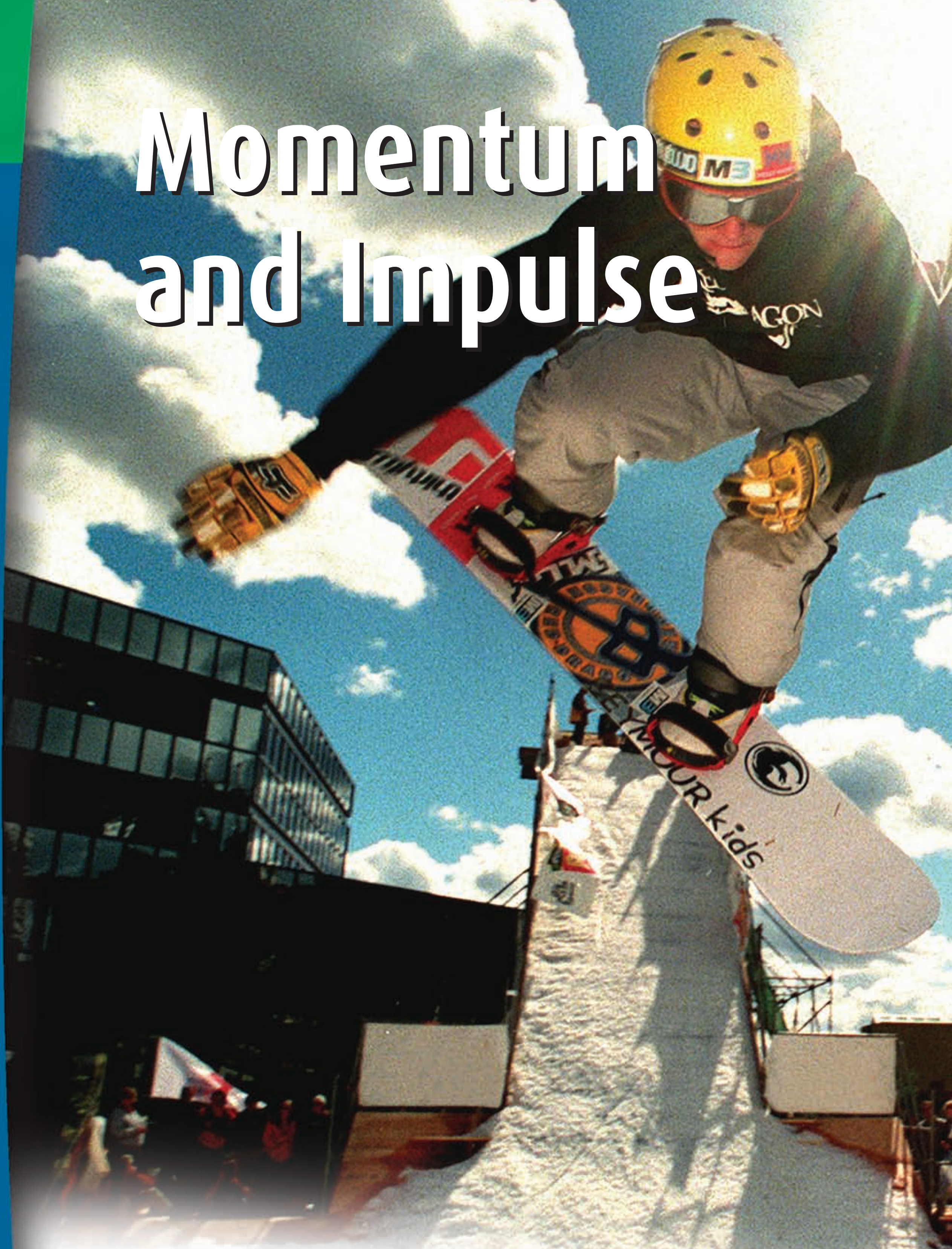
UNIT

V

# Momentum and Impulse



**Many situations and activities in the real world, such as snowboarding, involve an object gaining speed and momentum as it moves. Sometimes two or more objects collide, such as a hockey stick hitting a puck across the ice. What physics principles apply to the motion of colliding objects? How does the combination of the net force during impact and the interaction time affect an object during a collision?**

# Unit at a Glance

**CHAPTER 9 The momentum of an isolated system of interacting objects is conserved.**



## Unit Themes and Emphases

- Change and Systems
- Science and Technology

## Focussing Questions

In this study of momentum and impulse, you will investigate the motion of objects that interact, how the velocity of a system of objects is related before and after collision, and how safety devices incorporate the concepts of momentum and impulse.

As you study this unit, consider these questions:

- What characteristics of an object affect its momentum?
- How are momentum and impulse related?

## Unit Project

**An Impulsive Water Balloon**

- By the time you complete this unit, you will have the skills to design a model of an amusement ride that is suitable for a diverse group of people. You will first need to consider acceptable accelerations that most people can tolerate. To test your model, you will drop a water balloon from a height of 2.4 m to see if it will remain intact.

*e* **WEB**

Research the physics concepts that apply to collisions in sports. How do athletes apply these concepts when trying to score goals for their team? How do they apply these concepts to minimize injury? Write a summary of your findings. Begin your search at www.pearsoned.ca/school/physicssource.

CHAPTER

9

### Key Concepts

In this chapter, you will learn about:

- impulse
- momentum
- Newton's laws of motion
- elastic and inelastic collisions

### Learning Outcomes

When you have completed this chapter, you will be able to:

**Knowledge**

- define momentum as a vector quantity
- explain impulse and momentum using Newton's laws of motion
- explain that momentum is conserved in an isolated system
- explain that momentum is conserved in one- and two-dimensional interactions
- compare and contrast elastic and inelastic collisions

**Science, Technology, and Society**

- explain that technological problems lend themselves to multiple solutions

# The momentum of an isolated system of interacting objects is conserved.

Most sports involve objects colliding during the play. Hockey checks, curling takeouts, football tackles, skeet shooting, lacrosse catches, and interceptions of the ball in soccer are examples of collisions in sports action. Players, such as Randy Ferbey, who are able to accurately predict the resulting motion of colliding objects have a better chance of helping their team win (Figure 9.1).

When objects interact during a short period of time, they may experience very large forces. Evidence of these forces is the distortion in shape of an object at the moment of impact. In hockey, the boards become distorted for an instant when a player collides with them. Another evidence of these forces is a change in the motion of an object. If a goalie gloves a shot aimed at the net, you can see how the impact of the puck affects the motion of the goalie's hand.

In this chapter, you will examine how the net force on an object and the time interval during which the force acts affect the motion of the object. Designers of safety equipment for sports and vehicles use this type of analysis when developing new safety devices. In a system of objects, you will also investigate how their respective velocities change when the objects interact with each other.

◀ **Figure 9.1** Sports such as curling involve applying physics principles to change the score. Randy Ferbey, originally from Edmonton, won the Brier (Canadian) Curling Championship six times, and the World Curling Championship four times.

## 9-1 QuickLab

# Predicting Angles After Collisions

### Problem

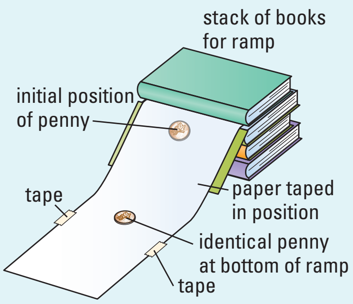
How do the masses of two objects affect the angle between their paths after they collide off centre?

### Materials

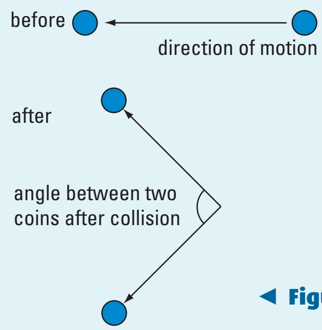
pennies and nickels with smooth, circular outer edges
marking devices for the paths (paper, tape, pencil, ruler)
stack of books
protractor (optional)

### Procedure

1. Set up the books and paper as shown in Figure 9.2. Open the cover of the book at the top of the stack for backing. Tape the paper securely to the lab bench.
2. Position one penny at the bottom of the ramp. Mark its initial position by drawing an outline on the paper.
3. Place the incoming penny at the top of the ramp as shown in Figure 9.2. Mark its initial position.
4. Predict the path each coin will take after they collide off centre. Lightly mark the predicted paths.
5. Send the coin down the ramp and mark the position of each coin after collision. Observe the relative velocities of the coins to each other both before and after collision.




▶ **Figure 9.2**




◀ **Figure 9.3**

6. Determine if the angle between the paths after collision is less than 90°, 90°, or greater than 90° (Figure 9.3).
7. Repeat steps 5 and 6, but have the incoming coin collide at a different contact point with the coin at the bottom of the ramp.
8. Repeat steps 2 to 7 using a penny as the incoming coin and a nickel at the bottom of the ramp.
9. Repeat steps 2 to 7 using a nickel as the incoming coin and a penny at the bottom of the ramp.

### Questions

**1.** What was the approximate angle formed by the paths of the two coins after collision when the coins were
 (a) the same mass?
 (b) of different mass?

**2.** Describe how the speeds of the two coins changed before and after collision.

**3.** How can you predict which coin will move faster after collision?

## Think About It

**1.** Under what circumstances could an object initially at rest be struck and move at a greater speed after collision than the incoming object?

**2.** Under what circumstances could a coin in 9-1 QuickLab rebound toward the ramp after collision?

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes to your ideas.

# 9.1 Momentum Is Mass Times Velocity

*info* **BIT**

A tragic avalanche occurred during the New Year's Eve party in the Inuit community of Kangiqsualujjuaq, formerly in Quebec and now part of Nunavut. At 1:30 a.m. on January 1, 1999, snow from the nearby 365-m mountain slope came cascading down, knocking out a wall and swamping those inside the gymnasium at the party. The snow on the mountain was initially about 1 m thick. After the avalanche was over, the school was covered with up to 3 m of snow.

Snow avalanches sliding down mountains involve large masses in motion. They can be both spectacular and catastrophic (Figure 9.4).

Unbalanced forces affect the motion of all objects. A mass of snow on the side of a mountain experiences many forces, such as wind, friction between the snow and the mountain, a normal force exerted by the mountain on the snow, and gravity acting vertically downward. Skiers and animals moving along the mountain slope also apply forces on the mass of snow.

When a large mass of snow becomes dislodged and slides down a mountain slope due to gravity, it not only gains speed but also more mass as additional snow becomes dislodged along the downward path.

*info* **BIT**

Most avalanches occur on slopes that form an angle of 30° to 45° with the horizontal, although they can occur on any slope if the right conditions exist. In North America, a large avalanche may release about 230 000 $m^3$ of snow.

▶ **Figure 9.4** When the risk of an avalanche seems imminent, ski patrols reduce the mass of snow along a mountain slope by forcing an avalanche to take place. They do this by targeting large masses of snow with guns or explosives to dislodge the snow.

## Momentum Is a Vector Quantity

All objects have mass. The **momentum**, $\vec{p}$, of an object is defined as the product of the mass of the object and its velocity. Since momentum is the product of a scalar (mass) and a vector (velocity), momentum is a vector quantity that has the same direction as the velocity.

**momentum:** product of the mass of an object and its velocity

$$\vec{p} = m\vec{v}$$

Momentum has units of kilogram-metres per second (kg•m/s).

When you compare the momenta of two objects, you need to consider both the mass and the velocity of each object (Figure 9.5). Although two identical bowling balls, A and B, have the same mass, they do not necessarily have the same momentum. If ball A is moving very slowly, it has a very small momentum. If ball B is moving much faster than ball A, ball B's momentum will have a greater magnitude than ball A's because of its greater speed.

▲ **Figure 9.5** The bowling ball in both photos is the same. However, the bowling ball on the left has less momentum than the ball on the right. What evidence suggests this?

In real life, almost no object in motion has constant momentum because its velocity is usually not constant. Friction opposes the motion of all objects and eventually slows them down. In most instances, it is more accurate to state the *instantaneous* momentum of an object if you can measure its instantaneous velocity and mass.

### Concept Check

How would the momentum of an object change if

(a) the mass is doubled but the velocity remains the same?

(b) the velocity is reduced to $\frac{1}{3}$ of its original magnitude?

(c) the direction of the velocity changes from [E] to [W]?

**e WEB**

Switzerland has a long history of studying avalanches. Find out what causes an avalanche. What physical variables do avalanche experts monitor? What models are scientists working on to better predict the likelihood and severity of avalanches? Begin your search at www.pearsoned.ca/school/physicssource.

## Relating Momentum to Newton's Second Law

**eWEB**

Research how momentum applies to cycling and other sports. Write a brief report of your findings. Begin your search at www.pearsoned.ca/school/physicssource.

The concept of momentum can be used to restate Newton's second law. From Unit II, Newton's second law states that an external non-zero net force acting on an object is equal to the product of the mass of the object and its acceleration, $\vec{F}_{\text{net}} = m\vec{a}$. Acceleration is defined as the rate of change of velocity. For constant acceleration, $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ or $\frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}$. If you substitute $\frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}$ for $\vec{a}$ in Newton's second law, you get

$$\begin{aligned}\vec{F}_{\text{net}} &= m\vec{a}\\ &= m\left(\frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}\right)\\ &= \frac{m\vec{v}_{\text{f}} - m\vec{v}_{\text{i}}}{\Delta t}\end{aligned}$$

The quantity $m\vec{v}$ is momentum. So the equation can be written as

$$\begin{aligned}\vec{F}_{\text{net}} &= \frac{\vec{p}_{\text{f}} - \vec{p}_{\text{i}}}{\Delta t}\\ &= \frac{\Delta\vec{p}}{\Delta t} \text{ where } \vec{F}_{\text{net}} \text{ is constant}\end{aligned}$$

Written this way, Newton's second law relates the net force acting on an object to its rate of change of momentum. It is interesting to note that Newton stated his second law of motion in terms of the rate of change of momentum. It may be worded as:

An external non-zero net force acting on an object is equal to the rate of change of momentum of the object.

$$\vec{F}_{\text{net}} = \frac{\Delta\vec{p}}{\Delta t} \text{ where } \vec{F}_{\text{net}} \text{ is constant}$$

**eSIM**

For a given net force, learn how the mass of an object affects its momentum and its final velocity. Follow the eSim links at www.pearsoned.ca/school/physicssource.

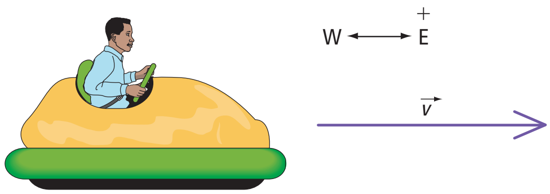
This form of Newton's law has some major advantages over the way it was written in Unit II. The equation $\vec{F}_{\text{net}} = m\vec{a}$ only applies to situations where the mass is constant. However, by using the concept of momentum, it is possible to derive another form for Newton's second law that applies to situations where the mass, the velocity, or both the mass and velocity are changing, such as an accelerating rocket where the mass is decreasing as fuel is being burned, while the velocity is increasing.

In situations where the net force changes over a time interval, the *average* net force is equal to the rate of change of momentum of the object.

$$\vec{F}_{\text{net}_{\text{ave}}} = \frac{\Delta\vec{p}}{\Delta t}$$

In Example 9.1, a person in a bumper car is moving at constant velocity. Since both the person and the car move together as a unit, both objects form a system. The momentum of the system is equal to the total mass of the system times the velocity of the system.

## Example 9.1




▲ **Figure 9.6**

A 180-kg bumper car carrying a 70-kg driver has a constant velocity of 3.0 m/s [E]. Calculate the momentum of the car-driver system. Draw both the velocity vector and the momentum vector.

***Given***

$m_c = 180$ kg $\qquad m_d = 70$ kg $\qquad \vec{v} = 3.0$ m/s [E]

***Required***

momentum of system ($\vec{p}$)

velocity and momentum vector diagrams

***Analysis and Solution***

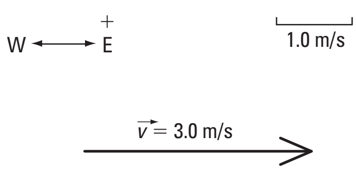
The driver and bumper car are a system because they move together as a unit. Find the total mass of the system.

$$\begin{aligned} m_T &= m_c + m_d \\ &= 180 \text{ kg} + 70 \text{ kg} \\ &= 250 \text{ kg} \end{aligned}$$

The momentum of the system is in the direction of the velocity of the system. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$\begin{aligned} p &= m_T v \\ &= (250 \text{ kg})(3.0 \text{ m/s}) \\ &= 7.5 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Draw the velocity vector to scale (Figure 9.7).



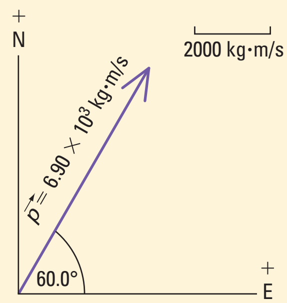

▲ **Figure 9.7**

## Practice Problems

1. A 65-kg girl is driving a 535-kg snowmobile at a constant velocity of 11.5 m/s [60.0° N of E].
   (a) Calculate the momentum of the girl-snowmobile system.
   (b) Draw the momentum vector for this situation.
2. The combined mass of a bobsled and two riders is 390 kg. The sled-rider system has a constant momentum of $4.68 \times 10^3$ kg·m/s [W]. Calculate the velocity of the sled.

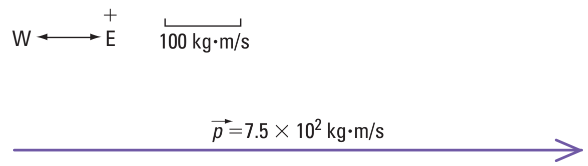
### Answers

1. (a) $6.90 \times 10^3$ kg·m/s [60.0° N of E]
   (b)




2. 12.0 m/s [W]

Draw the momentum vector to scale (Figure 9.8).




▲ **Figure 9.8**

*Paraphrase*
The momentum of the car-driver system is $7.5 \times 10^2$ kg·m/s [E].

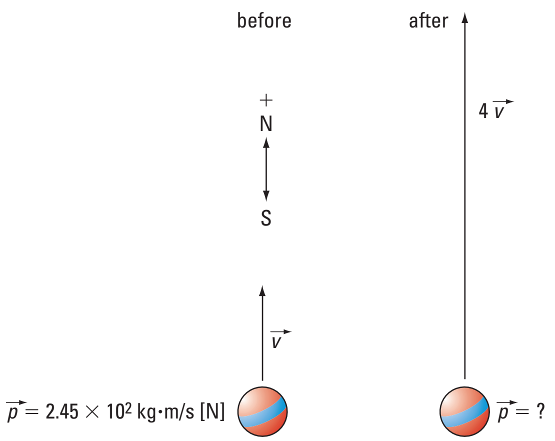
## Using Proportionalities to Solve Momentum Problems

Example 9.2 demonstrates how to solve momentum problems using proportionalities. In this example, both the mass and velocity of an object change.

### Example 9.2

An object has a constant momentum of $2.45 \times 10^2$ kg·m/s [N]. Determine the momentum of the object if its mass decreases to $\frac{1}{3}$ of its original value and an applied force causes the speed to increase by exactly four times. The direction of the velocity remains the same. Explain your reasoning.

*Analysis and Solution*
From the equation $\vec{p} = m\vec{v}$, $p \propto m$ and $p \propto v$.
Figure 9.9 represents the situation of the problem.




◀ **Figure 9.9**

$p \propto \frac{1}{3}m$ and $p \propto 4v$

Calculate the factor change of $p$.

$$\frac{1}{3} \times 4 = \frac{4}{3}$$

Calculate $p$.

$$\begin{aligned}\frac{4}{3}p &= \frac{4}{3} \times (2.45 \times 10^2 \text{ kg·m/s}) \\ &= 3.27 \times 10^2 \text{ kg·m/s}\end{aligned}$$

The new momentum will be $3.27 \times 10^2$ kg·m/s [N].

### Practice Problems

1. Many modern rifles use bullets that have less mass and reach higher speeds than bullets for older rifles, resulting in increased accuracy over longer distances. The momentum of a bullet is initially 8.25 kg·m/s [W]. What is the momentum if the speed of the bullet increases by a factor of $\frac{3}{2}$ and its mass decreases by a factor of $\frac{3}{4}$?
2. During one part of the liftoff of a model rocket, its momentum increases by a factor of four while its mass is halved. The velocity of the rocket is initially 8.5 m/s [up]. What is the final velocity during that time interval?

**Answers**

1. 9.28 kg·m/s [W]
2. 68 m/s [up]

## 9.1 Check and Reflect

### Knowledge

1. (a) Explain, in your own words, the concept of momentum.
   (b) State the SI units of momentum.
2. Explain why momentum is a vector quantity.
3. How is momentum related to Newton's second law?
4. Explain why stating Newton's second law in terms of momentum is more useful than stating it in terms of acceleration.
5. Explain, in your own words, the difference between momentum and inertia.
6. Provide three examples of situations in which
   (a) velocity is the dominant factor affecting the momentum of an object
   (b) mass is the dominant factor affecting the momentum of an object.

