energy level?

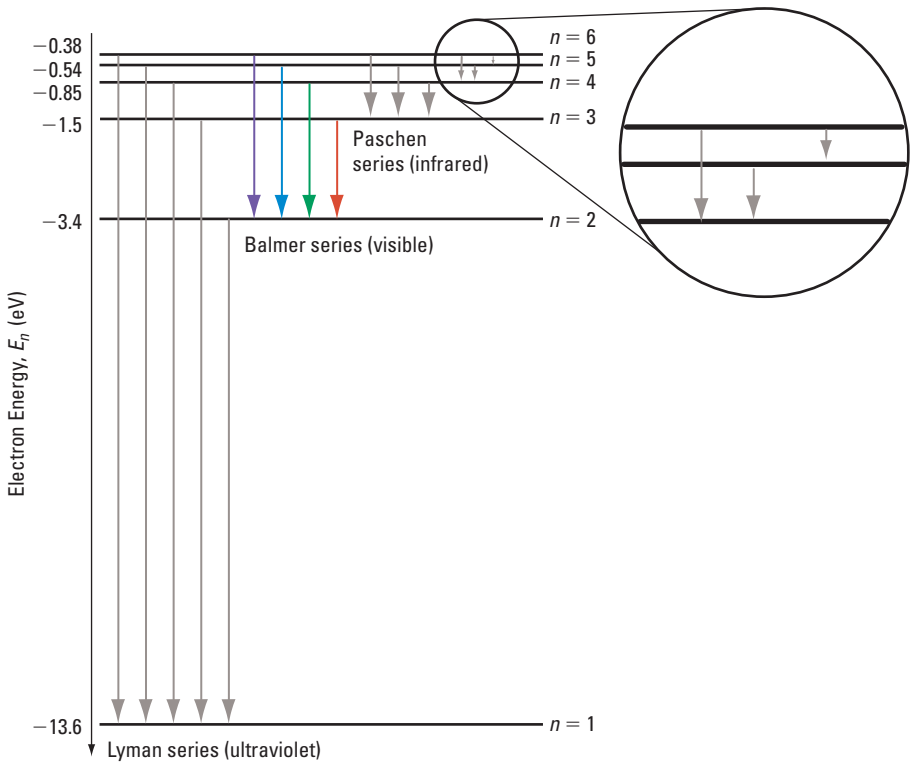
Answers

1. 2.04×10^{-18} J
2. 4.58×10^{-19} J

Energy Level Transitions and Line Spectra

The Bohr model explains why absorption and emission line spectra occur for hydrogen and other elements. To jump to a higher energy level, an electron in an atom must gain energy. The atom can gain this energy by absorbing a photon. For energy to be conserved, the photon's energy must match the difference between the electron's initial energy level and the higher one. Recall from section 14.1 that the energy and frequency of a photon are related by the equation $E = hf$. Thus, the atom can absorb only the frequencies that correspond to differences between the atom's energy levels. Absorption of light at these specific frequencies causes the discrete dark lines in absorption spectra.

Similarly, atoms can emit only photons that have energies corresponding to electron transitions from a higher energy level to a lower one. The arrows in Figure 15.21 illustrate all the possible downward transitions from the first six energy levels for hydrogen. Such energy level diagrams provide a way to predict the energies, and hence the wavelengths, of photons emitted by an atom.



▲ **Figure 15.21** The first six energy levels for hydrogen. Which arrow represents the transition that releases the most energy?

15-5 Inquiry Lab

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

An Emission Spectrum

Question

What is the energy difference between two energy states of atoms in a particular discharge tube?

Hypothesis

At a high temperature and low pressure, an atom emits light at the wavelengths predicted by the Bohr model.