### Applications

7. A Mexican jumping bean moves because an insect larva inside the shell wiggles. Would it increase the motion to have the mass of the insect greater or to have the mass of the shell greater? Explain.
8. What is the momentum of a 6.0-kg bowling ball with a velocity of 2.2 m/s [S]?
9. The momentum of a 75-g bullet is 9.00 kg•m/s [N]. What is the velocity of the bullet?
10. (a) Draw a momentum vector diagram for a 4.6-kg Canada goose flying with a velocity of 8.5 m/s [210°].
    (b) A 10.0-kg bicycle and a 54.0-kg rider both have a velocity of 4.2 m/s [40.0° N of E]. Draw momentum vectors for each mass and for the bicycle-rider system.
11. A hockey puck has a momentum of 3.8 kg•m/s [E]. If its speed is 24 m/s, what is the mass of the puck?
12. Draw a momentum vector diagram to represent a 425-g soccer ball flying at 18.6 m/s [214°].
13. At what velocity does a 0.046-kg golf ball leave the tee if the club has given the ball a momentum of 3.45 kg•m/s [S]?
14. (a) A jet flies west at 190 m/s. What is the momentum of the jet if its total mass is 2250 kg?
    (b) What would be the momentum of the jet if the mass was $\frac{3}{4}$ of its original value and the speed increased to $\frac{6}{5}$ of its original value?
15. The blue whale is the largest mammal ever to inhabit Earth. Calculate the mass of a female blue whale if, when alarmed, it swims at a velocity of 57.0 km/h [E] and has a momentum of $2.15 \times 10^6$ kg•m/s [E].

### Extensions

16. A loaded transport truck with a mass of 38 000 kg is travelling at 1.20 m/s [W]. What will be the velocity of a 1400-kg car if it has the same momentum?
17. Summarize the concepts and ideas associated with momentum using a graphic organizer of your choice. See Student References 4: Using Graphic Organizers on pp. 869–871 for examples of different graphic organizers. Make sure that the concepts and ideas are clearly presented and appropriately linked.

**e TEST**

To check your understanding of momentum, follow the eTest links at www.pearsoned.ca/school/physicssource.

# 9.2 Impulse Is Equivalent to a Change in Momentum

*info* **BIT**

Legendary stunt person Dar Robinson broke nine world records and made 21 "world firsts" during his career. One world record was a cable jump from the CN Tower in 1980 for the film *The World's Most Spectacular Stuntman.* While tied to a 3-mm steel cable, Robinson jumped more than 366 m and stopped only a short distance above the ground.

Stunt people take the saying, "It isn't the fall that hurts, it's the sudden stop at the end," very seriously. During the filming of a movie, when a stunt person jumps out of a building, the fall can be very dangerous. To minimize injury, stunt people avoid a sudden stop when landing by using different techniques to slow down more gradually out of sight of the cameras. These techniques involve reducing the peak force required to change their momentum.

Sometimes stunt people jump and land on a net. Other times, they may roll when they land. For more extreme jumps, such as from the roof of a tall building, a huge oversized, but slightly under-inflated, air mattress may be used (Figure 9.10). A hidden parachute may even be used to slow the jumper to a safer speed before impact with the surface below. Despite all these precautions, injuries occur as stunt people push the limits of what is possible in their profession.

Designers of safety equipment know that a cushioned surface can reduce the severity of an impact. Find out how the properties of a landing surface affect the shape of a putty ball that is dropped from a height of 1 m by doing 9-2 QuickLab.

▶ **Figure 9.10** The thick mattress on the ground provides a protective cushion for the stunt person when he lands. Why do you think the hardness of a surface affects the extent of injury upon impact?

## 9-2 QuickLab

# Softening the Hit

### Problem

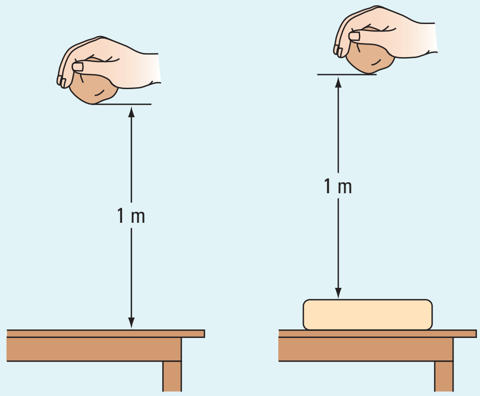
How is the change in the shape of a putty ball upon impact related to the structure of the landing surface?

### Materials

putty-type material
closed cell foam or felt pad
waxed paper or plastic wrap
metre-stick
urethane foam pad or pillow

### Procedure

1. Choose three surfaces of varying softness onto which to drop a putty ball. One of the surfaces should be either a lab bench or the floor. Cover each surface with some waxed paper or plastic wrap to protect it.
2. Knead or work the putty until you can form three pliable balls of equal size.
3. Measure a height of 1 m above the top of each surface. Then drop the balls, one for each surface (Figure 9.11).
4. Draw a side-view sketch of each ball after impact.




▲ **Figure 9.11**

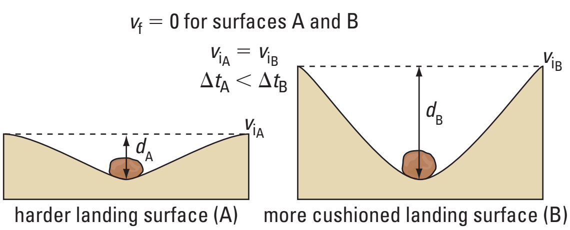
### Questions

1. Describe any differences in the shape of the putty balls after impact.
2. How does the amount of cushioning affect the deformation of the putty?
3. Discuss how the softness of the landing surface might be related to the time required for the putty ball to come to a stop. Justify your answer with an analysis involving the kinematics equations.

## Force and Time Affect Momentum

In 9-2 QuickLab, you found that the softer the landing surface, the less the shape of the putty ball changed upon impact. The more cushioned the surface, the more the surface became indented when the putty ball collided with it. In other words, the softer and more cushioned landing surface provided a greater stopping distance for the putty ball.

Suppose you label the speed of the putty ball at the instant it touches the landing surface $v_i$, and the speed of the putty ball after the impact $v_f$. For all the landing surfaces, $v_i$ was the same and $v_f = 0$. So the greater the stopping distance, the longer the time required for the putty ball to stop (Figure 9.12). In other words, the deformation of an object is less when the stopping time is increased.




◀ **Figure 9.12** The stopping distance of the putty ball was greater for the more cushioned landing surface (B). So the time interval of interaction was greater on surface B.

**PHYSICS INSIGHT**

To visualize the effect of how $F_{net}$ and $\Delta t$ can vary but $\Delta p$ remains the same, consider the effect of changing two numbers being multiplied together to give the same product.

$3 \times 12 = 36$
$6 \times 6 = 36$
$18 \times 2 = 36$

As the first number increases, the second number decreases in order to get the same product.

Apart from the different stopping times, what other differences were there between the drops that would have affected the shape of the putty ball upon impact? The answer to this question requires looking at Newton's second law written in terms of momentum.

From the previous section, the general form of Newton's second law states that the rate of change of momentum is equal to the net force acting on an object.

$$\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$$

If you multiply both sides of the equation by $\Delta t$, you get

$$\vec{F}_{net}\Delta t = \Delta\vec{p}$$

For all the landing surfaces, since $m$, $v_i$ (at the first instant of impact), and $v_f$ (after the impact is over) of the putty ball were the same, $p_i$ was the same and $p_f = 0$. So $\Delta\vec{p}$ was the same for all drops. But $\vec{F}_{net}\Delta t = \Delta\vec{p}$, so the product of net force and stopping time was the same for all drops.

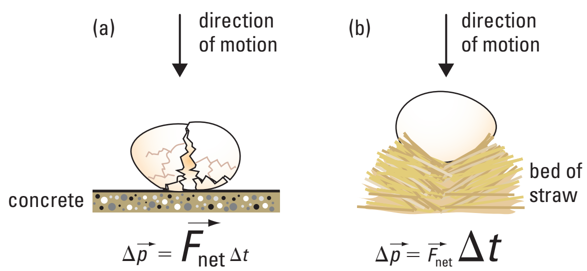
From Figure 9.12 on page 455, the stopping time varied depending on the amount of cushioning provided by the landing surface. If the stopping time was short, as on a hard landing surface, the magnitude of the net force acting on the putty ball was large. Similarly, if the stopping time was long, as on a very cushioned landing surface, the magnitude of the net force acting on the putty ball was small. This analysis can be used to explain why the putty ball became more deformed when it landed on a hard surface.

**Project LINK**

How will the net force and time interval affect the water balloon when it is brought to a stop?

What types of protective material will you use to surround the water balloon and why?

To minimize changes to the shape of an object being dropped, it is important to minimize $F_{net}$ required to stop the object, and this happens when you maximize $\Delta t$ (Figure 9.13). It is also important to note *where* $\vec{F}_{net}$ acts. If $\vec{F}_{net}$ acts on a large area, the result of the impact will have a different effect on the shape of the object than if $\vec{F}_{net}$ acts on only one small part on the surface of the object.




▲ **Figure 9.13** Identical eggs are dropped from a height of 2 m onto a concrete floor or a pile of straw. Although $\Delta\vec{p}$ is the same in both situations, the magnitude of the net force acting on the egg determines whether or not the egg will break.

***Concept Check***

In 9-2 QuickLab, was the momentum of the putty ball at the first instant of impact zero? Explain your reasoning.

## Impulse Is the Product of Net Force and Interaction Time

In the equation $\vec{F}_{net}\Delta t = \Delta\vec{p}$, the product of net force and interaction time is called **impulse**. Impulse is equivalent to the change in momentum that an object experiences during an interaction. Every time a net force acts on an object, the object is provided with an impulse because the force is applied for a specific length of time.

If you substitute the definition of momentum, $\vec{p} = m\vec{v}$, the equation $\vec{F}_{net}\Delta t = \Delta\vec{p}$ becomes

$$\vec{F}_{net}\Delta t = \Delta(m\vec{v})$$

If $m$ is constant, then the only quantity changing on the right-hand side of the equation is $\vec{v}$. So the equation becomes

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

So impulse can be calculated using either equation:

$$\vec{F}_{net}\Delta t = \Delta\vec{p} \qquad \text{or}$$

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

The unit of impulse is the newton-second (N•s). From Unit II, a newton is defined as 1 N = 1 kg•m/s². If you substitute the definition of a newton in the unit newton-seconds, you get

$$\begin{aligned} 1\ \text{N}\cdot\text{s} &= 1\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}}) \\ &= 1\ \frac{\text{kg}\cdot\text{m}}{\text{s}} \end{aligned}$$

which are the units for momentum. So the units on both sides of the impulse equation are equivalent. Since force is a vector quantity, impulse is also a vector quantity, and the impulse is in the same direction as the net force.

To better understand how net force and interaction time affect the change in momentum of an object, do 9-3 Design a Lab.

**impulse:** product of the net force on an object and the time interval during an interaction. Impulse causes a change in the momentum of the object.

*e* **SIM**

Learn how the mass and acceleration of an object affect its change in momentum. Follow the eSim links at www.pearsoned.ca/school/physicssource.

## 9-3 Design a Lab

# Providing Impulse

### The Question

What is the effect of varying either the net force or the interaction time on the momentum of an object?

### Design and Conduct Your Investigation

State a hypothesis to answer the question using an "if/then" statement. Then design an experiment to measure the change in momentum of an object. First vary $F_{net}$, then repeat the experiment and vary $\Delta t$ instead. List the materials you will use, as well as a detailed procedure. Check the procedure with your teacher and then do the investigation.

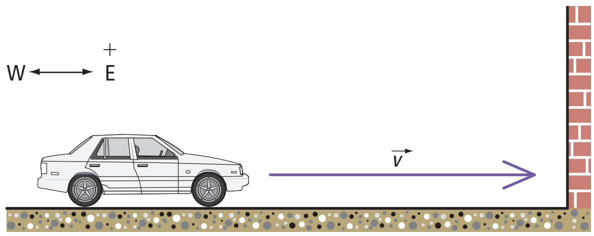
To find the net force, you may need to find the force of friction and add it, using vectors, to the applied force. The force of kinetic friction is the minimum force needed to keep an object moving at constant velocity once the object is in motion. Analyze your data and form conclusions. How well did your results agree with your hypothesis? Compare your results with those of other groups in your class. Account for any discrepancies.

*e* **LAB**

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Example 9.3 demonstrates how, for the same impulse, varying the interaction time affects the average net force on a car during a front-end collision (the net force on the car is not constant).

## Example 9.3




▲ **Figure 9.14**

To improve the safety of motorists, modern cars are built so the front end crumples upon impact. A 1200-kg car is travelling at a constant velocity of 8.0 m/s [E] (Figure 9.14). It hits an immovable wall and comes to a complete stop in 0.25 s.

(a) Calculate the impulse provided to the car.
(b) What is the average net force exerted on the car?
(c) For the same impulse, what would be the average net force exerted on the car if it had a rigid bumper and frame that stopped the car in 0.040 s?

***Given***

$m = 1200$ kg $\qquad \vec{v}_i = 8.0$ m/s [E]

(a) and (b) $\Delta t = 0.25$ s

(c) $\Delta t = 0.040$ s

***Required***

(a) impulse provided to car
(b) and (c) average net force on car ($\vec{F}_{net_{ave}}$)

***Analysis and Solution***

When the car hits the wall, the final velocity of the car is zero.

$$\vec{v}_f = 0 \text{ m/s}$$

During each collision with the wall, the net force on the car is not constant, but the mass of the car remains constant.

(a) Use the equation of impulse to calculate the impulse provided to the car.

$$\begin{aligned} \vec{F}_{net_{ave}}\Delta t &= m\Delta\vec{v} \\ &= m(\vec{v}_f - \vec{v}_i) \\ &= (1200 \text{ kg})[0 - (+8.0 \text{ m/s})] \\ &= (1200 \text{ kg})(-8.0 \text{ m/s}) \\ &= -9.6 \times 10^3 \text{ kg}\cdot\text{m/s} \\ \text{impulse} &= 9.6 \times 10^3 \text{ N}\cdot\text{s [W]} \end{aligned}$$

### Practice Problems

1. Two people push a car for 3.64 s with a combined net force of 200 N [S].
   (a) Calculate the impulse provided to the car.
   (b) If the car has a mass of 1100 kg, what will be its change in velocity?
2. A dog team pulls a 400-kg sled that has begun to slide backward. In 4.20 s, the velocity of the sled changes from 0.200 m/s [backward] to 1.80 m/s [forward]. Calculate the average net force the dog team exerts on the sled.

**Answers**

1. (a) 728 N•s [S], (b) 0.662 m/s [S]
2. 190 N [forward]

### *info* BIT

Some early cars were built with spring bumpers that tended to bounce off whatever they hit. These bumpers were used at a time when people generally travelled at much slower speeds. For safety reasons, cars today are built to crumple upon impact, not bounce. This results in a smaller change in momentum and a reduced average net force on motorists. The crushing also increases the time interval during the impulse, further decreasing the net force on motorists.

For (b) and (c), substitute the impulse from part (a) and solve for $\vec{F}_{net_{ave}}$.

$$\vec{F}_{net_{ave}}\Delta t = -9.6 \times 10^3 \text{ N}\cdot\text{s}$$

$$\vec{F}_{net_{ave}} = \frac{-9.6 \times 10^3 \text{ N}\cdot\text{s}}{\Delta t}$$

(b) $$\vec{F}_{net_{ave}} = \frac{-9.6 \times 10^3 \text{ N}\cdot\text{s}}{0.25 \text{ s}} = -3.8 \times 10^4 \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = -3.8 \times 10^4 \text{ N}$$

$$\vec{F}_{net_{ave}} = 3.8 \times 10^4 \text{ N [W]}$$

(c) $$\vec{F}_{net_{ave}} = \frac{-9.6 \times 10^3 \text{ N}\cdot\text{s}}{0.040 \text{ s}} = -2.4 \times 10^5 \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = -2.4 \times 10^5 \text{ N}$$

$$\vec{F}_{net_{ave}} = 2.4 \times 10^5 \text{ N [W]}$$

***Paraphrase and Verify***

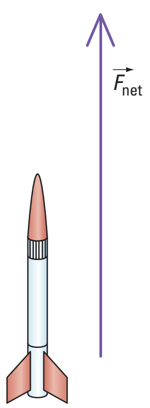
(a) The impulse provided to the car is $9.6 \times 10^3$ N•s [W].
The average net force exerted by the wall on the car is (b) $3.8 \times 10^4$ N [W] when it crumples, and (c) $2.4 \times 10^5$ N [W] when it is rigid.
The change in momentum is the same in parts (b) and (c), but the time intervals are different. So the average net force is different in both situations. The magnitude of $\vec{F}_{net_{ave}}$ on the car with the rigid frame is more than 6 times greater than when the car crumples.

**PHYSICS INSIGHT**

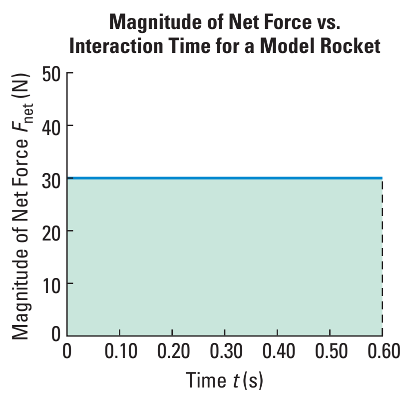
$\vec{F}_{net_{ave}}$ is in the *opposite* direction to the initial momentum of the car, because from Newton's third law, the wall is exerting a force directed west on the car.

## Impulse Can Be Calculated Using a Net Force-Time Graph

One way to calculate the impulse provided to an object is to graph the net force acting on the object as a function of the interaction time. Suppose a net force of magnitude 30 N acts on a model rocket for 0.60 s during liftoff (Figure 9.15). From the net force-time graph in Figure 9.16, the product $F_{net}\Delta t$ is equal to the magnitude of the impulse. But this product is also the area under the graph.




▲ **Figure 9.15** What forces act on the rocket during liftoff?



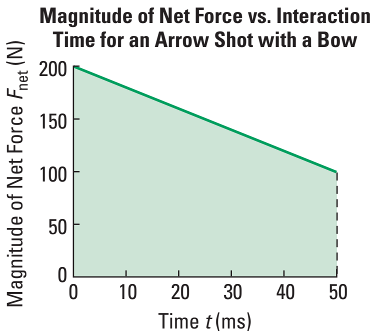

▲ **Figure 9.16** Magnitude of net force as a function of interaction time for a model rocket. The area under the graph is equal to the magnitude of the impulse provided to the rocket.

The magnitude of the impulse provided to the rocket is

$$\begin{aligned}\text{magnitude of impulse} &= F_{\text{net}}\Delta t\\ &= (30\ \text{N})(0.60\ \text{s})\\ &= 18\ \text{N}\cdot\text{s}\end{aligned}$$

In other words, the area under a net force-time graph gives the magnitude of the impulse. Note that a net force acting over a period of time *causes* a change in momentum.

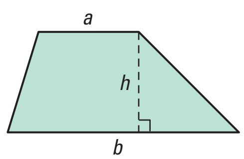
When $F_{\text{net}}$ is not constant, you can still calculate the impulse by finding the area under a net force-time graph. Figure 9.17 shows the magnitude of the net force exerted by a bow on an arrow during the first part of its release. The magnitude of the net force is greatest at the beginning and decreases linearly with time.




◀ **Figure 9.17** Magnitude of net force as a function of interaction time for an arrow shot with a bow.

*info* **BIT**

The area of a trapezoid is equal to $\frac{1}{2}(a + b)(h)$.

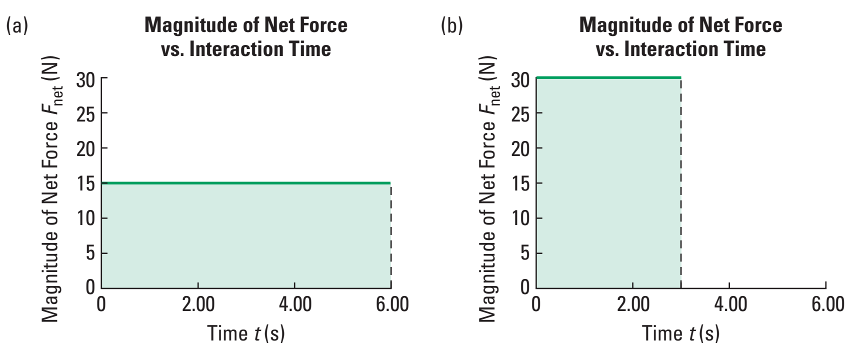



▲ **Figure 9.18**

In this case, the area under the graph could be divided into a rectangle and a triangle or left as a trapezoid (Figure 9.18). So the magnitude of the impulse provided to the arrow is

$$\begin{aligned}\text{magnitude of impulse} &= \frac{1}{2}(a + b)(h)\\ &= \frac{1}{2}(100\ \text{N} + 200\ \text{N})(0.050\ \text{s})\\ &= 7.5\ \text{N}\cdot\text{s}\end{aligned}$$

Sometimes two net force-time graphs may appear different but the magnitude of the impulse is the same in both cases. Figure 9.19 (a) shows a graph where $F_{\text{net}}$ is much smaller than in Figure 9.19 (b). The value of $\Delta t$ is different in each case, but the *area* under both graphs is the same. So the magnitude of the impulse is the same in both situations.

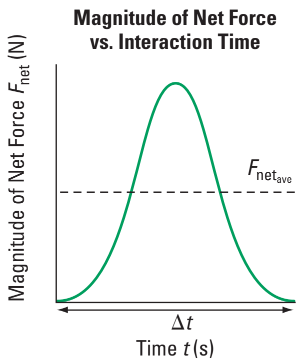



▲ **Figure 9.19** What other graph could you draw that has the same magnitude of impulse?

## Effect of a Non-linear Net Force on Momentum

In real life, many interactions occur during very short time intervals (Figure 9.20). If you tried to accurately measure the net force, you would find that it is difficult, if not impossible, to do. In addition, the relationship between $F_{net}$ and $\Delta t$ is usually non-linear, because $F_{net}$ increases from zero to a very large value in a short period of time (Figure 9.21).

▲ **Figure 9.20** When a baseball bat hits a ball, what evidence demonstrates that the force during the interaction is very large? What evidence demonstrates that the force on the ball changes at the instant the ball and bat separate?




▲ **Figure 9.21** The average net force gives some idea of the maximum instantaneous net force that an object actually experienced during impact.

From a practical point of view, it is much easier to measure the interaction time and the overall change in momentum of an object during an interaction, rather than $F_{net}$. In this case, the equation of Newton's second law expressed in terms of momentum is

$$\vec{F}_{net_{ave}} = \frac{\Delta \vec{p}}{\Delta t}$$

and the equation of impulse is

$$\vec{F}_{net_{ave}} \Delta t = \Delta \vec{p} \qquad \text{or}$$

$$\vec{F}_{net_{ave}} \Delta t = m \Delta \vec{v}$$

In all the above equations, $\vec{F}_{net_{ave}}$ represents the average net force that acted on the object during the interaction.

In Example 9.4, a golf club strikes a golf ball and an approximation of the net force-time graph is used to simplify the calculations for impulse. In reality, the net force-time graph for such a situation would be similar to that shown in Figure 9.21.

*info* **BIT**

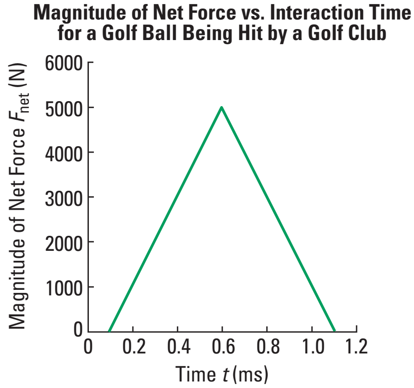
The fastest recorded speed for a golf ball hit by a golf club is 273 km/h.

## Example 9.4

A golfer hits a long drive sending a 45.9-g golf ball due east. Figure 9.22 shows an approximation of the net force as a function of time for the collision between the golf club and the ball.

(a) What is the impulse provided to the ball?

(b) What is the velocity of the ball at the moment the golf club and ball separate?




▲ **Figure 9.22**

*Given*

$m = 45.9$ g $\qquad t_i = 0.1$ ms

$t_f = 1.1$ ms $\qquad F_{net_i} = 0$ N

$F_{net_{max}} = 5000$ N $\qquad F_{net_f} = 0$ N

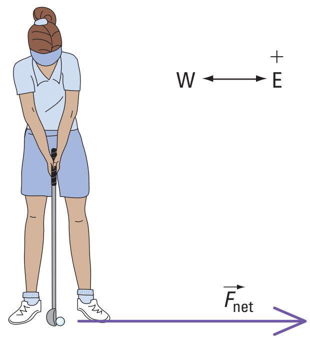
*Required*

(a) impulse provided to ball

(b) velocity of ball after impact ($\vec{v}_f$)

*Analysis and Solution*

The impulse and velocity after impact are in the east direction since the golfer hits the ball due east.




▲ **Figure 9.23**

(a)
$$\begin{aligned}\Delta t &= t_f - t_i\\ &= 1.1 \text{ ms} - 0.1 \text{ ms}\\ &= 1.0 \text{ ms or } 1.0 \times 10^{-3} \text{ s}\end{aligned}$$

$$\begin{aligned}\text{magnitude of impulse} &= \text{area under net force-time graph}\\ &= \frac{1}{2}(\Delta t)(F_{net_{max}})\\ &= \frac{1}{2}(1.0 \times 10^{-3} \text{ s})(5000 \text{ N})\\ &= 2.5 \text{ N}\cdot\text{s}\\ \text{impulse} &= 2.5 \text{ N}\cdot\text{s [E]}\end{aligned}$$

(b) Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$\begin{aligned}\text{But } \vec{v}_i &= 0 \text{ m/s}\\ \text{So, impulse} &= m(\vec{v}_f - 0)\\ +2.5 \text{ N}\cdot\text{s} &= m\vec{v}_f\\ \vec{v}_f &= \frac{+2.5 \text{ N}\cdot\text{s}}{m}\\ &= \frac{+2.50\left(\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^{\cancel{2}}}\right)\cdot\cancel{\text{s}}}{(45.9\ \cancel{\text{g}})\left(\frac{1\ \cancel{\text{kg}}}{1000\ \cancel{\text{g}}}\right)}\\ &= +54 \text{ m/s}\end{aligned}$$

*Paraphrase*

(a) The impulse provided to the ball is 2.5 N•s [E].

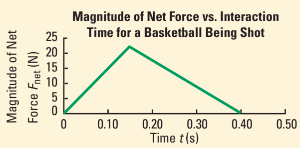
(b) The velocity of the ball after impact is 54 m/s [E].

## Practice Problems

1. (a) Draw a graph of net force as a function of time for a 0.650-kg basketball being shot. During the first 0.15 s, $F_{net}$ increases linearly from 0 N to 22 N. During the next 0.25 s, $F_{net}$ decreases linearly to 0 N.
   (b) Using the graph, calculate the magnitude of the impulse provided to the basketball.
   (c) What is the speed of the ball when it leaves the shooter's hands?
2. (a) A soccer player heads the ball with an average net force of 21 N [W] for 0.12 s. Draw a graph of the average net force on the ball as a function of time. Assume that $F_{net_{ave}}$ is constant during the interaction.
   (b) Calculate the impulse provided to the soccer ball.
   (c) The impulse changes the velocity of the ball from 4.0 m/s [E] to 2.0 m/s [W]. What is the mass of the ball?

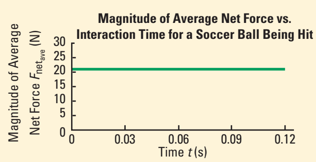
### Answers

1. (a)




(b) 4.4 N•s, (c) 6.8 m/s

2. (a)

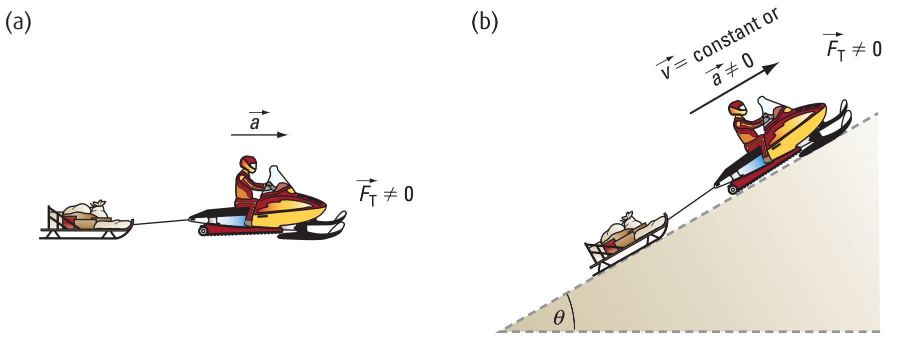



(b) 2.5 N•s [W], (c) 0.42 kg

## The Design of Safety Devices Involves Varying $F_{net_{ave}}$ and $\Delta t$

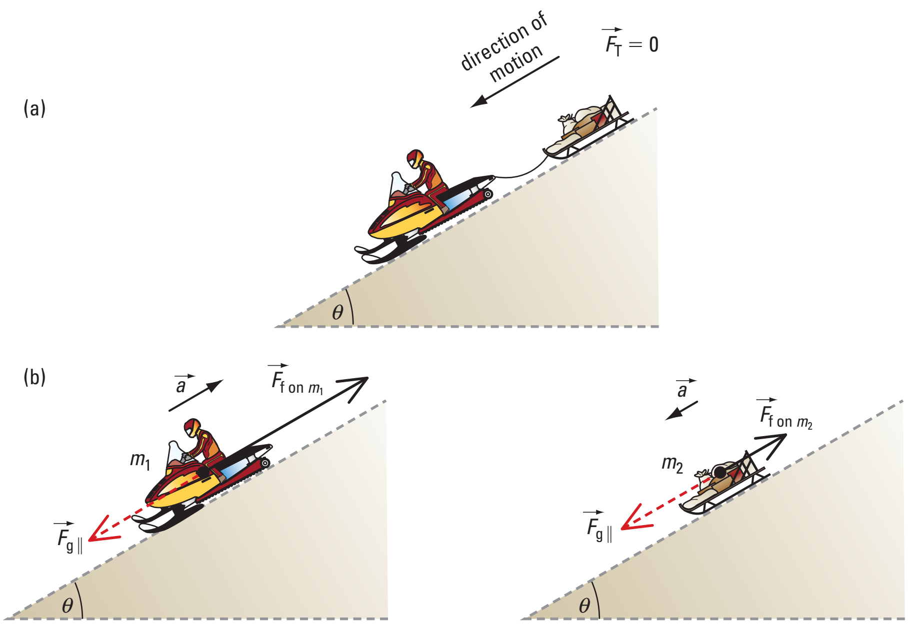
Many safety devices are based on varying both the average net force acting on an object and the interaction time for a given impulse. Suppose you attached a sled to a snowmobile with a rope hitch. As long as the sled is accelerating along a horizontal surface or is being pulled uphill, there is tension in the rope because the snowmobile applies a force on the sled (Figure 9.24).

If the driver in Figure 9.24 (a) brakes suddenly to slow down, the momentum of the snowmobile changes suddenly. However, the sled continues to move in a straight line until friction eventually slows it down to a stop. In other words, the only way that the momentum of the sled changes noticeably is if $\vec{F}_f$ acts for a long enough period of time.




▲ **Figure 9.24** (a) A snowmobile accelerating along a horizontal surface, and (b) the same snowmobile either moving at constant speed or accelerating uphill. In both (a) and (b), the tension in the rope is not zero.

Suppose the snowmobile driver is heading downhill and applies the brakes suddenly as in Figure 9.25 (a). $\vec{F}_{g\parallel}$ will cause the sled to accelerate downhill as shown in Figure 9.25 (b). The speed of the sled could become large enough to overtake the snowmobile, bump into it, or tangle the rope.




▲ **Figure 9.25** (a) The snowmobile is braking rapidly, and the tension in the rope is zero. (b) The free-body diagrams for the snowmobile and sled only show forces parallel to the incline.

The driver can change the momentum of the snowmobile suddenly by using the brakes. But, as before, the only way that the momentum of the sled can eventually become zero is if $\vec{F}_{f}$ acts for a long enough time interval. With experience, a driver learns to slow down gradually so that a towed sled remains in its proper position.

Some sleds are attached to snowmobiles using a metal tow bar, which alleviates this problem (Figure 9.26). Since the tow bar can never become slack like a rope, the sled always remains a fixed distance from the snowmobile.

Tow bars usually have a spring mechanism that increases the time during which a force can be exerted. So if the driver brakes or changes direction suddenly, the force exerted by the snowmobile on the sled acts for a longer period of time. Compared to a towrope, the spring mechanism in the tow bar can safely cause the momentum of the sled to decrease in a shorter period of time.

▶ **Figure 9.26** A rigid tow bar with a spring mechanism provides the impulse necessary to increase or decrease the momentum of a towed sled.

*e* **WEB**

During takeoff, the magnitude of Earth's gravitational field changes as a rocket moves farther away from Earth's surface. The mass of a rocket also changes because it is burning fuel to move upward. Research how impulse and momentum apply to the design and function of rockets and thrust systems. Write a brief report of your findings, including diagrams where appropriate. Begin your search at www.pearsoned.ca/school/physicssource.

Safety devices in vehicles are designed so that, for a given impulse such as in a collision, the interaction time is increased, thereby reducing the average net force. This is achieved by providing motorists with a greater distance to travel, which increases the time interval required to stop the motion of the motorist. Three methods are used to provide this extra distance and time:

- The dashboard is padded and the front end of the vehicle is designed to crumple.
- The steering column telescopes to collapse, providing an additional 15–20 cm of distance for the driver to travel forward.
- The airbag is designed to leak after inflation so that the fully inflated bag can decrease in thickness over time from about 30 cm to about 10 cm.

In fact, an inflated airbag distributes the net force experienced during a collision over the motorist's chest and head. By spreading the force over a greater area, the magnitude of the average net force at any one point on the motorist's body is reduced, lowering the risk of a major injury.

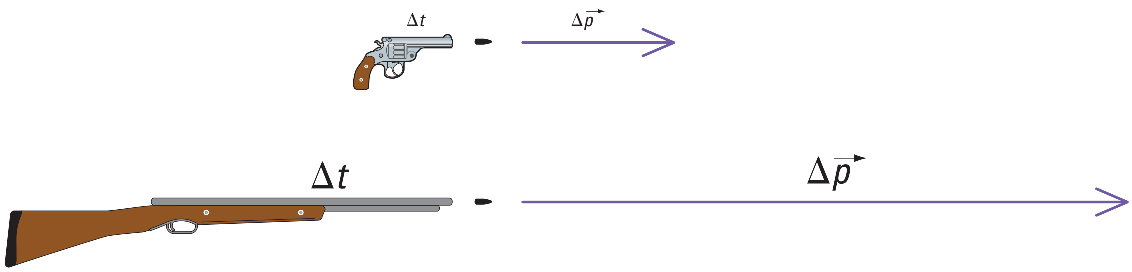
A similar reasoning applies to the cushioning in running shoes and the padding in helmets and body pads used in sports (Figure 9.27). For a given impulse, all these pieces of equipment increase the interaction time and decrease the average net force.

(a)

(b)

▲ **Figure 9.27** Padding in sports equipment helps reduce the risk of major injuries, because for a given impulse, the interaction time is increased and the average net force on the body part is reduced. (a) Team Canada in the World Women's hockey tournament in Sweden, 2005. (b) Calgary Stampeders (in red) playing against the B.C. Lions in 2005.

The effect of varying the average net force and the interaction time can be seen with projectiles. A bullet fired from a pistol with a short barrel does not gain the same momentum as another identical bullet fired from a rifle with a long barrel, assuming that each bullet experiences the same average net force (Figure 9.28). In the gun with the shorter barrel, the force from the expanding gases acts for a shorter period of time. So the change in momentum of the bullet is less.




▲ **Figure 9.28** For the same average net force on a bullet, a gun with a longer barrel increases the time during which this force acts. So the change in momentum is greater for a bullet fired from a long-barrelled gun.

## Improved Sports Performance Involves Varying $F_{\text{net}_{\text{ave}}}$ and $\Delta t$

In baseball, a skilled pitcher knows how to vary both the net force acting on the ball and the interaction time, so that the ball acquires maximum velocity before it leaves the pitcher's hand (Figure 9.29). To exert the maximum possible force on the ball, a pitcher uses his arms, torso, and legs to propel the ball forward. To maximize the time he can exert that force, the pitcher leans back using a windup and then takes a long step forward. This way, his hand can be in contact with the ball for a longer period of time. The combination of the greater net force and the longer interaction time increases the change in momentum of the ball.

▶ **Figure 9.29** When a pitcher exerts a force on the ball during a longer time interval, the momentum of a fastball increases even more.

In sports such as hockey, golf, and tennis, coaches emphasize proper "follow through." The reason is that it increases the time during which the puck or ball is in contact with the player's stick, club, or racquet. So the change in momentum of the object being propelled increases.

A similar reasoning applies when a person catches a ball. In this case, a baseball catcher should decrease the net force on the ball so that the ball doesn't cause injury and is easier to hold onto. Players soon learn to do this by letting their hands move with the ball. For the same impulse, the extra movement with the hands results in an increased interaction time, which reduces the net force.

This intentional flexibility when catching is sometimes referred to as having "soft hands," and it is a great compliment to a football receiver. Hockey goalies allow their glove hand to fly back when snagging a puck to reduce the impact and allow them a better chance of keeping the puck in their glove. Boxers are also taught to "roll with the punch," because if they move backward when hit, it increases the interaction time and decreases the average net force of an opponent's blow.

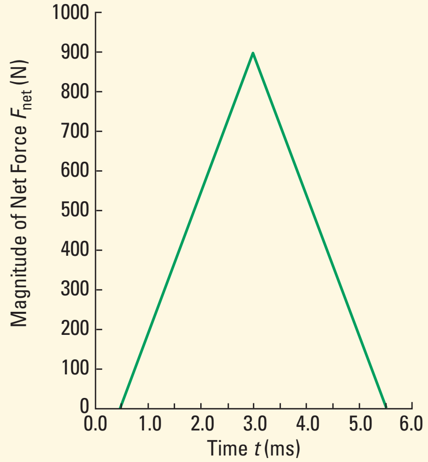
## 9.2 Check and Reflect

### Knowledge

1. (a) What quantities are used to calculate impulse?
   (b) State the units of impulse.
2. How are impulse and momentum related?
3. What graph could you use to determine the impulse provided to an object? Explain how to calculate the impulse using the graph.
4. What is the effect on impulse if
   (a) the time interval is doubled?
   (b) the net force is reduced to $\frac{1}{3}$ of its original magnitude?
5. Even though your mass is much greater than that of a curling stone, it is dangerous for a moving stone to hit your feet. Explain why.

### Applications

6. Using the concept of impulse, explain how a karate expert can break a board.
7. (a) From the graph below, what is the magnitude of the impulse provided to a 48-g tennis ball that is served due south?
   (b) What is the velocity of the ball when the racquet and ball separate?




8. What will be the magnitude of the impulse generated by a slapshot when an average net force of magnitude 520 N is applied to a puck for 0.012 s?
9. During competitive world-class events, a four-person bobsled experiences an average net force of magnitude 1390 N during the first 5.0 s of a run.
   (a) What will be the magnitude of the impulse provided to the bobsled?
   (b) If the sled has the maximum mass of 630 kg, what will be the speed of the sled?
10. An advertisement for a battery-powered 25-kg skateboard says that it can carry an 80-kg person at a speed of 8.5 m/s. If the skateboard motor can exert a net force of magnitude 75 N, how long will it take to attain that speed?
11. Whiplash occurs when a car is rear-ended and either there is no headrest or the headrest is not properly adjusted. The torso of the motorist is accelerated by the seat, but the head is jerked forward only by the neck, causing injury to the joints and soft tissue. What is the average net force on a motorist's neck if the torso is accelerated from 0 to 14.0 m/s [W] in 0.135 s? The mass of the motorist's head is 5.40 kg. Assume that the force acting on the head is the same magnitude as the force on the torso.
12. What will be the change in momentum of a shoulder-launched rocket that experiences a thrust of 2.67 kN [W] for 0.204 s?

### Extensions

13. Experienced curlers know how to safely stop a moving stone. What do they do and why?
14. Research one safety device used in sports that applies the concept of varying $F_{net_{ave}}$ and $\Delta t$ for a given impulse to prevent injury. Explain how the variables that affect impulse are changed by using this device. Begin your search at www.pearsoned.ca/school/physicssource.

**e TEST**

To check your understanding of impulse, follow the eTest links at www.pearsoned.ca/school/physicssource.

# 9.3 Collisions in One Dimension

*info* **BIT**

The horns of a bighorn ram can account for more than 10% of its mass, which is about 125 kg. Rams collide at about 9 m/s, and average about 5 collisions per hour. Mating contests between any two rams may last for more than 24 h in total.

During mating season each fall, adult bighorn rams compete for supremacy in an interesting contest. Two rams will face each other, rear up, and then charge, leaping into the air to butt heads with tremendous force (Figure 9.30). Without being consciously aware of it, each ram attempts to achieve maximum momentum before the collision, because herd structure is determined by the outcome of the contest. Often, rams will repeat the head-butting interaction until a clear winner is determined. While most other mammals would be permanently injured by the force experienced during such a collision, the skull and brain structure of bighorn sheep enables them to emerge relatively undamaged from such interactions.

In the previous section, many situations involved an object experiencing a change in momentum, or impulse, because of a collision with another object. When two objects such as bighorn rams collide, what relationship exists among the momenta of the objects both before and after collision? In order to answer this question, first consider one-dimensional collisions involving spheres in 9-4 QuickLab.

▲ **Figure 9.30** By lunging toward each other, these bighorn rams will eventually collide head-on. During the collision, each ram will be provided with an impulse.

**9-4 QuickLab**

# Observing Collinear Collisions

## Problem

What happens when spheres collide in one dimension?

## Materials

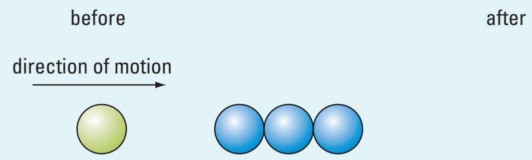
one set of four identical ball bearings or marbles (set A)
a second set of four identical ball bearings or marbles of double the mass (set B)
a third set of four identical ball bearings or marbles of half the mass (set C)
1-m length of an I-beam curtain rod or two metre-sticks with smooth edges
masking tape

## Procedure

1. Lay the curtain rod flat on a bench to provide a horizontal track for the spheres. Tape the ends of the rod securely. If you are using metre-sticks, tape them 5 mm apart to form a uniform straight horizontal track.

2. Using set A, place three of the spheres tightly together at the centre of the track.

3. Predict what will happen when one sphere of set A moves along the track and collides with the three stationary spheres.




▲ **Figure 9.31**

4. Test your prediction. Ensure that the spheres remain on the track after collision. Record your observations using diagrams similar to Figure 9.31.

5. Repeat steps 2 to 4, but this time use set B, spheres of greater mass.

6. Repeat steps 2 to 4, but this time use set C, spheres of lesser mass.

7. Repeat steps 2 to 4 using different numbers of stationary spheres. The stationary spheres should all be the same mass, but the moving sphere should be of a different mass in some of the trials.

## Questions

**1.** Describe the motion of the spheres in steps 4 to 6.

**2.** Explain what happened when
   (a) a sphere of lesser mass collided with a number of spheres of greater mass, and
   (b) a sphere of greater mass collided with a number of spheres of lesser mass.