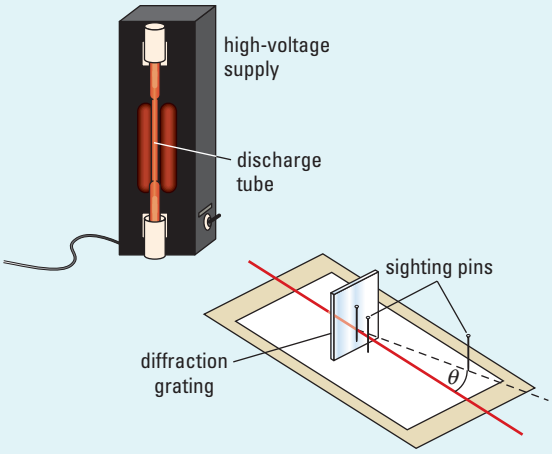
Variables

- diffraction angle
- wavelength

Materials and Equipment

a discharge tube
high-voltage power supply
diffraction grating
12 straight pins
sheet of paper
cardboard or foam-core sheet
protractor and straightedge
masking tape

CAUTION! Keep well clear of the power supply connections when the power is on.



▲ **Figure 15.22**

Procedure

1. Draw an *x*-axis and a *y*-axis that intersect in the middle of the sheet of paper. Then tape the paper to the cardboard or foam-core sheet (Figure 15.22).
2. Connect the high-voltage power supply to the discharge tube.
3. Switch on the power supply and look at the discharge tube through the diffraction grating. Orient the grating so that the spectral lines are vertical. You may want to darken the room to see the spectrum more clearly.
4. Align the bottom edge of the diffraction grating with the *x*-axis on the paper. Hold the grating vertical by pressing a straight pin into the *y*-axis on either side of the grating.
5. Sight along the *y*-axis, and turn the cardboard or foam-core base so that the *y*-axis points directly toward the discharge tube. Now tape the base to the tabletop.
6. For each spectral line, position a sighting pin so that it is on the line between the spectral line and the origin of your axes. Draw a line from the sighting pin to the origin.

Analysis

1. Decide how to record your data. You could use a table similar to the one below.

Colour of Line	Diffraction Angle, θ	Wavelength (nm)

2. Use the diffraction formula $n\lambda = d \sin \theta$ to calculate the wavelengths of the spectral lines (see section 13.5). Sometimes the spacing of the slits, *d*, is marked on the grating. If not, ask your teacher for this information.
3. Determine the energy difference between the two energy states.
4. Draw an energy level diagram showing the electron transition.

When an electron in an atom absorbs a photon, the final energy level of the atom's electron is greater than its initial energy level. From the law of conservation of energy, the energy of the photon can be expressed as:

$$E_{photon} = E_{final} - E_{initial}$$

Since the energy of an electron in a hydrogen atom at a given energy level, n, is given by the formula $E_n = -\frac{1}{n^2}\left(\frac{2\pi^2 mk^2e^4}{h^2}\right)$, it is possible (using algebra) to show that the expression $E_{photon} = E_{final} - E_{initial}$ becomes

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right)$$

This result is impressive! Bohr's model not only explains spectral lines, but leads to a generalized form of Balmer's formula, and predicts the value of Rydberg's constant for hydrogen.

Example 15.7

Find the wavelength of the light emitted by a hydrogen atom when its electron drops from the $n = 3$ to the $n = 2$ energy level.

Given

$n_{initial} = 3$ $\qquad$ $n_{final} = 2$

Required

wavelength (λ)

Analysis and Solution

Simply substitute the known values for n into the wavelength equation for an emitted photon:

$$\begin{aligned}\frac{1}{\lambda} &= R_H\left(\frac{1}{n_{final}^2} - \frac{1}{n_{initial}^2}\right)\\ &= R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\\ &= (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{1}{4} - \frac{1}{9}\right)\\ &= 1.524 \times 10^6\ \text{m}^{-1}\\ \lambda &= \frac{1}{1.524 \times 10^6\ \text{m}^{-1}}\\ &= 6.563 \times 10^{-7}\ \text{m}\end{aligned}$$

Paraphrase

The atom emits light with a wavelength of 656.3 nm.

Practice Problems

1. Find the wavelength of the light emitted by a hydrogen atom when its electron drops from the $n = 5$ to the $n = 2$ energy level.
2. Find the wavelength of light that a hydrogen atom will absorb when its electron moves from the $n = 3$ to the $n = 7$ energy level.