In 9-4 QuickLab, for each set of spheres A to C, when one sphere hit a row of three stationary ones from the same set, the last sphere in the row moved outward at about the same speed as the incoming sphere. But when one sphere from set A hit a row of spheres from set B, the last sphere in the row moved outward at a much *slower* speed than the incoming sphere, and the incoming sphere may even have rebounded. When one sphere from set A hit a row of spheres from set C, the last sphere in the row moved outward at a *greater* speed than the incoming sphere, and the incoming sphere continued moving forward.

To analyze these observations, it is important to first understand what a collision is. A **collision** is an interaction between two objects in which a force acts on each object for a period of time. In other words, the collision provides an impulse to each object.

**eMATH**

Explore how the masses of two colliding objects affect their velocities just after collision. Follow the eMath links at www.pearsoned.ca/school/physicssource.

**collision:** an interaction between two objects where each receives an impulse

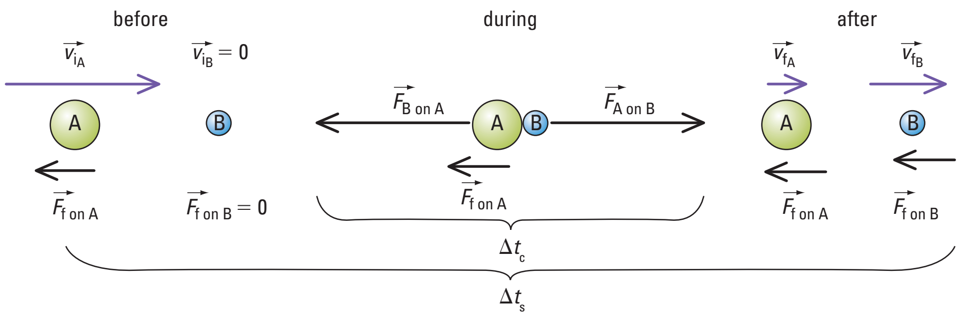
## Systems of Objects in Collisions

**system:** two or more objects that interact with each other

Each trial in 9-4 QuickLab involved two or more spheres colliding with each other. A group of two or more objects that interact is called a **system**. You encountered the concept of a system in Unit III in the context of energy.

For each system in 9-4 QuickLab, the total mass remained constant because the mass of each sphere did not change as a result of the interaction. However, friction was an external force that acted on the system. For example, in steps 4 to 6 of 9-4 QuickLab, you likely observed that the speed of the sphere moving outward was a little less than the speed of the incoming sphere.

In real life, a system of colliding objects is provided with two impulses: one due to external friction and the other due to the actual collision (Figure 9.32). External friction acts before, during, and after collision. The second impulse is only present *during the actual collision*. Since the actual collision time is very short, the impulse due to external friction *during* the collision is relatively small.




▲ **Figure 9.32** External friction acts throughout the entire time interval of the interaction $\Delta t_s$. But the action-reaction forces due to the objects only exist when the objects actually collide, and these forces only act for time interval $\Delta t_c$.

If you apply the form of Newton's second law that relates net force to momentum to analyze the motion of a system of objects, you get

$$(\vec{F}_{net})_{sys} = \frac{\Delta \vec{p}_{sys}}{\Delta t} \quad \text{where } \vec{p}_{sys} \text{ is the momentum of the system}$$

The **momentum of a system** is defined as the sum of the momenta of all the objects in the system. So if objects A, B, and C form a system, the momentum of the system is

$$\vec{p}_{sys} = \vec{p}_A + \vec{p}_B + \vec{p}_C$$

In the context of momentum, when the mass of a system is constant and no external net force acts on the system, the system is **isolated**. So $(\vec{F}_{net})_{sys} = 0$. In 9-5 Inquiry Lab, a nearly isolated system of objects is involved in a one-dimensional collision. Find a quantitative relationship for the momentum of such a system in terms of momenta before and after collision.

## 9-5 Inquiry Lab

# Relating Momentum Just Before and Just After a Collision

**Required Skills**

- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

### Question

How does the momentum of a system consisting of two objects compare just before and just after a collision?

### Hypothesis

State a hypothesis relating the momentum of a system immediately before and immediately after collision, where objects combine after impact. Remember to write an “if/then” statement.

### Variables

Read the procedure carefully and identify the manipulated variables, the responding variables, and the controlled variables.

### Materials and Equipment

one of these set-ups: air track, dynamics carts, Fletcher’s trolley, bead table or air table with linear guides
colliding objects for the set-up chosen: gliders, carts, discs, blocks, etc.
objects of different mass
fastening material (Velcro™ strips, tape, Plasticine™, magnets, etc.)
balance
timing device (stopwatch, spark-timer, ticker-tape timer, electronic speed-timing device, or time-lapse camera)
metre-sticks

### Procedure

1. Copy Tables 9.1 and 9.2 on page 472 into your notebook.
2. Set up the equipment in such a way that friction is minimized and the two colliding objects travel in the same straight line.
3. Attach some fastening material to the colliding objects, so that the two objects remain together after impact.
4. Measure and record the masses of the two objects. If necessary, change the mass of one object so that the two objects have significantly different masses.
5. Set up the timing device to measure the velocity of object 1 just before and just after collision. Object 2 will be stationary before collision. The velocities of both objects will be the same after collision because they will stick together.
6. Send object 1 at a moderate speed on a collision course with object 2. Ensure that both objects will stick together and that the timing device is working properly. Make adjustments if needed.
7. Send object 1 at a moderate speed on a collision course with stationary object 2, recording the relevant observations and the masses as trial 1.
8. Send object 1 at a different speed on a collision course with stationary object 2, recording the relevant observations and the masses as trial 2.
9. Change the mass of one of the objects and again send object 1 at a moderate speed on a collision course with stationary object 2, recording the relevant observations and the masses as trial 3.
10. If you can simultaneously measure the speed of two objects, run trials where both objects are in motion before the collision. Do one trial in which they begin moving toward each other and stick together upon impact, and another trial where they move apart after impact. If you remove the fastening material, you will have to remeasure the masses of the objects. Include the direction of motion for both objects before and after collision.
11. If you did not do step 10, do two more trials, changing the mass of one of the objects each time. Include the direction of motion for both objects before and after collision.

**eLAB**

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

## Analysis

1. Determine the velocities for each colliding object in each trial, and record them in your data table. Show your calculations.
2. For each trial, calculate the momentum of each object just before and just after collision. Show your calculations. Record the values in your data table.
3. Calculate the momentum of the system just before and just after collision for each trial. Show your calculations. Record the values in your data table.
4. Calculate the difference between the momentum of the system just before and just after collision. Show your calculations. Record the values in your data table.
5. What is the relationship between the momentum of the system just before and just after collision? Does this relationship agree with your hypothesis?
6. What effect did friction have on your results? Explain.
7. Check your results with other groups. Account for any discrepancies.

**▼ Table 9.1** Mass and Velocity

| | **Before and After for Object 1** | | | **Before and After for Object 2** | | |
|---|---|---|---|---|---|---|
| **Trial** | **Mass $m_1$ (g)** | **Initial Velocity $\vec{v}_{1_i}$ (m/s)** | **Final Velocity $\vec{v}_{1_f}$ (m/s)** | **Mass $m_2$ (g)** | **Initial Velocity $\vec{v}_{2_i}$ (m/s)** | **Final Velocity $\vec{v}_{2_f}$ (m/s)** |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |

**▼ Table 9.2** Momentum

| | **Before and After for Object 1** | | **Before and After for Object 2** | | **Before and After for System** | | |
|---|---|---|---|---|---|---|---|
| **Trial** | **Initial Momentum $\vec{p}_{1_i}\left(g \cdot \frac{m}{s}\right)$** | **Final Momentum $\vec{p}_{1_f}\left(g \cdot \frac{m}{s}\right)$** | **Initial Momentum $\vec{p}_{2_i}\left(g \cdot \frac{m}{s}\right)$** | **Final Momentum $\vec{p}_{2_f}\left(g \cdot \frac{m}{s}\right)$** | **Initial Momentum of System $\vec{p}_{sys_i}\left(g \cdot \frac{m}{s}\right)$** | **Final Momentum of System $\vec{p}_{sys_f}\left(g \cdot \frac{m}{s}\right)$** | **Change in Momentum of System $\Delta\vec{p}_{sys}\left(g \cdot \frac{m}{s}\right)$** |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |

## Momentum Is Conserved in One-dimensional Collisions

In 9-5 Inquiry Lab, you discovered that, in one-dimensional collisions, the momentum of a system immediately before collision is about the same as the momentum of the system immediately after collision.

If the external force of friction acting on the system is negligible, the momentum of the system is constant. This result is true no matter how many objects are in the system, how many of those objects collide, how massive the objects are, or how fast they are moving.

The general form of Newton's second law for a system is

$$(\vec{F}_{\text{net}})_{\text{sys}} = \frac{\Delta \vec{p}_{\text{sys}}}{\Delta t}$$

In an isolated system, the external net force on the system is zero, $(\vec{F}_{\text{net}})_{\text{sys}} = 0$. So

$$0 = \frac{\Delta \vec{p}_{\text{sys}}}{\Delta t}$$

In order for $\frac{\Delta \vec{p}_{\text{sys}}}{\Delta t}$ to be zero, the change in momentum of the system must be zero.

$$\Delta \vec{p}_{\text{sys}} = 0$$

$$\vec{p}_{\text{sys}_\text{f}} - \vec{p}_{\text{sys}_\text{i}} = 0$$

$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{sys}_\text{f}}$$

In other words, $\vec{p}_{\text{sys}} = \text{constant}$.

This is a statement of the **law of conservation of momentum**. In Unit III, you encountered another conservation law, that is, in an isolated system the total energy of the system is conserved. Conservation laws always have one quantity that remains unchanged. In the law of conservation of momentum, it is momentum that remains unchanged.

**law of conservation of momentum:** momentum of an isolated system is constant

When no external net force acts on a system, the momentum of the system remains constant.

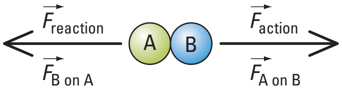
$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{sys}_\text{f}} \text{ where } (\vec{F}_{\text{net}})_{\text{sys}} = 0$$

### *Concept Check*

Why did cannons on 16th- to 19th-century warships need a rope around the back, tying them to the side of the ship (Figure 9.33)?

◀ **Figure 9.33**

## Writing the Conservation of Momentum in Terms of Mass and Velocity




▲ **Figure 9.34**
The action-reaction forces when two objects collide

Suppose a system consists of two objects, A and B. If the system is isolated, $(\vec{F}_{\text{net}})_{\text{sys}} = 0$. Consider the internal forces of the system. At collision time, object A exerts a force on object B and object B exerts a force on object A (Figure 9.34). From Newton's third law, these action-reaction forces are related by the equation

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

Objects A and B interact for the same time interval $\Delta t$. If you multiply both sides of the equation by $\Delta t$, you get an equation in terms of impulse:

$$\vec{F}_{\text{A on B}}\,\Delta t = -\vec{F}_{\text{B on A}}\,\Delta t$$

Since impulse is equivalent to a change in momentum, the equation can be rewritten in terms of the momenta of each object:

$$\Delta\vec{p}_{\text{B}} = -\Delta\vec{p}_{\text{A}}$$

$$\Delta\vec{p}_{\text{A}} + \Delta\vec{p}_{\text{B}} = 0$$

$$\vec{p}_{\text{A}_\text{f}} - \vec{p}_{\text{A}_\text{i}} + \vec{p}_{\text{B}_\text{f}} - \vec{p}_{\text{B}_\text{i}} = 0$$

$$\vec{p}_{\text{A}_\text{i}} + \vec{p}_{\text{B}_\text{i}} = \vec{p}_{\text{A}_\text{f}} + \vec{p}_{\text{B}_\text{f}}$$

If the mass of each object is constant during the interaction, the equation can be written in terms of $m$ and $\vec{v}$:

$$m_\text{A}\vec{v}_{\text{A}_\text{i}} + m_\text{B}\vec{v}_{\text{B}_\text{i}} = m_\text{A}\vec{v}_{\text{A}_\text{f}} + m_\text{B}\vec{v}_{\text{B}_\text{f}}$$

This equation is the law of conservation of momentum in terms of the momenta of objects A and B. So if two bighorn rams head-butt each other, the sum of the momenta of both rams is constant during the collision, even though the momentum of each ram changes. The law of conservation of momentum has no known exceptions, and holds even when particles are travelling close to the speed of light, or when the mass of the colliding particles is very small, as in the case of electrons.

**eSIM**

Learn how the momentum of a system just before and just after a one-dimensional collision are related. Vary the ratio of the mass of two pucks. Follow the eSim links at www.pearsoned.ca/school/physicssource.

In real life, when objects collide, external friction acts on nearly all systems and the instantaneous forces acting on each object are usually not known (Figure 9.35). Often, the details of the interaction are also unknown.

However, you do not require such information to apply the law of conservation of momentum. Instead, it is the mass and instantaneous velocity of the objects immediately before and immediately after collision that are important, so that the effects of external friction are minimal, and do not significantly affect the outcome.

▲ **Figure 9.35** During a vehicle collision, many forces cause a change in the velocity and shape of each vehicle.

## Conservation of Momentum Applied to Rockets

In Unit II, the motion of a rocket was explained using Newton's third law. However, the conservation of momentum can be used to explain why a rocket can accelerate even in a vacuum. When the engines of a rocket burn fuel, the escaping exhaust gas has mass and considerable speed.

When a rocket is in outer space, external friction is negligible. So the rocket-exhaust gas system is an isolated system. For a two-object system, the equation for the conservation of momentum is

$$\Delta\vec{p}_{\text{rocket}} + \Delta\vec{p}_{\text{gas}} = 0$$

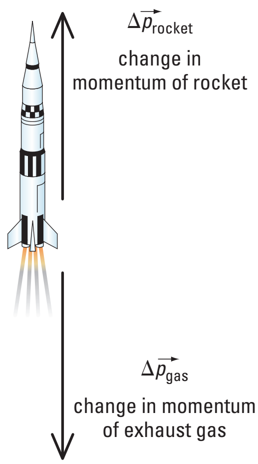
$$\Delta\vec{p}_{\text{rocket}} = -\Delta\vec{p}_{\text{gas}}$$

where, during time interval $\Delta t$, $\Delta\vec{p}_{\text{rocket}}$ is the change in momentum of the rocket including any unspent fuel and $\Delta\vec{p}_{\text{gas}}$ is the change in momentum of the fuel that is expelled in the form of exhaust gas. It is the change in momentum of the exhaust gas that enables a rocket to accelerate (Figure 9.36). In the case of a very large rocket, such as a Saturn V, the magnitude of $\Delta\vec{p}_{\text{gas}}$ would be very large (Figure 9.37).

**info BIT**

None of the 32 Saturn rockets that were launched ever failed. Altogether 15 Saturn V rockets were built. Three Saturn V rockets are on display, one at each of these locations: the Johnson Space Center, the Kennedy Space Center, and the Alabama Space and Rocket Center. Of these three, only the rocket at the Johnson Space Center is made up entirely of former flight-ready, although mismatched, parts.

▲ **Figure 9.36** With a height of about 112 m, the Saturn V rocket was the largest and most powerful rocket ever built.




▲ **Figure 9.37** From the law of conservation of momentum, the magnitude of $\Delta\vec{p}_{\text{gas}}$ is equal to the magnitude of $\Delta\vec{p}_{\text{rocket}}$. That is why a rocket can accelerate on Earth or in outer space.

**info BIT**

Design of the Saturn V began in the 1950s with the intent to send astronauts to the Moon. In the early 1970s, this type of rocket was used to launch the Skylab space station. The rocket engines in the first stage burned a combination of kerosene and liquid oxygen, producing a total thrust of magnitude $3.34 \times 10^{7}$ N. The rocket engines in the second and third stages burned a combination of liquid hydrogen and liquid oxygen. The magnitude of the total thrust produced by the second-stage engines was $5.56 \times 10^{6}$ N, and the third-stage engine produced $1.11 \times 10^{6}$ N of thrust.

### Concept Check

(a) Refer to the second infoBIT on this page. Why is less thrust needed by the second-stage engines of a rocket?
(b) Why is even less thrust needed by the third-stage engine?

In Example 9.5 on the next page, the conservation of momentum is applied to a system of objects that are initially stationary. This type of interaction is called an explosion.

## Example 9.5

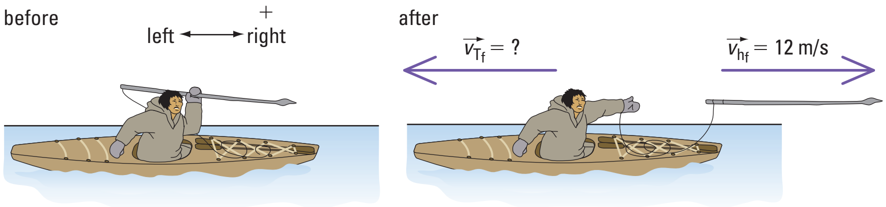
A 75-kg hunter in a stationary kayak throws a 0.72-kg harpoon at 12 m/s [right]. The mass of the kayak is 10 kg. What will be the velocity of the kayak and hunter immediately after the harpoon is released?

*Given*

$m_p = 75$ kg  $m_k = 10$ kg  $m_h = 0.72$ kg

$\vec{v}_{p_i} = 0$ m/s  $\vec{v}_{k_i} = 0$ m/s  $\vec{v}_{h_i} = 0$ m/s

$\vec{v}_{h_f} = 12$ m/s [right]




▲ **Figure 9.38**

*Required*

final velocity of hunter and kayak

*Analysis and Solution*

Choose the kayak, hunter, and harpoon as an isolated system. The hunter and kayak move together as a unit after the harpoon is released. So find the total mass of the hunter and kayak.

$$\begin{aligned} m_T &= m_p + m_k \\ &= 75 \text{ kg} + 10 \text{ kg} \\ &= 85 \text{ kg} \end{aligned}$$

The hunter, kayak, and harpoon each have an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

Apply the law of conservation of momentum.

$$\begin{aligned} \vec{p}_{sys_i} &= \vec{p}_{sys_f} \\ \vec{p}_{sys_i} &= \vec{p}_{T_f} + \vec{p}_{h_f} \\ 0 &= m_T\vec{v}_{T_f} + m_h\vec{v}_{h_f} \\ \vec{v}_{T_f} &= -\left(\frac{m_h}{m_T}\right)\vec{v}_{h_f} \\ &= -\left(\frac{0.72 \cancel{\text{ kg}}}{85 \cancel{\text{ kg}}}\right)(+12 \text{ m/s}) \\ &= -0.10 \text{ m/s} \\ \vec{v}_{T_f} &= 0.10 \text{ m/s [left]} \end{aligned}$$

*Paraphrase and Verify*

The velocity of the kayak and hunter will be 0.10 m/s [left] immediately after the harpoon is released. Since the harpoon is thrown right, from Newton's third law, you would expect the hunter and kayak to move left after the throw. So the answer is reasonable.

## Practice Problems

1. A 110-kg astronaut and a 4000-kg spacecraft are attached by a tethering cable. Both masses are motionless relative to an observer a slight distance away from the spacecraft. The astronaut wants to return to the spacecraft, so he pulls on the cable until his velocity changes to 0.80 m/s [toward the spacecraft] relative to the observer. What will be the change in velocity of the spacecraft?
2. A student is standing on a stationary 2.3-kg skateboard. If the student jumps at a velocity of 0.37 m/s [forward], the velocity of the skateboard becomes 8.9 m/s [backward]. What is the mass of the student?

### Answers

1. 0.022 m/s [toward the astronaut]
2. 55 kg

In Example 9.6, a dart is fired at a stationary block sitting on a glider. This situation involves two objects (dart and block) that join together and move as a unit after interaction. This type of interaction is called a hit-and-stick interaction.

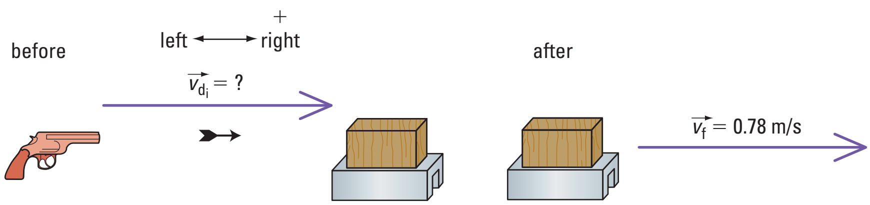
## Example 9.6

A wooden block attached to a glider has a combined mass of 0.200 kg. Both the block and glider are at rest on a frictionless air track. A dart gun shoots a 0.012-kg dart into the block. The velocity of the block-dart system after collision is 0.78 m/s [right]. What was the velocity of the dart just before it hit the block?

***Given***

$m_{\mathrm{b}} = 0.200 \text{ kg}$ $\quad$ $m_{\mathrm{d}} = 0.012 \text{ kg}$

$\vec{v}_{\mathrm{b_i}} = 0 \text{ m/s}$ $\quad$ $\vec{v}_{\mathrm{f}} = 0.78 \text{ m/s [right]}$




▲ **Figure 9.39**

***Required***

initial velocity of dart ($\vec{v}_{\mathrm{d_i}}$)

***Analysis and Solution***

Choose the block, glider, and dart as an isolated system. The dart, block, and glider move together as a unit after collision. The block-glider unit has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{\mathrm{b_i}} = 0$$

Apply the law of conservation of momentum.

$$\begin{aligned}
\vec{p}_{\mathrm{sys_i}} &= \vec{p}_{\mathrm{sys_f}} \\
\vec{p}_{\mathrm{b_i}} + \vec{p}_{\mathrm{d_i}} &= \vec{p}_{\mathrm{sys_f}} \\
0 + m_{\mathrm{d}}\vec{v}_{\mathrm{d_i}} &= (m_{\mathrm{b}} + m_{\mathrm{d}})\vec{v}_{\mathrm{f}} \\
\vec{v}_{\mathrm{d_i}} &= \left(\frac{m_{\mathrm{b}} + m_{\mathrm{d}}}{m_{\mathrm{d}}}\right)\vec{v}_{\mathrm{f}} \\
&= \left(\frac{0.200 \text{ kg} + 0.012 \text{ kg}}{0.012 \text{ kg}}\right)(+0.78 \text{ m/s}) \\
&= \left(\frac{0.212 \cancel{\text{kg}}}{0.012 \cancel{\text{kg}}}\right)(+0.78 \text{ m/s}) \\
&= +14 \text{ m/s} \\
\vec{v}_{\mathrm{d_i}} &= 14 \text{ m/s [right]}
\end{aligned}$$

***Paraphrase***

The dart had a velocity of 14 m/s [right] just before it hit the block.