Answers

1. 434 nm
2. 1005 nm

Concept Check

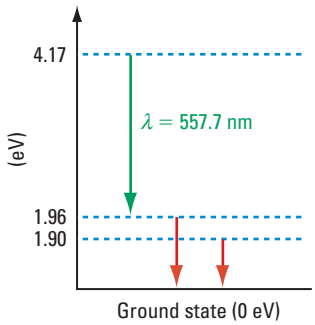
What features do the Bohr model of the atom and the planetary model have in common? In what critical ways do these two models differ?

The Northern Lights and the Emission Line Spectrum of Oxygen

Alberta skies often display one of nature's most beautiful phenomena — the aurora borealis, or northern lights, which you first studied in Chapter 12. At altitudes between 100 km and 400 km above the surface of Earth, high-energy electrons, trapped by Earth's magnetic field, interact with oxygen and nitrogen atoms. During these interactions, the electrons in these atoms are excited and move into higher energy levels. Eventually, the excited electrons return to their ground states. In doing so, they emit light that forms the characteristic colours of the aurora borealis. Figure 15.23 shows a display of the aurora borealis above northern Alberta. You can use the quantized energy levels to help explain the characteristic green colour of the aurora (as well as the subtler shades of red and blue).

(a)

(b)



◀ **Figure 15.23** The photo in (a) shows a bright, mostly green aurora. The green colour is due to an energy level transition in oxygen atoms in Earth's atmosphere. The diagram in (b) shows some of the energy levels in the oxygen atom, including the one that produces the green light that is mostly seen in the aurora. In this diagram, the ground state energy equals 0 eV.

From Figure 15.23(b), the green colour of the aurora occurs when an excited electron drops from an energy level $E_{\text{initial}} = 4.17$ eV to a lower energy level $E_{\text{final}} = 1.96$ eV. Using the equations $\Delta E = E_{\text{final}} - E_{\text{intial}}$ and $\Delta E = \dfrac{hc}{\lambda}$, you can show that this change in energy level produces the colour you see.

info BIT

The emission spectra of elements act like nature's fingerprints. By vaporizing a small sample and looking at the emission lines produced, you can determine the chemical composition of a substance. This technique, called *atomic emission spectroscopy*, is used routinely in forensics labs around the world.

Concept Check

Use the energy level diagram for oxygen, given in Figure 15.23(b), to verify that a transition from $E_{initial}$ = 4.17 eV to E_{final} = 1.96 eV will produce green light (λ = 558 nm). Two other possible transitions are also shown in the diagram. What colours would these transitions produce? Determine their wavelengths.

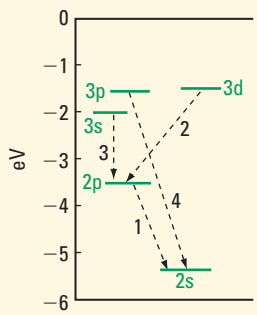
15.4 Check and Reflect

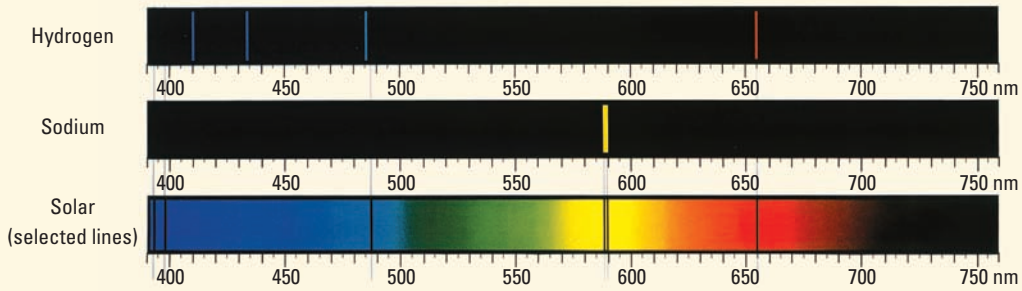
Knowledge

1. Explain what the phrase "quantized process" means.
2. Sketch the first five orbits in a hydrogen atom. Indicate on your sketch which transitions cause the blue, green, and red lines shown in Figure 15.21.
3. List three quantities predicted by Bohr's model of the atom.
4. Why do electrons in hydrogen atoms emit infrared light when they make transitions to the n = 3 energy level, and ultraviolet light when they make transitions to the n = 1 energy level?