### Practice Problems

1. A student on a skateboard, with a combined mass of 78.2 kg, is moving east at 1.60 m/s. As he goes by, the student skilfully scoops his 6.4-kg backpack from the bench where he had left it. What will be the velocity of the student immediately after the pickup?
2. A 1050-kg car at an intersection has a velocity of 2.65 m/s [N]. The car hits the rear of a stationary truck, and their bumpers lock together. The velocity of the car-truck system immediately after collision is 0.78 m/s [N]. What is the mass of the truck?

**Answers**

1. 1.5 m/s [E]
2. $2.5 \times 10^3$ kg

Example 9.7 involves a basketball player, initially moving with some velocity, colliding with a stationary player. After the interaction, both players move in different directions.

## Example 9.7

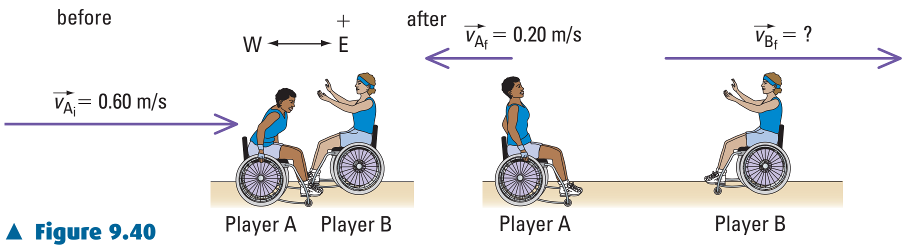
A basketball player and her wheelchair (player A) have a combined mass of 58 kg. She moves at 0.60 m/s [E] and pushes off a stationary player (player B) while jockeying for a position near the basket. Player A ends up moving at 0.20 m/s [W]. The combined mass of player B and her wheelchair is 85 kg. What will be player B's velocity immediately after the interaction?

***Given***

$m_A = 58$ kg  $m_B = 85$ kg

$\vec{v}_{A_i} = 0.60$ m/s [E]  $\vec{v}_{B_i} = 0$ m/s

$\vec{v}_{A_f} = 0.20$ m/s [W]




▲ **Figure 9.40**

***Required***

final velocity of player B ($\vec{v}_{B_f}$)

***Analysis and Solution***

Choose players A and B as an isolated system.
Player B has an initial velocity of zero. So her initial momentum is zero.

$$\vec{p}_{B_i} = 0$$

Apply the law of conservation of momentum.

$$\begin{aligned}
\vec{p}_{sys_i} &= \vec{p}_{sys_f} \\
\vec{p}_{A_i} + \vec{p}_{B_i} &= \vec{p}_{A_f} + \vec{p}_{B_f} \\
m_A\vec{v}_{A_i} + 0 &= m_A\vec{v}_{A_f} + m_B\vec{v}_{B_f} \\
\vec{v}_{B_f} &= \left(\frac{m_A}{m_B}\right)(\vec{v}_{A_i} - \vec{v}_{A_f}) \\
&= \left(\frac{58\ \cancel{\text{kg}}}{85\ \cancel{\text{kg}}}\right)[+0.60\text{ m/s} - (-0.20\text{ m/s})] \\
&= \left(\frac{58}{85}\right)(0.60\text{ m/s} + 0.20\text{ m/s}) \\
&= +0.55\text{ m/s} \\
\vec{v}_{B_f} &= 0.55\text{ m/s [E]}
\end{aligned}$$

***Paraphrase***

Player B's velocity is 0.55 m/s [E] just after collision.

## Practice Problems

1. A 0.25-kg volleyball is flying west at 2.0 m/s when it strikes a stationary 0.58-kg basketball dead centre. The volleyball rebounds east at 0.79 m/s. What will be the velocity of the basketball immediately after impact?
2. A 9500-kg rail flatcar moving forward at 0.70 m/s strikes a stationary 18 000-kg boxcar, causing it to move forward at 0.42 m/s. What will be the velocity of the flatcar immediately after collision if they fail to connect?

### Answers

1. 1.2 m/s [W]
2. 0.096 m/s [backward]

In Example 9.8, two football players in motion collide with each other. After the interaction, the players bounce apart.

## Example 9.8

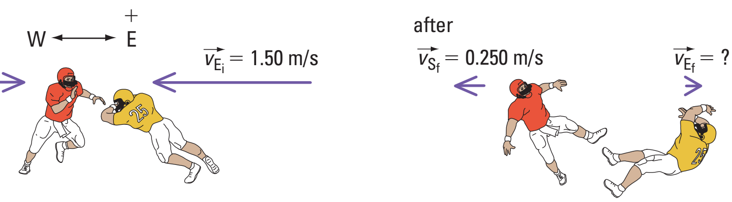
A 110-kg Stampeders football fullback moving east at 1.80 m/s on a snowy playing field is struck by a 140-kg Eskimos defensive lineman moving west at 1.50 m/s. The fullback is bounced west at 0.250 m/s. What will be the velocity of the Eskimos defensive lineman just after impact?

*Given*

$m_S = 110$ kg $\qquad m_E = 140$ kg

$\vec{v}_{S_i} = 1.80$ m/s [E] $\qquad \vec{v}_{E_i} = 1.50$ m/s [W]

$\vec{v}_{S_f} = 0.250$ m/s [W]




▲ **Figure 9.41**

*Required*

final velocity of Eskimos lineman ($\vec{v}_{E_f}$)

*Analysis and Solution*

Choose the fullback and lineman as an isolated system.
Apply the law of conservation of momentum.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{S_i} + \vec{p}_{E_i} = \vec{p}_{S_f} + \vec{p}_{E_f}$$

$$m_S\vec{v}_{S_i} + m_E\vec{v}_{E_i} = m_S\vec{v}_{S_f} + m_E\vec{v}_{E_f}$$

$$m_E\vec{v}_{E_f} = m_S\vec{v}_{S_i} + m_E\vec{v}_{E_i} - m_S\vec{v}_{S_f}$$

$$\begin{aligned}
\vec{v}_{E_f} &= \left(\frac{m_S}{m_E}\right)\vec{v}_{S_i} + \vec{v}_{E_i} - \left(\frac{m_S}{m_E}\right)\vec{v}_{S_f} \\
&= \left(\frac{m_S}{m_E}\right)(\vec{v}_{S_i} - \vec{v}_{S_f}) + \vec{v}_{E_i} \\
&= \left(\frac{110\ \cancel{\text{kg}}}{140\ \cancel{\text{kg}}}\right)[(+1.80\text{ m/s}) \\
&\quad - (-0.250\text{ m/s})] + (-1.50\text{ m/s}) \\
&= \left(\frac{110}{140}\right)(1.80\text{ m/s} + 0.250\text{ m/s}) \\
&\quad - 1.50\text{ m/s} \\
&= +0.111\text{ m/s} \\
\vec{v}_{E_f} &= 0.111\text{ m/s [E]}
\end{aligned}$$

*Paraphrase*

The velocity of the Eskimos defensive lineman immediately after impact is 0.111 m/s [E].

### Practice Problems

1. A 72-kg snowboarder gliding at 1.6 m/s [E] bounces west at 0.84 m/s immediately after colliding with an 87-kg skier travelling at 1.4 m/s [W]. What will be the velocity of the skier just after impact?
2. A 125-kg bighorn ram butts heads with a younger 122-kg ram during mating season. The older ram is rushing north at 8.50 m/s immediately before collision, and bounces back at 0.11 m/s [S]. If the younger ram moves at 0.22 m/s [N] immediately after collision, what was its velocity just before impact?

#### Answers

1. 0.62 m/s [E]
2. 8.6 m/s [S]

## Elastic and Inelastic Collisions in One Dimension

In Examples 9.3 to 9.8, some of the collisions involved hard objects, such as the golf club hitting the golf ball. Other collisions, such as the block and dart, involved a dart that became embedded in a softer material (a block of wood). In all these collisions, it was possible to choose an isolated system so that the total momentum of the system was conserved.

When objects collide, they sometimes deform, make a sound, give off light, or heat up a little at the moment of impact. Any of these observations indicate that the kinetic energy of the system before collision is not the same as after collision. However, the total energy of the system is constant.

**PHYSICS INSIGHT**

The law of conservation of energy states that the *total* energy of an isolated system remains constant. The energy may change into several different forms. This law has no known exceptions.

**Concept Check**

(a) Is it possible for an object to have energy and no momentum? Explain, using an example.
(b) Is it possible for an object to have momentum and no energy? Explain, using an example.

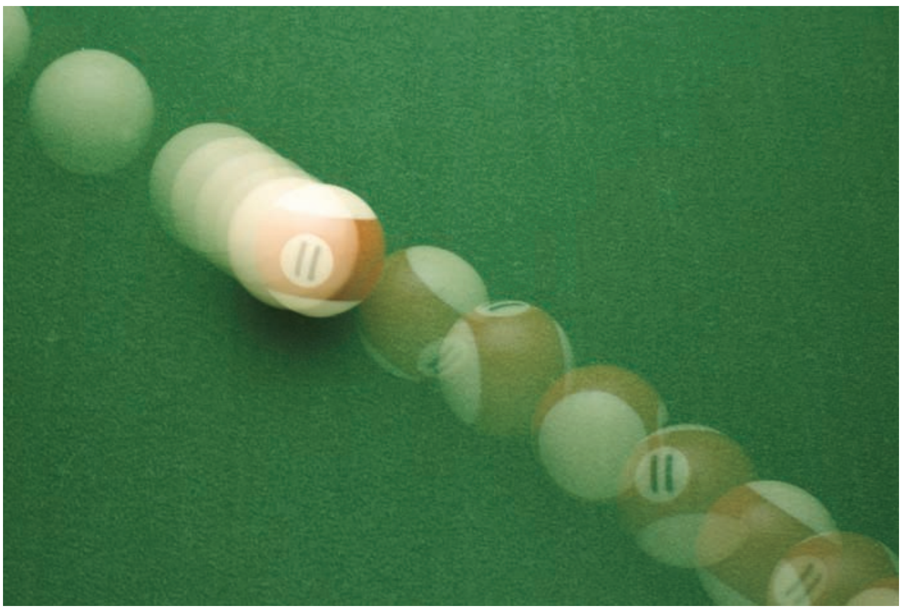
### Elastic Collisions

Suppose you hit a stationary pool ball dead centre with another pool ball so that the collision is collinear and the balls move without spinning immediately after impact. What will be the resulting motion of both balls (Figure 9.42)?

The ball that was initially moving will become stationary upon impact, while the other ball will start moving in the same direction as the incoming ball. If you measure the speed of both balls just before and just after collision, you will find that the speed of the incoming ball is almost the same as that of the outgoing ball. Since $E_k = \frac{1}{2}mv^2$, the final kinetic energy of the system is *almost* the same as the initial kinetic energy of the system.

**eSIM**

Predict the speed of two pucks just after a one-dimensional collision using momentum and energy concepts. Follow the eSim links at www.pearsoned.ca/school/physicssource.



▶ **Figure 9.42** Many collisions take place during a game of pool. What evidence suggests that momentum is conserved during the collision shown in the photo? What evidence suggests that energy is conserved?

If the initial kinetic energy of a system is equal to the final kinetic energy of the system after collision, the collision is **elastic**.

**elastic collision:** a collision in which $E_{k_i} = E_{k_f}$

In an elastic collision, the total kinetic energy of the system is conserved.

$$E_{k_i} = E_{k_f}$$

Most macroscopic interactions in the real world involve some of the initial kinetic energy of the system being converted to sound, light, or deformation (Figure 9.43). When deformation occurs, some of the initial kinetic energy of the system is converted to heat because friction acts on objects in almost all situations. These factors make it difficult to achieve an elastic collision.

Even if two colliding objects are hard and do not appear to deform, energy is still lost in the form of sound, light, and/or heat due to friction. Usually, the measured speed of an object after interaction is a little less than the predicted speed, which indicates that the collision is inelastic.

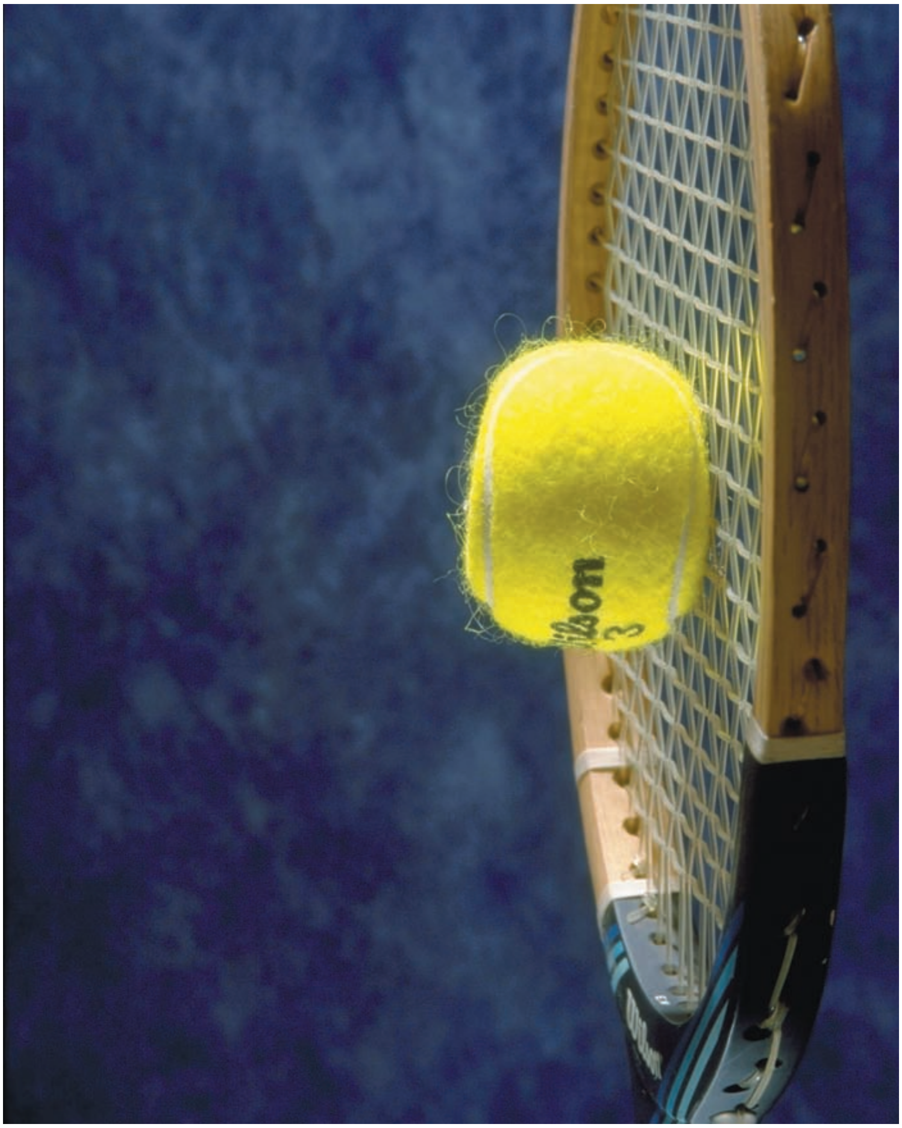
Example 9.9 demonstrates how to determine if the collision between a billiard ball and a snooker ball is elastic.

**Project LINK**

How will you apply the concepts of conservation of momentum and conservation of energy to the design of the water balloon protection?

**info BIT**

A steel sphere will bounce as high on a steel anvil as a rubber ball will on concrete. However, when a steel sphere is dropped on linoleum or hardwood, even more kinetic energy is lost and the sphere hardly bounces at all. The kinetic energy of the sphere is converted to sound, heat, and the deformation of the floor surface. To try this, use flooring samples. Do not try this on floors at home or at school.




◀ **Figure 9.43** Is the collision shown in this photo elastic? What evidence do you have to support your answer?

## Example 9.9

A 0.160-kg billiard ball travelling at 0.500 m/s [N] strikes a stationary 0.180-kg snooker ball and rebounds at 0.0230 m/s [S]. The snooker ball moves off at 0.465 m/s [N]. Ignore possible rotational effects. Determine if the collision is elastic.

***Given***

$m_b = 0.160$ kg

$m_s = 0.180$ kg

$\vec{v}_{b_i} = 0.500$ m/s [N]

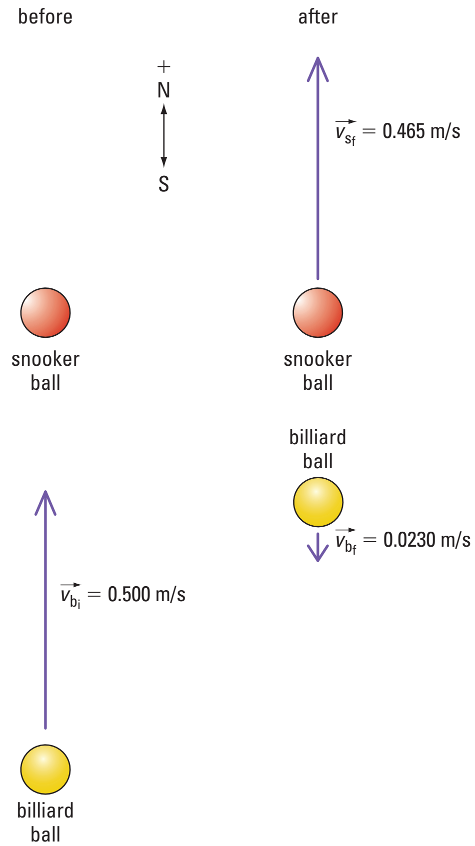
$\vec{v}_{s_i} = 0$ m/s

$\vec{v}_{b_f} = 0.0230$ m/s [S]

$\vec{v}_{s_f} = 0.465$ m/s [N]

***Required***

determine if the collision is elastic




▶ **Figure 9.44**

***Analysis and Solution***

Choose the billiard ball and the snooker ball as an isolated system. Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$\begin{aligned} E_{k_i} &= \frac{1}{2}m_b(v_{b_i})^2 + \frac{1}{2}m_s(v_{s_i})^2 \\ &= \frac{1}{2}(0.160 \text{ kg})(0.500 \text{ m/s})^2 + 0 \\ &= 0.0200 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\ &= 0.0200 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2}m_b(v_{b_f})^2 + \frac{1}{2}m_s(v_{s_f})^2 \\ &= \frac{1}{2}(0.160 \text{ kg})(0.0230 \text{ m/s})^2 \\ &\quad + \frac{1}{2}(0.180 \text{ kg})(0.465 \text{ m/s})^2 \\ &= 0.0195 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\ &= 0.0195 \text{ J} \end{aligned}$$

Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.

***Paraphrase***

The collision between the billiard ball and the snooker ball is inelastic.

### Practice Problems

1. A 45.9-g golf ball is stationary on the green when a 185-g golf club face travelling horizontally at 1.24 m/s [E] strikes it. After impact, the club face continues moving at 0.760 m/s [E] while the ball moves at 1.94 m/s [E]. Assume that the club face is vertical at the moment of impact so that the ball does not spin. Determine if the collision is elastic.
2. An argon atom with a mass of $6.63 \times 10^{-26}$ kg travels at 17 m/s [right] and strikes another identical argon atom dead centre travelling at 20 m/s [left]. The first atom rebounds at 20 m/s [left], while the second atom moves at 17 m/s [right]. Determine if the collision is elastic.

### Answers

1. inelastic
2. elastic

## Inelastic Collisions

In 9-2 QuickLab on page 455, after the putty ball collided with a hard surface, the putty ball became stationary and had no kinetic energy. Upon impact, the putty ball deformed and the kinetic energy of the putty ball was converted mostly to thermal energy.

Although the *total* energy of the system was conserved, the total initial kinetic energy of the system was not equal to the total final kinetic energy of the system after collision. This type of collision is **inelastic**.

**inelastic collision:** a collision in which $E_{k_i} \neq E_{k_f}$

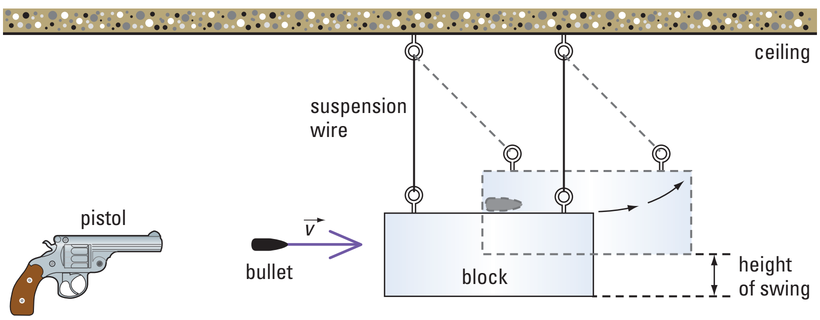
In an inelastic collision, the total kinetic energy of the system is *not* conserved.

$$E_{k_i} \neq E_{k_f}$$

One type of inelastic collision occurs when two objects stick together after colliding. However, this type of interaction does not necessarily mean that the final kinetic energy of the system is zero. For example, consider a **ballistic pendulum**, a type of pendulum used to determine the speed of bullets before electronic timing devices were invented (Figure 9.45).