Applications

5. The wavelengths of the first four visible lines in the hydrogen spectrum are 410, 434, 486, and 656 nm.
   (a) Show that Balmer's formula predicts these wavelengths.
   (b) Which of the wavelengths corresponds to a transition from the n = 4 energy level to the n = 2 energy level?
   (c) Use this wavelength to calculate the energy difference between the n = 4 and the n = 2 stationary states.
6. A helium-neon laser produces photons of wavelength 633 nm when an electron in a neon atom drops from an excited energy state to a lower state. What is the energy difference between these two states? Express your answer in electron volts.
7. The diagram shown below shows some of the energy levels for the lithium atom. The designations 2s, 2p, etc., are a common notation used in spectroscopy.
   (a) Without doing any calculations, sort the four transitions shown from shortest wavelength to longest wavelength. Explain your reasoning.
   (b) Estimate the energy of each of the four energy levels shown.
   (c) Calculate the wavelengths produced in these transitions. Indicate which ones are in the visible part of the spectrum, along with their colours.





8. What can you conclude about the composition of the Sun from the spectra given above? Explain your reasoning.

9. (a) Find the difference in energy between the $n = 2$ and $n = 3$ energy levels in hydrogen.

(b) Find the energy difference between the $n = 5$ and $n = 6$ energy levels in hydrogen.

(c) What happens to the energy difference between successive orbits as the distance from the nucleus increases?

10. The ionization energy for an atom is the energy required to remove an electron completely from an atom. Show why the ionization energy for hydrogen is equal to E_1. (Hint: Consider going from the ground state to an energy level with $n > 1000$.)

11. When an atom's energy levels are closely spaced, the atom "de-excites" by having one of its electrons drop through a series of energy levels. This process is called fluorescence and is often seen when a high-energy photon, such as an X ray or a UV photon, excites an atom, which then de-excites through a series of longer-wavelength emissions. A common example of fluorescence occurs in the colours produced when rocks and minerals containing mercury and many other elements are illuminated by UV photons. Below is the energy level diagram for some of the energy levels in mercury (Hg). The ground state has been assigned an energy of 0 eV.

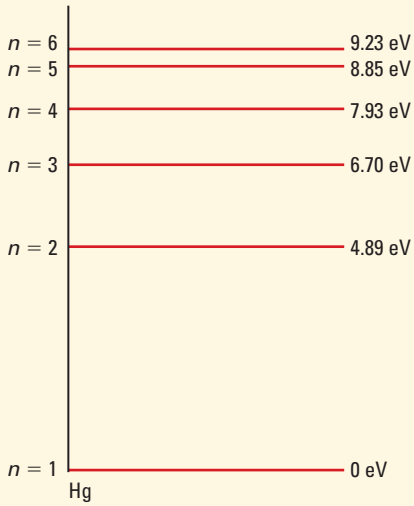
Determine:

(a) the wavelength of the photon needed to excite the mercury atom from its ground state to the $n = 5$ energy level

(b) the longest wavelength of photon that will be emitted as the mercury atom de-excites

(c) the number of possible downward transitions that can occur

(d) If an atom had 100 possible energy transitions that were very close together, what would the spectrum produced by this atom look like?



Extension

12. A laser produces light that is monochromatic, coherent, and collimated.

(a) Explain each of these three properties.

(b) Describe the spectrum of a laser.

eTEST

To check your understanding of Bohr's model of the atom, follow the eTEST links at www.pearsoned.ca/school/physicssource.

15.5 The Quantum Model of the Atom

PHYSICS INSIGHT

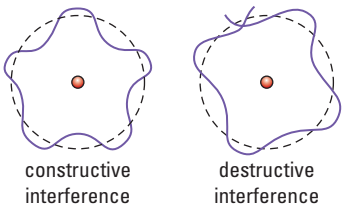
Recall from section 13.5 that for constructive interference to occur, the path difference between waves must be a whole number of wavelengths, or $n\lambda$.

Bohr's model explains the spectral lines of hydrogen and accurately predicts the size and ionization energy of the hydrogen atom. Despite these remarkable accomplishments, the theory has several serious failings:

- It does not really explain why energy is quantized, nor why orbiting electrons do not radiate electromagnetic energy.
- It is not accurate for atoms that have two or more electrons.
- It does not explain why a magnetic field splits the main spectral lines into multiple closely spaced lines. The Dutch physicist Pieter Zeeman discovered this effect in 1896. It is known as the Zeeman effect.

Physicists solved these problems within 15 years, but the solutions were even more radical than Bohr's theory!

The Wave Nature of Electrons



▲ **Figure 15.24** A standing wave is possible only if a whole number of electron wavelengths fit exactly along the circumference of an orbit.

In 1924, Louis de Broglie developed his theory that particles have wave properties. As described in section 14.4, diffraction experiments confirmed that electrons behave like waves that have the wavelength predicted by de Broglie. So, the principles of interference and standing waves apply for electrons orbiting a nucleus.

For most sizes of orbit, successive cycles of the electron wave will be out of phase, and destructive interference will reduce the amplitude of the wave (Figure 15.24). For constructive interference to occur, the circumference of the orbit must be equal to a whole number of wavelengths:

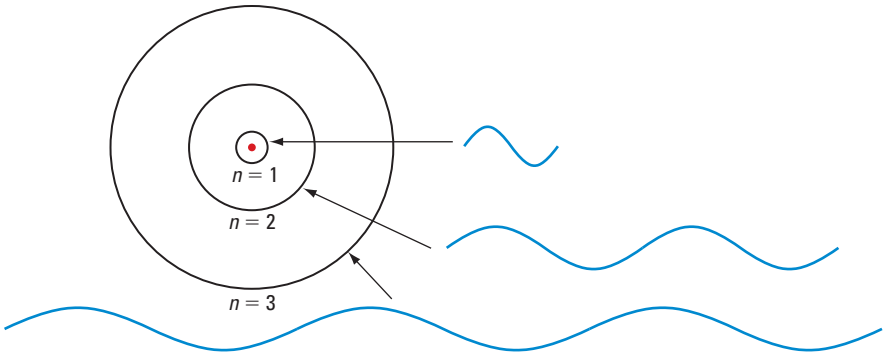
$$2\pi r_n = n\lambda$$

where n is a positive integer, λ is the electron wavelength, and r_n is the radius of the nth energy level.

By substituting de Broglie's definition for wavelength, $\lambda = \frac{h}{mv}$, into the above equation, the condition for constructive interference becomes

$$2\pi r_n = \frac{nh}{mv} \quad \text{or} \quad mvr_n = \frac{nh}{2\pi}$$

This condition is fundamentally the same one that Bohr found was necessary for the energy levels in his model of the atom. Thus, the wave nature of matter provides a natural explanation for quantized energy levels. Figure 15.25 shows the standing waves corresponding to the first three energy levels in an atom. Note that the de Broglie wavelength is longer in each successive energy level because the electron's speed decreases as the radius of the orbit increases.



◀ **Figure 15.25** Standing waves in the first three energy levels, where $2\pi r_1 = 1\lambda$, $2\pi r_2 = 2\lambda$, and $2\pi r_3 = 3\lambda$

In 1926, Erwin Schrödinger (1887–1961) derived an equation for determining how electron waves behave in the electric field surrounding a nucleus. The solutions to Schrödinger's equation are functions that define the amplitude of the electron wave in the space around a nucleus. Max Born (1882–1970) showed that the square of the amplitude of these wave functions at any point is proportional to the probability of finding an electron at that point. Each wave function defines a different probability distribution or **orbital**.

info BIT

Although Born won a Nobel Prize for his work on quantum theory, Schrödinger never accepted Born's interpretation of electron wave functions.

orbital: probability distribution of an electron in an atom

Quantum Indeterminacy

Unlike the Bohr model, the quantum model does not have electrons orbiting at precisely defined distances from the nucleus. Instead, the electrons behave as waves, which do not have a precise location. The orbitals in the quantum model show the likelihood of an electron being at a given point. They are not paths that the electrons follow.