*e* **WEB**

Research examples of elastic and inelastic one-dimensional collisions. Then analyze how the momentum and energy change in those collisions. Begin your search at www.pearsoned.ca/school/physicssource.




▲ **Figure 9.45** When a bullet is fired into the block, both the block and bullet move together as a unit after impact.

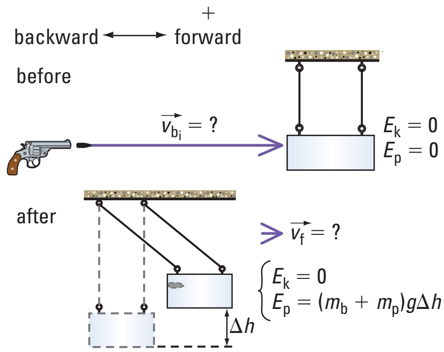
The pendulum consists of a stationary block of wood suspended from the ceiling by light ropes or cables. When a bullet is fired at the block, the bullet becomes embedded in the wood upon impact. The kinetic energy of the bullet is converted to sound, thermal energy, deformation of the wood and bullet, and the kinetic energy of the pendulum-bullet system.

The initial momentum of the bullet causes the pendulum to move upon impact, but since the pendulum is suspended by cables, it swings upward just after the bullet becomes embedded in the block. As the pendulum-bullet system swings upward, its kinetic energy is converted to gravitational potential energy.

Example 9.10 involves a ballistic pendulum. By using the conservation of energy, it is possible to determine the speed of the pendulum-bullet system immediately after impact. By applying the conservation of momentum to the collision, it is possible to determine the initial speed of the bullet.

## Example 9.10

A 0.0149-kg bullet from a pistol strikes a 2.0000-kg ballistic pendulum. Upon impact, the pendulum swings forward and rises to a height of 0.219 m. What was the velocity of the bullet immediately before impact?




▲ **Figure 9.46**

*Given*

$m_b = 0.0149$ kg  $m_p = 2.0000$ kg  $\Delta h = 0.219$ m

*Required*

initial velocity of bullet ($\vec{v}_{b_i}$)

*Analysis and Solution*

Choose the pendulum and the bullet as an isolated system.
Since the pendulum is stationary before impact, its initial velocity is zero. So its initial momentum is zero.

$$\vec{p}_{p_i} = 0$$

Immediately after collision, the bullet and pendulum move together as a unit. The kinetic energy of the pendulum-bullet system just after impact is converted to gravitational potential energy.

$$E_k = E_p$$

Apply the law of conservation of energy to find the speed of the pendulum-bullet system just after impact.

$$\begin{aligned}
E_k &= E_p \\
\frac{1}{2}\cancel{(m_b + m_p)}(v_f)^2 &= \cancel{(m_b + m_p)}g(\Delta h) \\
(v_f)^2 &= 2g(\Delta h) \\
v_f &= \sqrt{2g(\Delta h)} \\
&= \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.219\ \text{m})} \\
&= 2.073\ \text{m/s} \\
\vec{v}_f &= 2.073\ \text{m/s [forward]}
\end{aligned}$$

Apply the law of conservation of momentum to find the initial velocity of the bullet.

$$\begin{aligned}
\vec{p}_{sys_i} &= \vec{p}_{sys_f} \\
\vec{p}_{b_i} + \vec{p}_{p_i} &= \vec{p}_{sys_f} \\
m_b\vec{v}_{b_i} + 0 &= (m_b + m_p)\vec{v}_f \\
\vec{v}_{b_i} &= \left(\frac{m_b + m_p}{m_b}\right)\vec{v}_f \\
&= \left(\frac{0.0149\ \text{kg} + 2.0000\ \text{kg}}{0.0149\ \text{kg}}\right)(+2.073\ \text{m/s}) \\
&= \left(\frac{2.0149\ \cancel{\text{kg}}}{0.0149\ \cancel{\text{kg}}}\right)(+2.073\ \text{m/s}) \\
&= +280\ \text{m/s} \\
\vec{v}_{b_i} &= 280\ \text{m/s [forward]}
\end{aligned}$$

### Practice Problems

1. A 2.59-g bullet strikes a stationary 1.00-kg ballistic pendulum, causing the pendulum to swing up to 5.20 cm from its initial position. What was the speed of the bullet immediately before impact?
2. A 7.75-g bullet travels at 351 m/s before striking a stationary 2.5-kg ballistic pendulum. How high will the pendulum swing?

**Answers**

1. 391 m/s
2. 6.0 cm

*Paraphrase*
The initial velocity of the bullet immediately before impact was 280 m/s [forward].

Example 9.11 demonstrates how to determine if the collision in Example 9.10 is elastic or inelastic by comparing the kinetic energy of the system just before and just after collision.

## Example 9.11

Determine if the collision in Example 9.10 is elastic or inelastic.

*Given*
$m_b = 0.0149$ kg $\quad \vec{v}_{b_i} = 280$ m/s [forward] from Example 9.10
$m_p = 2.0000$ kg $\quad \vec{v}_f = 2.073$ m/s [forward] from Example 9.10

*Required*
initial and final kinetic energies ($E_{k_i}$ and $E_{k_f}$) to find if the collision is elastic

*Analysis and Solution*
Choose the pendulum and the bullet as an isolated system.
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$
\begin{aligned}
E_{k_i} &= \frac{1}{2}m_b(v_{b_i})^2 + \frac{1}{2}m_p(v_{p_i})^2 \\
&= \frac{1}{2}(0.0149 \text{ kg})(280 \text{ m/s})^2 + 0 \\
&= 585 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\
&= 585 \text{ J}
\end{aligned}
\qquad
\begin{aligned}
E_{k_f} &= \frac{1}{2}(m_b + m_p)(v_f)^2 \\
&= \frac{1}{2}(0.0149 \text{ kg} + 2.0000 \text{ kg})(2.073 \text{ m/s})^2 \\
&= 4.33 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\
&= 4.33 \text{ J}
\end{aligned}
$$

Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.

*Paraphrase and Verify*
Since the kinetic energy of the system just before impact is much greater than the kinetic energy of the system just after impact, the collision is inelastic. This result makes sense since the bullet became embedded in the pendulum upon impact.

### Practice Problems

1. In Example 9.6 on page 477, how much kinetic energy is lost immediately after the interaction?
2. (a) Determine if the interaction in Example 9.8 on page 479 is elastic.
   (b) What percent of kinetic energy is lost?

#### Answers

1. 1.1 J
2. (a) inelastic
   (b) 98.7%

## 9.3 Check and Reflect

### Knowledge

1. In your own words, state the law of conservation of momentum.
2. (a) In the context of momentum, what is an isolated system?
   (b) Why is it necessary to choose an isolated system when solving a momentum problem?
3. Explain the difference between an elastic and an inelastic collision. Include an example of each type of collision in your answer.
4. What evidence suggests that a collision is
   (a) elastic?
   (b) inelastic?

### Applications

5. Give two examples, other than those in the text, of possible collinear collisions between two identical masses. Include a sketch of each situation showing the velocity of each object immediately before and immediately after collision.
6. A student is sitting in a chair with nearly frictionless rollers. Her homework bag is in an identical chair right beside her. The chair and bag have a combined mass of 20 kg. The student and her chair have a combined mass of 65 kg. If she pushes her homework bag away from her at 0.060 m/s relative to the floor, what will be the student's velocity immediately after the interaction?
7. At liftoff, a space shuttle has a mass of $2.04 \times 10^6$ kg. The rocket engines expel $3.7 \times 10^3$ kg of exhaust gas during the first second of liftoff, giving the rocket a velocity of 5.7 m/s [up]. At what velocity is the exhaust gas leaving the rocket engines? Ignore the change in mass due to the fuel being consumed. The exhaust gas needed to counteract the force of gravity has already been accounted for and should not be part of this calculation.
8. A 60.0-kg student on a 4.2-kg skateboard is travelling south at 1.35 m/s. A friend throws a 0.585-kg basketball to him with a velocity of 12.6 m/s [N]. What will be the velocity of the student and skateboard immediately after he catches the ball?
9. A hockey forward with a mass of 95 kg skates in front of the net at 2.3 m/s [E]. He is met by a 104-kg defenceman skating at 1.2 m/s [W]. What will be the velocity of the resulting tangle of players if they stay together immediately after impact?
10. A 75.6-kg volleyball player leaps toward the net to block the ball. At the top of his leap, he has a horizontal velocity of 1.18 m/s [right], and blocks a 0.275-kg volleyball moving at 12.5 m/s [left]. The volleyball rebounds at 6.85 m/s [right].
    (a) What will be the horizontal velocity of the player immediately after the block?
    (b) Determine if the collision is elastic.
11. A 220-kg bumper car (A) going north at 0.565 m/s hits another bumper car (B) and rebounds at 0.482 m/s [S]. Bumper car B was initially travelling south at 0.447 m/s, and after collision moved north at 0.395 m/s.
    (a) What is the mass of bumper car B?
    (b) Determine if the collision is elastic.

### Extension

12. Summarize the concepts and ideas associated with one-dimensional collisions using a graphic organizer of your choice. See Student References 4: Using Graphic Organizers on pp. 869–871 for examples of different graphic organizers. Make sure that the concepts and ideas are clearly presented and appropriately linked.

**e TEST**

To check your understanding of the conservation of momentum and one-dimensional collisions, follow the eTest links at www.pearsoned.ca/school/physicssource.

# 9.4 Collisions in Two Dimensions

Many interactions in the universe involve collisions. Comets, asteroids, and meteors sometimes collide with celestial bodies. Molecules and atoms are constantly colliding during chemical reactions throughout the universe: in stars, in Earth's atmosphere, and even within your body.

An interesting collision in recent history occurred on June 30, 1908, at Tunguska, Siberia, between a cosmic object and Earth (Figure 9.47). Eyewitnesses reported seeing a giant fireball that moved rapidly across the sky and eventually collided with the ground. Upon impact, a tremendous explosion occurred producing an atmospheric shock wave that circled Earth twice. About 2000 $km^2$ of forest were levelled and thousands of trees were burned. In fact, there was so much fine dust in the atmosphere that people in London, England, could read a newspaper at night just from the scattered light.

**info BIT**

Scientists speculate that the cosmic object that hit Tunguska was about 100 m across and had a mass of about $1 \times 10^6$ t. The estimated speed of the object was about 30 km/s, which is $1.1 \times 10^5$ km/h. After the collision at Tunguska, a large number of diamonds were found scattered all over the impact site. So the cosmic object contained diamonds as well as other materials.

◄ **Figure 9.47** The levelled trees and charred remnants of a forest at Tunguska, Siberia, after a cosmic object collided with Earth in 1908. Although the chance that a similar collision with Earth during your lifetime may seem remote, such collisions have happened throughout Earth's history.

In real life, most collisions occur in three dimensions. Only in certain situations, such as those you studied in section 9.3, does the motion of the interacting objects lie along a straight line. In this section, you will examine collisions that occur in two dimensions. These interactions occur when objects in a plane collide off centre. In 9-1 QuickLab on page 447, you found that when two coins collide off centre, the resulting path of each coin is in a different direction from its initial path. You may have noticed that certain soccer or hockey players seem to be at the right place at the right time whenever there is a rebound from the goalie. How do these players know where to position themselves so that they can score on the rebound? Find out by doing 9-6 Inquiry Lab.

## 9-6 Inquiry Lab

# Analyzing Collisions in Two Dimensions

**Required Skills**
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

### Question

How does the momentum of a two-body system in the *x* and *y* directions compare just before and just after a collision?

### Hypothesis

State a hypothesis relating the momentum of a system in each direction immediately before and immediately after collision. Remember to write an “if/then” statement.

### Variables

Read the procedure and identify the controlled, manipulated, and responding variables in the experiment.

### Materials and Equipment

air table or bead table
pucks
spark-timer or camera set-up to measure velocities
rulers or metre-sticks
protractors

### Procedure

1. Copy Tables 9.3 and 9.4 on page 489 into your notebook.
2. Label the pucks as “puck 1” and “puck 2” respectively. Measure the mass of each puck and record it in Table 9.3.
3. Set up the apparatus so that puck 2 is at rest near the centre of the table.
4. Have each person in your group do one trial. Each time, send puck 1 aimed at the left side of puck 2, recording the paths of both pucks. Make sure the recording tracks of both pucks can be used to accurately measure their velocities before and after collision.
5. Have each person in your group measure and analyze one trial. Help each other as needed to ensure the measurements and calculations for each trial are accurate.
6. Find a suitable point on the recorded tracks to be the impact location.
7. On the path of puck 1 before collision, choose an interval where the speed is constant. Choose the positive *x*-axis to be in the initial direction of puck 1.
8. Using either the spark dots, the physical centre of the puck, or the leading or trailing edge of the puck, measure the distance and the time interval. Record those values in Table 9.3.
9. On the path of each puck after collision, choose an interval where the speed is constant. Measure the distance, direction of motion relative to the positive *x*-axis, and time interval. Record those values in Table 9.3.

### Analysis

**1.** Calculate the initial velocity and initial momentum of puck 1. Record the values in Table 9.4.

**2.** Calculate the velocity of puck 1 after collision. Resolve the velocity into *x* and *y* components. Record the values in Table 9.4.

**3.** Use the results of question 2 to calculate the *x* and *y* components of the final momentum of puck 1. Record the values in Table 9.4.

**4.** Repeat questions 2 and 3 but this time use the data for puck 2. Explain why the *y* component of the momentum of puck 2 is negative.

**5.** Record the calculated values from each member of your group as a different trial in Table 9.4.

**6.** For each trial, state the relationship between the initial momentum of the system in the *x* direction and the final momentum of the system in the *x* direction. Remember to consider measurement errors. Write this result as a mathematical statement.

**7.** The initial momentum of the system in the *y* direction was zero. For each trial, what was the final momentum of the system in the *y* direction? Remember to consider measurement errors. Write this result as a mathematical statement.

**8.** Compare your answers to questions 6 and 7 with other groups. Does this relationship agree with your hypothesis? Account for any discrepancies.

**eLAB**

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

▼ **Table 9.3** Mass, Distance, Time Elapsed, and Angle

| | Before and After for Puck 1 | | | | | | After for Puck 2 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Trial** | **Mass $m_1$ (g)** | **Initial Distance $d_{1_i}$ (m)** | **Initial Time Elapsed $\Delta t_{1_i}$ (s)** | **Final Distance $d_{1_f}$ (m)** | **Final Time Elapsed $\Delta t_{1_f}$ (s)** | **Final Angle $\theta_{1_f}$ (°)** | **Mass $m_2$ (g)** | **Final Distance $d_{2_f}$ (m)** | **Final Time Elapsed $\Delta t_{2_f}$ (s)** | **Final Angle $\theta_{2_f}$ (°)** |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |

▼ **Table 9.4** Velocity and Momentum

| | Before and After for Puck 1 | | | | | | After for Puck 2 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Trial** | **Initial x Velocity $v_{1_{ix}}$ (m/s)** | **Initial x Momentum $p_{1_{ix}}\left(\mathrm{g\cdot\frac{m}{s}}\right)$** | **Final x Velocity $v_{1_{fx}}$ (m/s)** | **Final y Velocity $v_{1_{fy}}$ (m/s)** | **Final x Momentum $p_{1_{fx}}\left(\mathrm{g\cdot\frac{m}{s}}\right)$** | **Final y Momentum $p_{1_{fy}}\left(\mathrm{g\cdot\frac{m}{s}}\right)$** | **Final x Velocity $v_{2_{fx}}$ (m/s)** | **Final y Velocity $v_{2_{fy}}$ (m/s)** | **Final x Momentum $p_{2_{fx}}\left(\mathrm{g\cdot\frac{m}{s}}\right)$** | **Final y Momentum $p_{2_{fy}}\left(\mathrm{g\cdot\frac{m}{s}}\right)$** |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |

## Momentum Is Conserved in Two-dimensional Collisions

In 9-6 Inquiry Lab, you found that along each direction, $x$ and $y$, the momentum of the system before collision is about the same as the momentum of the system immediately after collision. In other words, momentum is conserved in two-dimensional collisions. This result agrees with what you saw in 9-5 Inquiry Lab, where only one-dimensional collisions were examined.

As in one-dimensional collisions, the law of conservation of momentum is valid only when no external net force acts on the system. In two dimensions, the motion of each object in the system must be analyzed in terms of two perpendicular axes. To do this, you can either use a vector addition diagram drawn to scale or vector components.

The law of conservation of momentum can be stated using components in the $x$ and $y$ directions.

In two-dimensional collisions where no external net force acts on the system, the momentum of the system in both the $x$ and $y$ directions remains constant.

$$p_{\text{sys}_{\text{ix}}} = p_{\text{sys}_{\text{fx}}} \text{ and } p_{\text{sys}_{\text{i}y}} = p_{\text{sys}_{\text{f}y}} \text{ where } (\vec{F}_{\text{net}})_{\text{sys}} = 0$$

***e*SIM**

Apply the law of conservation of momentum to two-dimensional collisions. Go to www.pearsoned.ca/school/physicssource.

## Concept Check

(a) Will the magnitude of the momentum of an object always increase if a non-zero net force acts on it? Explain, using an example.
(b) How can the momentum of an object change but its speed remain the same? Explain, using an example.

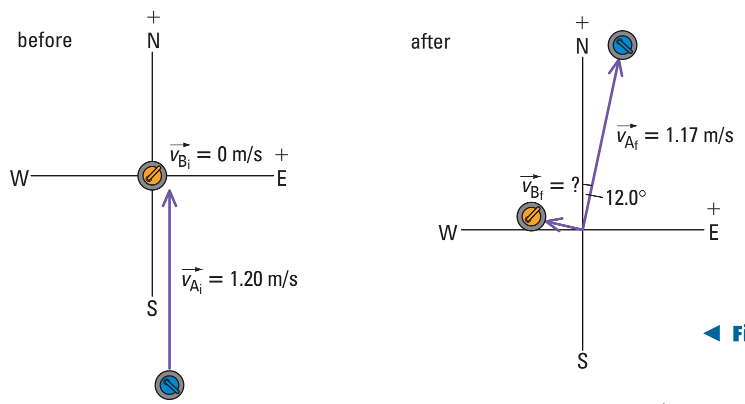
Example 9.12 involves a curling stone colliding off centre with an identical stone that is at rest. The momentum of each stone is analyzed in two perpendicular directions.

**info BIT**
In championship curling, rebound angles and conservation of momentum are crucial for placing stones in counting position behind guards. Just nudging a stone several centimetres can make all the difference.

## Example 9.12

A 19.6-kg curling stone (A) moving at 1.20 m/s [N] strikes another identical stationary stone (B) off centre, and moves off with a velocity of 1.17 m/s [12.0° E of N]. What will be the velocity of stone B after the collision? Ignore frictional and rotational effects.

*Given*
$m_A = 19.6$ kg $\quad m_B = 19.6$ kg $\quad \vec{v}_{A_i} = 1.20$ m/s [N]
$\vec{v}_{B_i} = 0$ m/s $\quad \vec{v}_{A_f} = 1.17$ m/s [12.0° E of N]




◀ **Figure 9.48**

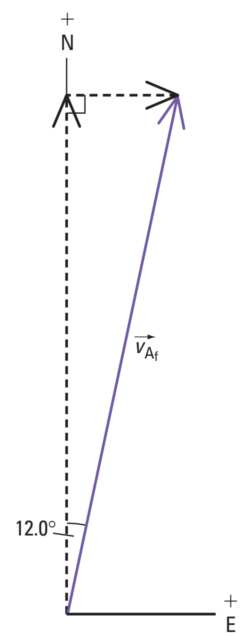
*Required*
final velocity of stone B ($\vec{v}_{B_f}$)

*Analysis and Solution*
Choose both curling stones as an isolated system. Stone B has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{B_i} = 0$$

Resolve all velocities into east and north components (Figure 9.49).

| Vector | East component | North component |
|---|---|---|
| $\vec{v}_{A_i}$ | 0 | 1.20 m/s |
| $\vec{v}_{B_i}$ | 0 | 0 |
| $\vec{v}_{A_f}$ | (1.17 m/s)(sin 12.0°) | (1.17 m/s)(cos 12.0°) |




▲ **Figure 9.49**

Apply the law of conservation of momentum to the system in the east and north directions.

*E* direction

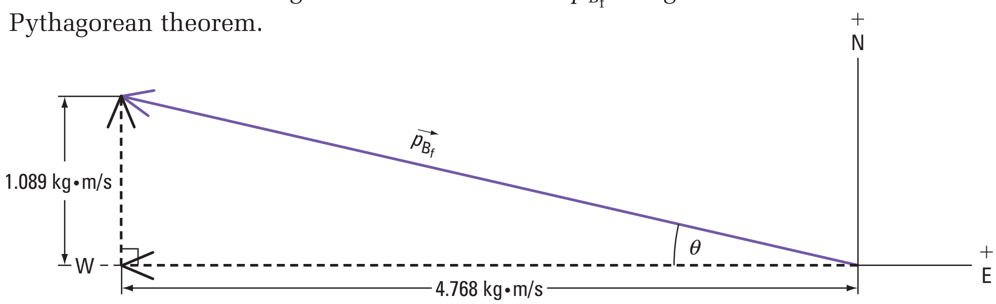
$$p_{\text{sys}_{\text{i}E}} = p_{\text{sys}_{\text{f}E}}$$
$$p_{\text{A}_{\text{i}E}} + p_{\text{B}_{\text{i}E}} = p_{\text{A}_{\text{f}E}} + p_{\text{B}_{\text{f}E}}$$
$$p_{\text{B}_{\text{f}E}} = p_{\text{A}_{\text{i}E}} + p_{\text{B}_{\text{i}E}} - p_{\text{A}_{\text{f}E}}$$
$$= m_{\text{A}}v_{\text{A}_{\text{i}E}} + m_{\text{B}}v_{\text{B}_{\text{i}E}} - m_{\text{A}}v_{\text{A}_{\text{f}E}}$$
$$= 0 + 0 - (19.6\ \text{kg})(1.17\ \text{m/s})(\sin 12.0°)$$
$$= -4.768\ \text{kg}\cdot\text{m/s}$$

The negative sign indicates the vector is directed west.