The idea that electrons within an atom behave as waves rather than as orbiting particles explains why these electrons do not radiate electromagnetic energy continuously.

Some physicists, including Einstein and Schrödinger, had difficulty accepting a quantum model that could predict only probabilities rather than clearly defined locations for electrons in an atom. As Niels Bohr noted, "Anyone who is not shocked by quantum theory has not understood a single word." Despite its challenging concepts, quantum theory is the most comprehensive and accurate model of atoms and molecules yet developed.

Concept Check

Soon after Bohr's model was published, physicists discovered that the spectral lines in hydrogen and other elements were not distinct, but could themselves be split into numerous, very closely spaced spectral lines. How does the splitting of spectral lines show that Bohr's concept of energy levels is incomplete?

15.5 Check and Reflect

Knowledge

1. Describe three failings of the Bohr model.
2. What is the Zeeman effect?
3. What is an orbital?

Applications

4. (a) Find the de Broglie wavelength for an electron in the ground state for hydrogen. Recall that the ground-state radius for hydrogen is 5.29×10^{-11} m.
   (b) Find the momentum of the electron in part (a). (Hint: Use the formula for the de Broglie wavelength.)
   (c) Find the kinetic energy and speed for the electron in part (a).
5. (a) Express the de Broglie wavelength of the $n = 2$ and $n = 3$ energy levels of an atom in terms of λ_1, the de Broglie wavelength for the $n = 1$ energy level. (Hint: Apply Bohr's equation for the orbital radii.)
   (b) Make a scale diagram or model of the atom showing the electron waves for the first three energy levels.

Extensions

6. In more advanced treatments of quantum mechanics, the wave solutions to the Schrödinger equation can describe complex-looking orbitals around atoms. These solutions will sometimes have nodes. As you learned in Chapter 13, a node is a location where waves combine destructively and have zero amplitude. According to Born's interpretation of a wave function, what meaning would you give to a node in a wave function?
7. Radio astronomers use the 21-cm line to study hydrogen gas clouds in our galaxy. Use a library or the Internet to research this spectral line. What unusual transition causes the 21-cm line? Why does this transition occur naturally in hydrogen in deep space, but not in hydrogen on Earth?

eTEST

To check your understanding of the quantum model, follow the eTEST links at www.pearsoned.ca/school/physicssource.

Key Terms and Concepts

cathode ray
elementary unit of charge
planetary model
spectroscopy
emission line spectrum
absorption line spectrum
spectrometer
Fraunhofer line
energy level
stationary state
principal quantum number
Bohr radius
ground state
excited state
ionization energy
orbital

Key Equations

Velocity selection: no deflection if $v = \frac{|\vec{E}|}{|\vec{B}|}$

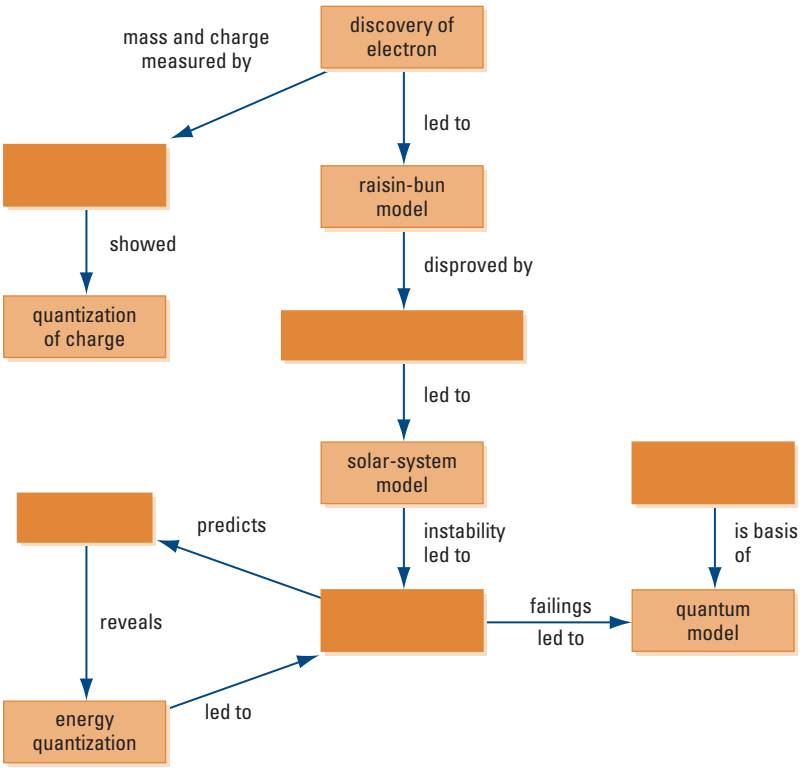
Bohr model for hydrogen: $r_n = r_1 n^2$, where $r_1 = 5.29 \times 10^{-11}$ m $\quad E_n = -\frac{E_1}{n^2}$, where $E_1 = 2.18 \times 10^{-18}$ J

Emission lines: $E_{\text{photon}} = E_{\text{final}} - E_{\text{initial}}$ and $\frac{1}{\lambda} = R_H\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)$, where $R_H = 1.097 \times 10^7\ \text{m}^{-1}$

Quantum model: $mvr_n = \frac{nh}{2\pi}$

Conceptual Overview

Summarize the chapter by copying and completing this concept map.

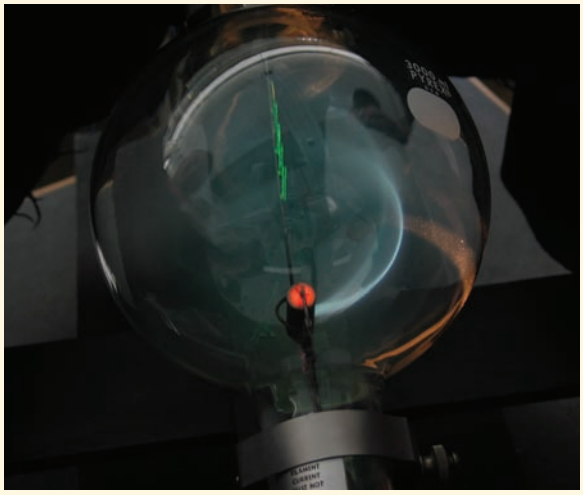


Knowledge

1. (15.1) What is a cathode ray? What type of electric charge does a cathode ray carry?

2. (15.1) (a) Describe the force acting on a cathode ray moving to the right in an electric field directed down (toward the bottom of the page).
   (b) Describe the force acting on a cathode ray moving to the right in a magnetic field directed down (toward the bottom of the page).
   (c) How could the electric and magnetic fields be directed so that the net force on the cathode ray is zero?

3. (15.1) The glowing gas shows the path of the electrons in this gas discharge tube. How can you tell that the beam of electrons is travelling through a magnetic field?



4. (15.1) Why was Thomson's experiment able to determine only the charge-to-mass ratio for an electron?

5. (15.2) Explain how the results of Millikan's oil-drop experiment also enabled physicists to determine the mass of the electron.

6. (15.2) What is the electrical charge on a dust particle that has lost 23 electrons?

7. (15.2) Calculate the electrical charge carried by 1.00 kg of electrons.

8. (15.3) What is an alpha particle?

9. (15.3) Why is the Rutherford gold-foil experiment sometimes called a "scattering experiment"?

10. (15.3) Explain how Thomson's model of the atom was inconsistent with the results of Rutherford's gold-foil experiment.

11. (15.4) (a) What is an emission line spectrum?
    (b) How could you produce an emission line spectrum?

12. (15.4) What are Fraunhofer lines?

13. (15.4) Explain how emission line spectra demonstrate Planck's concept of energy quantization.

14. (15.4) What is the difference between the ground state of an atom and an excited state?