*N* direction

$$p_{\text{sys}_{\text{i}N}} = p_{\text{sys}_{\text{f}N}}$$
$$p_{\text{A}_{\text{i}N}} + p_{\text{B}_{\text{i}N}} = p_{\text{A}_{\text{f}N}} + p_{\text{B}_{\text{f}N}}$$
$$p_{\text{B}_{\text{f}N}} = p_{\text{A}_{\text{i}N}} + p_{\text{B}_{\text{i}N}} - p_{\text{A}_{\text{f}N}}$$
$$= m_{\text{A}}v_{\text{A}_{\text{i}N}} + m_{\text{B}}v_{\text{B}_{\text{i}N}} - m_{\text{A}}v_{\text{A}_{\text{f}N}}$$
$$= (19.6\ \text{kg})(1.20\ \text{m/s}) + 0 - (19.6\ \text{kg})(1.17\ \text{m/s})(\cos 12.0°)$$
$$= 1.089\ \text{kg}\cdot\text{m/s}$$

Draw a vector diagram of the components of the final momentum of stone B and find the magnitude of the resultant $\vec{p}_{\text{B}_\text{f}}$ using the Pythagorean theorem.




▲ **Figure 9.50**

$$p_{\text{B}_\text{f}} = \sqrt{(-4.768\ \text{kg}\cdot\text{m/s})^2 + (1.089\ \text{kg}\cdot\text{m/s})^2}$$
$$= 4.8906\ \text{kg}\cdot\text{m/s}$$

Use the magnitude of the momentum and the mass of stone B to find its final speed.

$$p_{\text{B}_\text{f}} = m_{\text{B}}v_{\text{B}_\text{f}}$$
$$v_{\text{Bf}} = \frac{p_{\text{B}_\text{f}}}{m_{\text{B}}}$$
$$v_{\text{B}_\text{f}} = \frac{4.8906\ \text{kg}\cdot\text{m/s}}{19.6\ \text{kg}}$$
$$= 0.250\ \text{m/s}$$

Use the tangent function to find the direction of the final momentum.

$$\tan\theta = \frac{p_{\text{B}_{\text{f}N}}}{p_{\text{B}_{\text{f}W}}}$$
$$\theta = \tan^{-1}\left(\frac{1.089\ \text{kg}\cdot\text{m/s}}{4.768\ \text{kg}\cdot\text{m/s}}\right)$$
$$= 12.9°$$

The final velocity will be in the same direction as the final momentum.

***Paraphrase***

The velocity of stone B after the collision is 0.250 m/s [12.9° N of W].

## Practice Problems

1. A 97.0-kg hockey centre stops momentarily in front of the net. He is checked from the side by a 104-kg defenceman skating at 1.82 m/s [E], and bounces at 0.940 m/s [18.5° S of E]. What is the velocity of the defenceman immediately after the check?
2. A 1200-kg car, attempting to run a red light, strikes a stationary 1350-kg vehicle waiting to make a turn. Skid marks show that after the collision, the 1350-kg vehicle moved at 8.30 m/s [55.2° E of N], and the other vehicle at 12.8 m/s [36.8° W of N]. What was the velocity of the 1200-kg vehicle just before collision? Note this type of calculation is part of many vehicle collision investigations where charges may be pending.

### Answers

1. 1.03 m/s [15.7° N of E]
2. 15.6 m/s [N]

Example 9.13 involves a football tackle with two players. Each player has an initial velocity, but when they collide, both players move together as a unit. To analyze the motion, the centre of mass of the combination of both players must be used.

The **centre of mass** is a point that serves as an average location of the total mass of an object or system. Depending on the distribution of mass, the centre of mass may be located even *outside* the object. Generally, momentum calculations are made using the centre of mass of an object.

No matter where any external forces are acting on an object, whether the object is rotating or not, or whether the object is deformable or rigid, the translational motion of the object can be easily analyzed in terms of its centre of mass.

**centre of mass:** point where the total mass of an object can be assumed to be concentrated

*info* **BIT**

When an object is symmetric and has uniform density, its centre of mass is in the same location as the physical centre of the object.

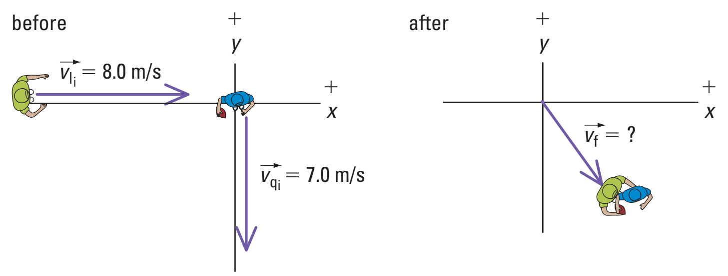
## Example 9.13

A 90-kg quarterback moving at 7.0 m/s [270°] is tackled by a 110-kg linebacker running at 8.0 m/s [0°]. What will be the velocity of the centre of mass of the combination of the two players immediately after impact?

*Given*

$m_q = 90$ kg $\qquad m_l = 110$ kg

$\vec{v}_{q_i} = 7.0$ m/s [270°] $\qquad \vec{v}_{l_i} = 8.0$ m/s [0°]




▲ **Figure 9.51**

*Required*

final velocity of centre of mass of the two players ($\vec{v}_f$)

*Analysis and Solution*

Choose the quarterback and the linebacker as an isolated system. The linebacker tackled the quarterback. So both players have the same final velocity.

Resolve all velocities into *x* and *y* components.

| Vector | *x* component | *y* component |
|---|---|---|
| $\vec{v}_{q_i}$ | 0 | −7.0 m/s |
| $\vec{v}_{l_i}$ | 8.0 m/s | 0 |

Apply the law of conservation of momentum to the system in the *x* and *y* directions.

## Practice Problems

1. A 2000-kg car travelling at 20.0 m/s [90.0°] is struck at an intersection by a 2500-kg pickup truck travelling at 14.0 m/s [180°]. If the vehicles stick together upon impact, what will be the velocity of the centre of mass of the car-truck combination immediately after the collision?
2. A 100-kg hockey centre is moving at 1.50 m/s [W] in front of the net. He is checked by a 108-kg defenceman skating at 4.20 m/s [S]. Both players move off together after collision. What will be the velocity of the centre of mass of the combination of the two players immediately after the check?

**Answers**

1. 11.8 m/s [131°]
2. 2.30 m/s [71.7° S of W]

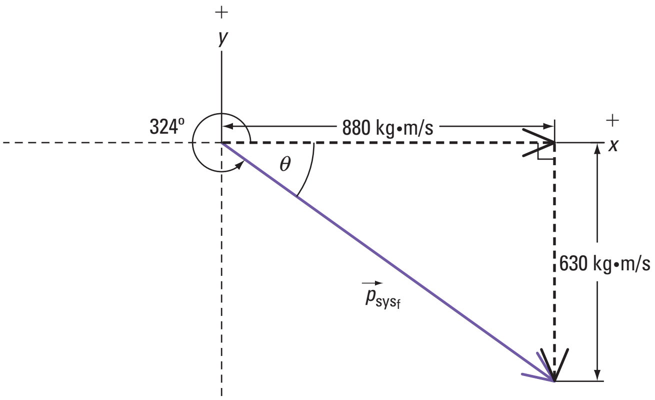
$x$ direction

$$\begin{aligned} p_{\text{sys}_{\text{i}x}} &= p_{\text{sys}_{\text{f}x}} \\ p_{\text{q}_{\text{i}x}} + p_{\text{l}_{\text{i}x}} &= p_{\text{sys}_{\text{f}x}} \\ p_{\text{sys}_{\text{f}x}} &= p_{\text{q}_{\text{i}x}} + p_{\text{l}_{\text{i}x}} \\ &= m_{\text{q}}v_{\text{q}_{\text{i}x}} + m_{\text{l}}v_{\text{l}_{\text{i}x}} \\ &= 0 + (110 \text{ kg})(8.0 \text{ m/s}) \\ &= 880 \text{ kg}\cdot\text{m/s} \end{aligned}$$

$y$ direction

$$\begin{aligned} p_{\text{sys}_{\text{i}y}} &= p_{\text{sys}_{\text{f}y}} \\ p_{\text{q}_{\text{i}y}} + p_{\text{l}_{\text{i}y}} &= p_{\text{sys}_{\text{f}y}} \\ p_{\text{sys}_{\text{f}y}} &= p_{\text{q}_{\text{i}y}} + p_{\text{l}_{\text{i}y}} \\ &= m_{\text{q}}v_{\text{q}_{\text{i}y}} + m_{\text{l}}v_{\text{l}_{\text{i}y}} \\ &= (90 \text{ kg})(-7.0 \text{ m/s}) + 0 \\ &= -630 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Draw a vector diagram of the components of the final momentum of the two players and find the magnitude of the resultant $\vec{p}_{\text{sys}_\text{f}}$ using the Pythagorean theorem.




◀ **Figure 9.52**

$$\begin{aligned} p_{\text{sys}_\text{f}} &= \sqrt{(880 \text{ kg}\cdot\text{m/s})^2 + (630 \text{ kg}\cdot\text{m/s})^2} \\ &= 1082 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Use the magnitude of the momentum and combined masses of the two football players to find their final speed.

$$\begin{aligned} p_{\text{sys}_\text{f}} &= (m_{\text{q}} + m_{\text{l}})v_{\text{f}} \\ v_{\text{f}} &= \frac{p_{\text{sys}_\text{f}}}{(m_{\text{q}} + m_{\text{l}})} \\ &= \frac{1082 \text{ kg}\cdot\text{m/s}}{(90 \text{ kg} + 110 \text{ kg})} \\ &= 5.4 \text{ m/s} \end{aligned}$$

Use the tangent function to find the direction of the final momentum.

$$\tan\theta = \frac{p_{\text{sys}_{\text{f}y}}}{p_{\text{sys}_{\text{f}x}}}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{630 \text{ kg}\cdot\text{m/s}}{880 \text{ kg}\cdot\text{m/s}}\right) \\ &= 35.6° \end{aligned}$$

The final velocity will be in the same direction as the final momentum.

From Figure 9.52, $\theta$ is below the positive $x$-axis. So the direction of $\vec{v}_\text{f}$ measured *counterclockwise* from the positive $x$-axis is $360° - 35.6° = 324.4°$.

$\vec{v}_\text{f} = 5.4$ m/s [324°]

***Paraphrase***

The final velocity of both players is 5.4 m/s [324°].

Example 9.14 deals with a fireworks bundle that is initially stationary. After it explodes, three fragments (A, B, and C) fly in different directions in a plane. To demonstrate an alternative method of solving collision problems, a vector addition diagram is used to determine the momentum of fragment C. This quantity is then used to calculate its final velocity.

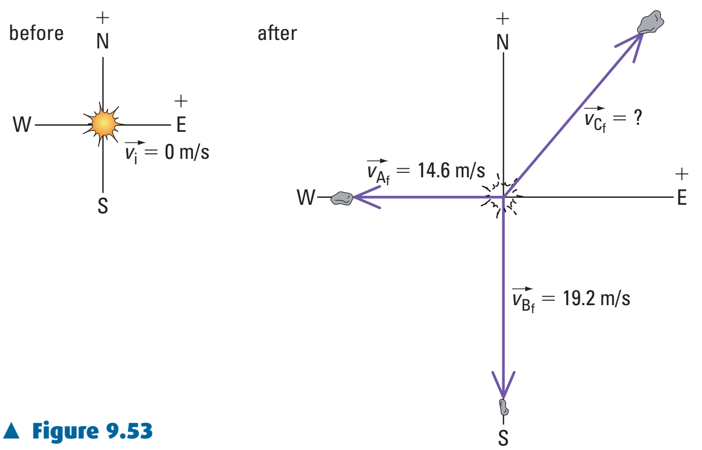
## Example 9.14

A 0.60-kg fireworks bundle is at rest just before it explodes into three fragments. A 0.20-kg fragment (A) flies at 14.6 m/s [W], and a 0.18-kg fragment (B) moves at 19.2 m/s [S]. What is the velocity of the third fragment (C) just after the explosion? Assume that no mass is lost, and that the motion of the fragments lies in a plane.

***Given***

$m_T = 0.60$ kg $\quad m_A = 0.20$ kg $\quad m_B = 0.18$ kg

$\vec{v}_i = 0$ m/s $\quad \vec{v}_{A_f} = 14.6$ m/s [W] $\quad \vec{v}_{B_f} = 19.2$ m/s [S]




▲ **Figure 9.53**

***Required***

final velocity of fragment C ($\vec{v}_{C_f}$)

***Analysis and Solution***

Choose fragments A, B, and C as an isolated system.
Since no mass is lost, find the mass of fragment C.

$$\begin{aligned} m_C &= m_T - (m_A + m_B) \\ &= 0.60 \text{ kg} - (0.20 \text{ kg} + 0.18 \text{ kg}) \\ &= 0.22 \text{ kg} \end{aligned}$$

The original firework has an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

The momentum of each fragment is in the same direction as its velocity. Calculate the momentum of fragments A and B.

$$\begin{aligned} p_{A_f} &= m_A v_{A_f} \\ &= (0.20 \text{ kg})(14.6 \text{ m/s}) \\ &= 2.92 \text{ kg}\cdot\text{m/s} \\ \vec{p}_{A_f} &= 2.92 \text{ kg}\cdot\text{m/s [W]} \end{aligned}$$

$$\begin{aligned} p_{B_f} &= m_B v_{B_f} \\ &= (0.18 \text{ kg})(19.2 \text{ m/s}) \\ &= 3.46 \text{ kg}\cdot\text{m/s} \\ \vec{p}_{B_f} &= 3.46 \text{ kg}\cdot\text{m/s [S]} \end{aligned}$$

### Practice Problems

1. A 0.058-kg firecracker that is at rest explodes into three fragments. A 0.018-kg fragment moves at 2.40 m/s [N] while a 0.021-kg fragment moves at 1.60 m/s [E]. What will be the velocity of the third fragment? Assume that no mass is lost, and that the motion of the fragments lies in a plane.
2. A 65.2-kg student on a 2.50-kg skateboard moves at 0.40 m/s [W]. He jumps off the skateboard with a velocity of 0.38 m/s [30.0° S of W]. What will be the velocity of the skateboard immediately after he jumps? Ignore friction between the skateboard and the ground.

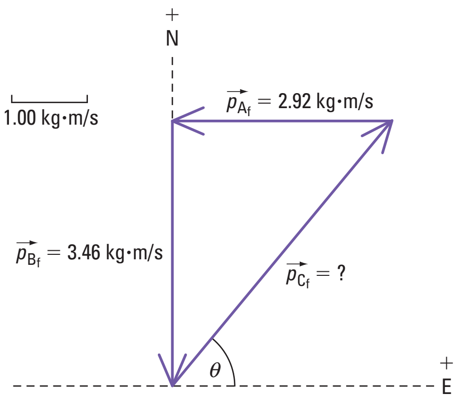
#### Answers

1. 2.9 m/s [52° S of W]
2. 5.4 m/s [66° N of W]

Apply the law of conservation of momentum to the system.

$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{sys}_\text{f}}$$
$$0 = \vec{p}_{\text{A}_\text{f}} + \vec{p}_{\text{B}_\text{f}} + \vec{p}_{\text{C}_\text{f}}$$

Use a vector addition diagram to determine the momentum of fragment C.




◀ **Figure 9.54**

From Figure 9.54, careful measurements give $p_{\text{C}_\text{f}} = 4.53$ kg·m/s and $\theta = 50°$ N of E.

Divide the momentum of fragment C by its mass to find the velocity.

$$p_{\text{C}_\text{f}} = m_\text{C} v_{\text{C}_\text{f}}$$
$$v_{\text{C}_\text{f}} = \frac{p_{\text{C}_\text{f}}}{m_\text{C}}$$
$$= \frac{4.53\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}}}{0.22\ \cancel{\text{kg}}}$$
$$= 21\ \text{m/s}$$
$$\vec{v}_{\text{C}_\text{f}} = 21\ \text{m/s}\ [50°\ \text{N of E}]$$

***Paraphrase***

The velocity of the third fragment just after the explosion is 21 m/s [50° N of E].

# Elastic and Inelastic Collisions in Two Dimensions

As with one-dimensional collisions, collisions in two dimensions may be either elastic or inelastic. The condition for an elastic two-dimensional collision is the same as for an elastic one-dimensional collision, $E_{\text{k}_\text{i}} = E_{\text{k}_\text{f}}$. To determine if a collision is elastic, the kinetic energy values before and after collision must be compared.

The kinetic energy of an object only depends on the *magnitude* of the velocity vector. So it does not matter if the velocity vector has only an $x$ component, only a $y$ component, or both $x$ and $y$ components. If you can determine the magnitude of the velocity vector, it is possible to calculate the kinetic energy.

An example of an inelastic collision occurs when two objects join together and move as a unit immediately after impact. If two objects bounce apart after impact, the collision may be either elastic or inelastic, depending on the initial and final kinetic energy of the system. Usually, if one or both colliding objects deform upon impact, the collision is inelastic.

*e* **SIM**

Predict $v_{\text{f}_x}$ and $v_{\text{f}_y}$ for an object just after a two-dimensional collision using momentum and energy concepts. Follow the eSim links at www.pearsoned.ca/school/physicssource.

**e WEB**

Research some design improvements in running shoes. Use momentum and collision concepts to explain how these features affect athletes. Write a brief report of your findings, including diagrams where appropriate. Begin your search at www.pearsoned.ca/school/physicssource.

In track sports, the material used for the track has a profound effect on the elasticity of the collision between a runner's foot and the running surface (Figure 9.55). If a track is made of a very hard material such as concrete, it experiences very little deformation when a runner's foot comes in contact with it. The collision is more elastic than if the track were made of a more compressible material such as cork. So less kinetic energy of the runner is converted to other forms of energy upon impact.

However, running on harder tracks results in a decreased interaction time and an increase in the net force acting on each foot, which could result in more injuries to joints, bones, and tendons. But a track that is extremely compressible is not desirable either, because it slows runners down. With all the pressure to achieve faster times in Olympic and world competitions, researchers and engineers continue to search for the optimum balance between resilience and safety in track construction. On the other hand, some collisions in sporting events present a very low risk of injury to contestants.

Example 9.15 involves determining if the collision between two curling stones is elastic.


▶ **Figure 9.55** Canadian runner Diane Cummins (far right) competing in the 2003 World Championships. The material of a running surface affects the interaction time and the net force acting on a runner's feet. How would the net force change if the track were made of a soft material that deforms easily?

## Example 9.15

Determine if the collision in Example 9.12 on pages 490 and 491 is elastic. If it is not, what percent of the kinetic energy is retained?

***Given***

$m_A = 19.6$ kg $\quad m_B = 19.6$ kg $\quad \vec{v}_{A_i} = 1.20$ m/s [N]

$\vec{v}_{B_i} = 0$ m/s $\quad \vec{v}_{A_f} = 1.17$ m/s [12.0° E of N]

$\vec{v}_{B_f} = 0.2495$ m/s [77.1° W of N]

***Required***

determine if the collision is elastic

***Analysis and Solution***

Choose the two curling stones as an isolated system.

Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$E_{k_i} = \frac{1}{2}m_A(v_{A_i})^2 + \frac{1}{2}m_B(v_{B_i})^2$$
$$= \frac{1}{2}(19.6\text{ kg})(1.20\text{ m/s})^2 + 0$$
$$= 14.11\text{ kg·m}^2/\text{s}^2$$
$$= 14.11\text{ J}$$

$$E_{k_f} = \frac{1}{2}m_A(v_{A_f})^2 + \frac{1}{2}m_B(v_{B_f})^2$$
$$= \frac{1}{2}(19.6\text{ kg})(1.17\text{ m/s})^2 + \frac{1}{2}(19.6\text{ kg})(0.2495\text{ m/s})^2$$
$$= 14.03\text{ kg·m}^2/\text{s}^2$$
$$= 14.03\text{ J}$$

Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.
Find the percent of $E_k$ retained.

$$\%\ E_k\text{ retained} = \frac{E_{k_f}}{E_{k_i}} \times 100\%$$
$$= \frac{14.03\ \cancel{\text{J}}}{14.11\ \cancel{\text{J}}} \times 100\%$$
$$= 99.4\%$$

*Paraphrase*

The collision is inelastic, and 99.4% of the kinetic energy is retained. (Collisions like this, where very little kinetic energy is lost, may be called "near elastic collisions.")

### Practice Problems

1. A 0.168-kg hockey puck flying at 45.0 m/s [252°] is trapped in the pads of an 82.0-kg goalie moving at 0.200 m/s [0°]. The velocity of the centre of mass of the goalie, pads, and puck immediately after collision is 0.192 m/s [333°]. Was the collision elastic? If not, calculate the percent of total kinetic energy retained.
2. A 19.0-kg curling stone collides with another identical stationary stone. Immediately after collision, the first stone moves at 0.663 m/s. The second stone, which was stationary, moves at 1.31 m/s. If the collision was elastic, what would have been the speed of the first stone just before collision?