15. (15.4) Here are four energy-level transitions for an electron in a hydrogen atom:
    $n_i = 1 \rightarrow n_f = 5$ $\quad$ $n_i = 4 \rightarrow n_f = 3$
    $n_i = 2 \rightarrow n_f = 5$ $\quad$ $n_i = 8 \rightarrow n_f = 3$
    (a) For which of these transition(s) does the atom gain energy?
    (b) For which transition does the atom gain the most energy?
    (c) Which transition emits the photon with the longest wavelength?

16. (15.4) Calculate the radius of a hydrogen atom in the $n = 3$ state.

17. (15.5) Describe the difference between an orbit in Bohr's model of the atom and an orbital in the quantum model.

18. (15.5) How does the quantum model explain why electrons in an atom do not continuously radiate energy?

Applications

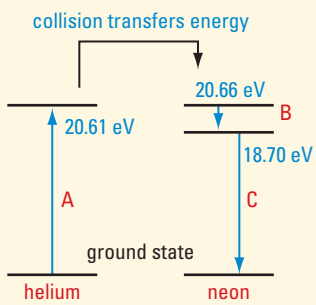
19. An electron is moving at a speed of 1.0 km/s perpendicular to a magnetic field with a magnitude of 1.5 T. How much force does the magnetic field exert on the electron?

20. (a) Use the Bohr model to predict the speed of an electron in the $n = 2$ energy level of a hydrogen atom.
    (b) Explain why the quantum model can predict the energy of this electron, but not its speed.

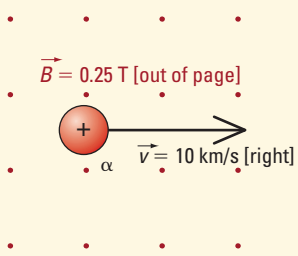
21. Calculate the electric field that will suspend an oil droplet that has a mass of 2.0×10^{-15} kg and a charge of +3e.

22. Transitions to or from the $n = 3$ energy level produce the Paschen series of lines in the hydrogen spectrum.
    (a) Find the change in energy level for the first three Paschen transitions.
    (b) Find the wavelengths and frequencies of the first three Paschen transitions.
    (c) Use $E = hf$ to calculate the energy of the photon produced by a transition from the $n = 5$ to the $n = 3$ energy level. Is your calculation consistent with your answer to part (a)? Why or why not?
    (d) In what part of the electromagnetic spectrum would you find the Paschen lines?

23. The helium-neon laser produces a red-coloured light by exciting a gas that contains a mixture of both helium and neon atoms. The energy level diagram below shows three transitions, A, B, and C, that are involved. Two of these transitions are produced by collisions with other atoms or electrons, and the third is the result of photon emission.
    (a) Explain which process is involved in transitions A, B, and C.
    (b) For the transition that produces a photon, determine the wavelength of the photon.



24. Determine the electric field that will stop this alpha particle from being deflected as it travels at 10 km/s through a 0.25-T magnetic field.



Extensions

25. Classical electromagnetic theory predicts that an electron orbiting a nucleus of charge q will radiate energy at a rate of $P = -\dfrac{2kq^2a^2}{3c^3}$, where k is Coulomb's constant, a is the electron's acceleration, and c is the speed of light.
    (a) Determine the kinetic energy of the electron in the ground state of a hydrogen atom.
    (b) Use the Bohr model to calculate the acceleration of an electron in the ground state of a hydrogen atom. (Hint: Apply the equations for circular motion.)
    (c) Show that P has the units of energy divided by time.
    (d) How long will it take the electron to give off all of its kinetic energy as electromagnetic radiation, assuming that the electron's acceleration remains constant?
    (e) Explain how your answer to part (d) shows that classical models of the atom are invalid.

Consolidate Your Understanding

1. Explain how the Rutherford gold-foil experiment radically changed the understanding of atomic structure.
2. Explain how Bohr linked spectral lines to Planck's idea of energy quantization.
3. Outline the successes and failures of the Bohr model.
4. Describe three fundamental differences between the quantum model of the atom and the Bohr model.

Think About It

Review your answers to the Think About It questions on page 753. How would you answer each question now?

eTEST

To check your understanding of atomic structure, follow the eTEST links at www.pearsoned.ca/school/physicssource.