#### Answers

1. inelastic, 0.882%
2. 1.47 m/s

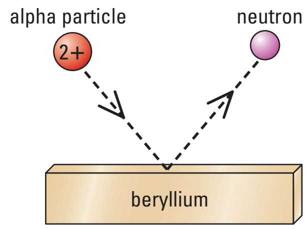
## Conservation Laws and the Discovery of Subatomic Particles

Based on the results of experiments, scientists have gained great confidence in the laws of conservation of momentum and of conservation of energy, and have predicted that there are no known exceptions. This confidence has enabled scientists to make discoveries about the existence of particles within atoms as well. You will learn more about subatomic particles in Units VII and VIII.

In 1930, German scientists Walther Bothe and Wilhelm Becker produced a very penetrating ray of unknown particles when they bombarded the element beryllium with alpha particles (Figure 9.56). An **alpha particle** is two protons and two neutrons bound together to form a stable particle.

In 1932, British scientist James Chadwick (1891–1974) directed rays of these unknown particles at a thin paraffin strip and found that protons were emitted from the paraffin. He analyzed the speeds and angles of the emitted protons and, by using the conservation of momentum, he showed that the protons were being hit by particles of approximately the same mass. In other related experiments, Chadwick was able to determine the mass of these unknown particles very accurately using the conservation of momentum.

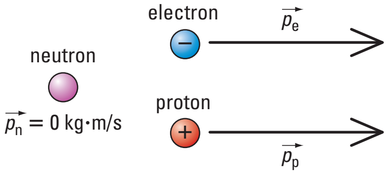
Earlier experiments had shown that the unknown particles were neutral because they were unaffected by electric or magnetic fields. You will learn about electric and magnetic fields in Unit VI. Chadwick had attempted for several years to find evidence of a suggested neutral particle that was believed to be located in the nucleus of an atom. The discovery of these neutral particles, now called neutrons, resulted in Chadwick winning the Nobel Prize for Physics in 1935.




▲ **Figure 9.56** The experiment of Bothe and Becker using beryllium paved the way for Chadwick who later discovered the existence of neutrons by creating an experiment where he could detect them.

**eSIM**

Practise solving problems involving two-dimensional collisions. Follow the eSim links at www.pearsoned.ca/school/physicssource.

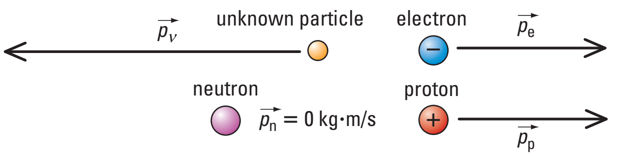



▲ **Figure 9.57** If a neutron is initially stationary, $\vec{p}_{sys_i} = 0$. If the neutron becomes transformed into a proton and an electron moving in the same direction, the momentum of the system is no longer zero.

Scientists later found that the neutron, when isolated, soon became transformed into a proton and an electron. Sometimes the electron and proton were both ejected in the same direction, which seemed to contradict the law of conservation of momentum (Figure 9.57). Furthermore, other experiments showed that the total energy of the neutron before transformation was greater than the total energy of both the proton and electron. It seemed as if the law of conservation of energy was not valid either.

Austrian physicist Wolfgang Pauli (1900–1958) insisted that the conservation laws of momentum and of energy were still valid, and in 1930, he proposed that an extremely tiny neutral particle produced during the transformation must be moving in the opposite direction at an incredibly high speed. This new particle accounted for the missing momentum and missing energy (Figure 9.58).

Many other scientists accepted Pauli's explanation because they were convinced that the conservation laws of momentum and of energy were valid. For 25 years, they held their belief in the existence of this tiny particle, later called a neutrino, with no other evidence. Then in 1956, the existence of neutrinos was finally confirmed experimentally, further strengthening the universal validity of conservation laws.




▲ **Figure 9.58** The existence of another particle accounted for the missing momentum and missing energy observed when a neutron transforms itself into a proton and an electron.

## THEN, NOW, AND FUTURE Neutrino Research in Canada

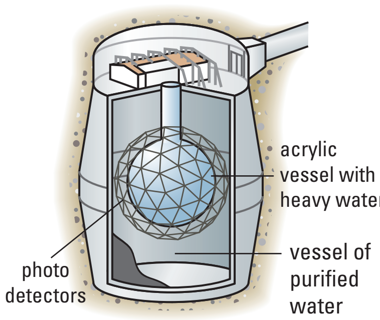
Canada is a world leader in neutrino research. The SNO project (Sudbury Neutrino Observatory) is a special facility that allows scientists to gather data about these extremely tiny particles that are difficult to detect.

The observatory is located in INCO's Creighton Mine near Sudbury, Ontario, 2 km below Earth's surface. Bedrock above the mine shields the facility from cosmic rays that might interfere with the observation of neutrinos.

The experimental apparatus consists of 1000 t of heavy water encased in an acrylic vessel shaped like a 12-m diameter boiling flask (Figure 9.59). The vessel is surrounded by an array of about 1000 photo detectors, all immersed in a 10-storey chamber of purified water.

When a neutrino collides with a heavy water molecule, a tiny burst of light is emitted, which the photo detectors pick up.

Despite all that equipment, scientists only detect an average of about 10 neutrinos a day. So experiments must run for a long time in order to collect enough useful data.




▲ **Figure 9.59** The Sudbury Neutrino Observatory is a collaborative effort among 130 scientists from Canada, the U.S., and the U.K.

Scientists are interested in neutrinos originating from the Sun and other distant parts of the universe. At first, it seemed that the Sun was not emitting as many neutrinos as expected. Scientists thought they would have to modify their theories about the reactions taking place within the core of the Sun.

Tripling the sensitivity of the detection process, by adding 2 t of salt to the heavy water, showed that $\frac{2}{3}$ of the neutrinos from the Sun were being transformed into different types of neutrinos as they travelled. This discovery has important implications about the basic properties of a neutrino, including its mass.

It now appears that scientists' theories about the reactions within the core of the Sun are very accurate. Continued research at this facility will help answer fundamental questions about matter and the universe.

### Questions

1. Why was Sudbury chosen as the site for this type of observatory?
2. Explain why, at first, it appeared that the Sun was not emitting the expected number of neutrinos.
3. If a neutrino has a very small mass and travels very fast, why doesn't it run out of energy?

## 9.4 Check and Reflect

### Knowledge

1. How is a two-dimensional collision different from a one-dimensional collision? Explain, using examples.
2. In your own words, state the law of conservation of momentum for two-dimensional collisions. Show how the law relates to *x* and *y* components by using an example.
3. In your own words, define the centre of mass of an object.
4. Explain why scientists accepted the existence of the neutrino for so long when there was no direct evidence for it.

### Applications

5. A cue ball travelling at 0.785 m/s [270°] strikes a stationary five-ball, causing it to move at 0.601 m/s [230°]. The cue ball and the five-ball each have a mass of 160 g. What will be the velocity of the cue ball immediately after impact? Ignore frictional and rotational effects.
6. A stationary 230-kg bumper car in a carnival is struck off centre from behind by a 255-kg bumper car moving at 0.843 m/s [W]. The more massive car bounces off at 0.627 m/s [42.0° S of W]. What will be the velocity of the other bumper car immediately after collision?
7. A 0.25-kg synthetic rubber ball bounces to a height of 46 cm when dropped from a height of 50 cm. Determine if this collision is elastic. If not, how much kinetic energy is lost?
8. A football halfback carrying the ball, with a combined mass of 95 kg, leaps toward the goal line at 4.8 m/s [S]. In the air at the goal line, he collides with a 115-kg linebacker travelling at 4.1 m/s [N]. If the players move together after impact, will the ball cross the goal line?
9. A 0.160-kg pool ball moving at 0.563 m/s [67.0° S of W] strikes a 0.180-kg snooker ball moving at 0.274 m/s [39.0° S of E]. The pool ball glances off at 0.499 m/s [23.0° S of E]. What will be the velocity of the snooker ball immediately after collision?
10. A 4.00-kg cannon ball is flying at 18.5 m/s [0°] when it explodes into two fragments. One 2.37-kg fragment (A) goes off at 19.7 m/s [325°]. What will be the velocity of the second fragment (B) immediately after the explosion? Assume that no mass is lost during the explosion, and that the motion of the fragments lies in the *xy* plane.
11. A 0.952-kg baseball bat moving at 35.2 m/s [0°] strikes a 0.145-kg baseball moving at 40.8 m/s [180°]. The baseball rebounds at 37.6 m/s [64.2°]. What will be the velocity of the centre of mass of the bat immediately after collision if the batter exerts no force on the bat during and after the instant of impact?

### Extensions

12. Some running shoe designs contain springs. Research these types of shoes and the controversy surrounding them. How do momentum and impulse apply to these shoes? Write a brief report of your findings. Include diagrams where appropriate. Begin your search at www.pearsoned.ca/school/physicssource.
13. Italian physicist Enrico Fermi gave the name "neutrino" to the elusive particle that scientists were at first unable to detect. Research Fermi's contributions to physics. Write a brief report of your findings. Begin your search at www.pearsoned.ca/school/physicssource.

**eTEST**

To check your understanding of two-dimensional collisions, follow the eTest links at www.pearsoned.ca/school/physicssource.

## Key Terms and Concepts

momentum
impulse
collision
system
law of conservation of momentum
elastic collision
inelastic collision
centre of mass

## Key Equations

Momentum: $\vec{p} = m\vec{v}$

Impulse: $\vec{F}_{\text{net}}\Delta t = \Delta\vec{p}$ or $\vec{F}_{\text{net}}\Delta t = m\Delta\vec{v}$

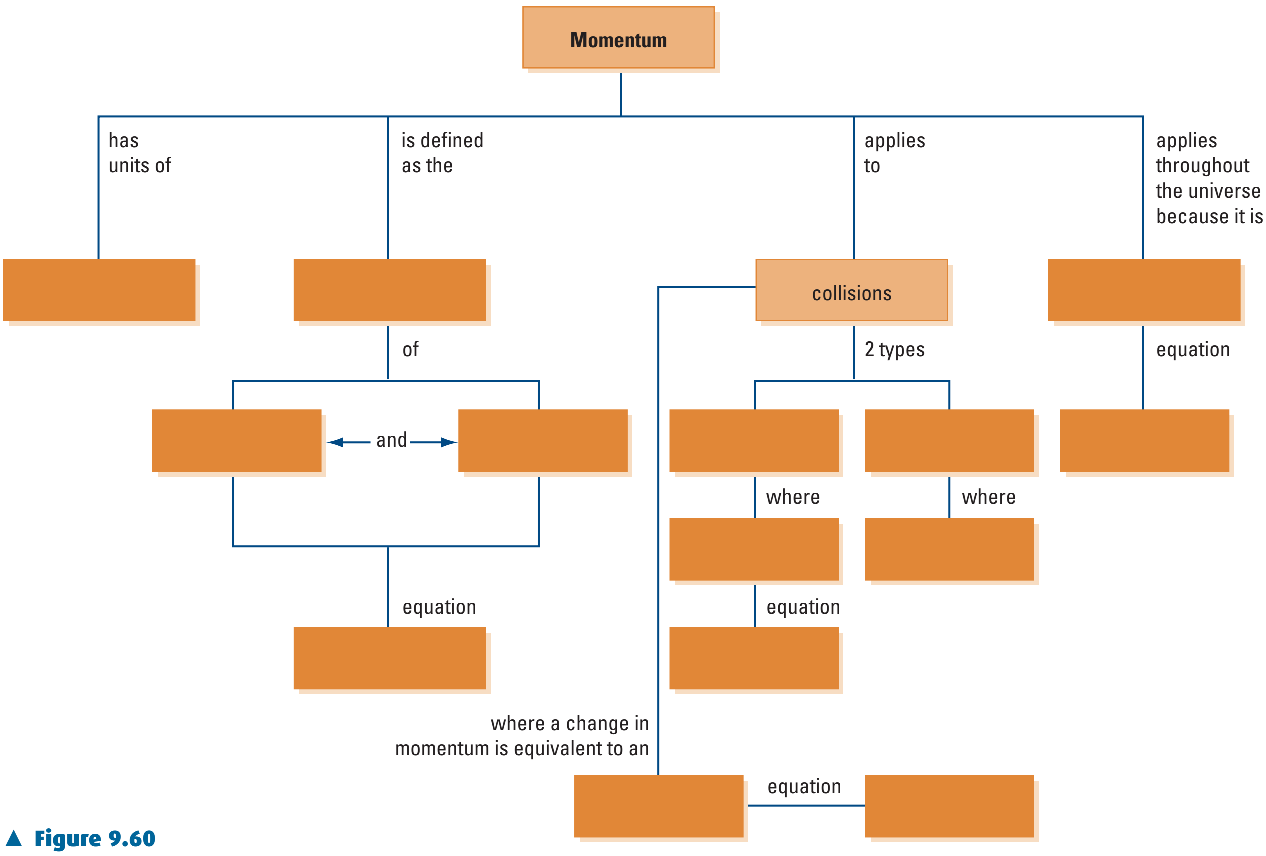
$\vec{F}_{\text{net}_{\text{ave}}}\Delta t = \Delta\vec{p}$ or $\vec{F}_{\text{net}_{\text{ave}}}\Delta t = m\Delta\vec{v}$

Conservation of momentum: $\vec{p}_{\text{sys}_{\text{i}}} = \vec{p}_{\text{sys}_{\text{f}}}$

Elastic collisions: $E_{\text{k}_{\text{i}}} = E_{\text{k}_{\text{f}}}$

## Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.




▲ **Figure 9.60**

# An Impulsive Water Balloon

## Scenario

Imagine you are part of a creative engineering design team commissioned to design and build an amusement ride that will make the West Edmonton Mall 14-storey "Space Shot" obsolete. The ride must allow patrons to experience the thrill of free fall in a safe environment. Accelerations have to remain within appropriate limits during a required change from vertical to horizontal motion, and as the people are brought to rest. A model must be constructed for sales demonstrations using a water balloon that experiences a minimum 2.4-m drop, where the impulse changes the magnitude and direction of the momentum while maintaining the integrity of the balloon. The water balloon must begin with a vertical drop equivalent to eight storeys. Then for the equivalent height of six storeys, the balloon must change direction and come to a stop horizontally.

## Planning

Form a design team of three to five members. Plan and assign roles so that each team member has at least one major task. Roles may include researcher, engineer to perform mathematical calculations, creative designer, construction engineer, materials acquisition officer, and writer, among others. One person may need to perform several roles in turn. Ensure that all team members help along the way. Prepare a time schedule for each task, and for group planning and reporting sessions.

## Materials

- small balloon filled with water
- plastic zip-closing bag to contain the water balloon
- cardboard and/or wooden frame for apparatus
- vehicle or container for balloon
- cushioning material
- braking device
- art materials

**CAUTION: Test your design in an appropriate area. Make sure no one is in the way during the drop.**

### Assessing Results

After completing the project, assess its success based on a rubric designed in class* that considers

- research strategies
- experiment and construction techniques
- clarity and thoroughness of the written report
- effectiveness of the team's presentation
- quality and fairness of the teamwork

## Procedure

1. Research the range of acceptable accelerations that most people can tolerate.
2. Calculate the maximum speed obtained when an object is dropped from an eight-storey building (equivalent to 24.6 m).
3. Calculate the impulse necessary to change the direction of motion of a 75.0-kg person from vertical to horizontal in the remaining height of six storeys (equivalent to 18.4 m). The person must come to a stop at the end. Assume that the motion follows the arc of a circle.
4. Determine the time required so that the change in the direction of motion and stopping the person meets the maximum acceptable acceleration in step 1.
5. Include your calculations in a report that shows your design and method of changing the motion.
6. Build a working model and test it. Make modifications as necessary to keep the water balloon intact for a fall. Present the project to your teacher and the class.

## Thinking Further

1. Explain why eight storeys was used in the calculation in step 2, instead of 14 storeys.
2. What other amusement rides has your team thought of while working on this project? What would make each of these rides thrilling and appealing?
3. In what ways could your ideas have a practical use, such as getting people off a high oil derrick or out of a high-rise building quickly and safely?
4. What conditions would cause a person to be an unacceptable candidate for your ride? Write out a list of rules or requirements that would need to be posted.

*Note: Your instructor will assess the project using a similar assessment rubric.

# UNIT V SUMMARY

## Unit Concepts and Skills: Quick Reference

| Concepts | Summary | Resources and Skill Building |
|---|---|---|
| **CHAPTER 9** | **The momentum of an isolated system of interacting objects is conserved.** | |
| | **9.1 Momentum Is Mass Times Velocity** | |
| Momentum | Momentum is the product of the mass of an object and its velocity. Momentum is a vector quantity measured in kilogram-metres per second (kg•m/s). | Examples 9.1 & 9.2 |
| Newton's second law | Newton's second law states that the net force on an object is equal to the rate of change of its momentum. | |
| | **9.2 Impulse Is Equivalent to a Change in Momentum** | |
| Impulse | The impulse provided to an object is defined as the product of the net force (or average net force if $\vec{F}_{net} \neq$ constant) acting on the object during an interaction and the interaction time. Impulse is equivalent to the change in momentum of the object. | 9-2 QuickLab |
| Effects of varying the net force and time interval for a given impulse | The magnitude of the net force during an interaction and the interaction time determine whether or not injuries or damage to an object occurs. | Figures 9.12 & 9.13<br>9-3 Design a Lab<br>Example 9.3 |
| Net force-time graph | Impulse can be determined by calculating the area under a net force-time graph. | Example 9.4 |
| | **9.3 Collisions in One Dimension** | |
| Conservation of momentum in one dimension | Momentum is conserved when objects in an isolated system interact in one dimension. A system is the group of objects that interact with each other, and it is isolated if no external net force acts on these objects. | 9-4 QuickLab<br>9-5 Inquiry Lab<br>Examples 9.5–9.8, 9.10 |
| Elastic collisions | Elastic collisions are collisions in which a system of objects has the same initial and final kinetic energy. So both the momentum and kinetic energy of the system are conserved. | Example 9.9 |
| Inelastic collisions | Inelastic collisions are collisions in which a system of objects has different initial and final kinetic energy values. | Example 9.11 |
| | **9.4 Collisions in Two Dimensions** | |
| Conservation of momentum in two dimensions | Momentum is conserved when objects in an isolated system interact in two dimensions. An isolated system has no external net force acting on it. | 9-6 Inquiry Lab<br>Examples 9.12–9.14 |
| Elastic and inelastic collisions | Elastic collisions in two dimensions satisfy the same conditions as one-dimensional elastic collisions, that is, $E_{k_i} = E_{k_f}$. | Example 9.15 |

# UNIT V REVIEW

## Vocabulary

1. Using your own words, define these terms, concepts, principles, or laws.
   momentum
   impulse
   one-dimensional collisions
   conservation of momentum
   conservation of energy
   elastic collisions
   inelastic collisions
   two-dimensional collisions
   centre of mass

## Knowledge

**CHAPTER 9**

2. Compare and contrast momentum and impulse.
3. Explain the relationship between the units in which momentum and impulse are measured.
4. A student calculated the answer to a problem and got 40 kg•m/s [W]. Which quantities could the student have calculated?
5. In your own words, restate Newton's second law in terms of momentum.
6. What difference does it make that momentum is a vector quantity rather than a scalar quantity?
7. Compare and contrast a net force and an average net force acting on an object during an interaction.
8. Statistics show that less massive vehicles tend to have fewer accidents than more massive vehicles. However, the survival rate for accidents in more massive vehicles is much greater than for less massive ones. How could momentum be used to explain these findings?
9. Using the concept of impulse, explain how the shocks on a high-end mountain bike reduce the chance of strain injuries to the rider.
10. State the quantities, including units, you would need to measure to determine the momentum of an object.
11. State the quantities that are conserved in one- and two-dimensional collisions. Give an example of each type of collision.
12. How do internal forces affect the momentum of a system?
13. What instructions would you give a young gymnast so that she avoids injury when landing on a hard surface?
14. Will the magnitude of the momentum of an object always increase if a net force acts on it? Explain, using an example.
15. What quantity do you get when $\Delta\vec{p}$ is divided by mass?
16. For a given impulse, what is the effect of
    (a) increasing the time interval?
    (b) decreasing the net force during interaction?
17. For each situation, explain how you would effectively provide the required impulse.
    - to catch a water balloon tossed from some distance
    - to design a hiking boot for back-country hiking on rough ground
    - to shoot an arrow with maximum velocity using a bow
    - for an athlete to win the gold medal in the javelin event with the longest throw
    - for a car to accelerate on an icy road
18. Why does a hunter always press the butt of a shotgun tight against the shoulder before firing?
19. Describe a method to find the components of a momentum vector.
20. Explain why the conservation of momentum and the conservation of energy are universal laws.
21. Why does the law of conservation of momentum require an isolated system?
22. Suppose a problem involves a two-dimensional collision between two objects, and the initial momentum of one object is unknown. Explain how to solve this problem using
    (a) a vector addition diagram drawn to scale
    (b) vector components
23. Explain, in terms of momentum, why a rocket does not need an atmosphere to push against when it accelerates.
24. If a firecracker explodes into two fragments of unequal mass, which fragment will have the greater speed? Why?

**25.** When applying the conservation of momentum to a situation, why is it advisable to find the velocities of all objects in the system immediately after collision, instead of several seconds later?

**26.** Which physics quantities are conserved in a collision?

**27.** A curling stone hits another stationary stone off centre. Draw possible momentum vectors for each stone immediately before and immediately after collision, showing both the magnitude and direction of each vector.

**28.** A Superball™ of rubber-like plastic hits a wall perpendicularly and rebounds elastically. Explain how momentum is conserved.

**29.** What two subatomic particles were discovered using the conservation of momentum and the conservation of energy?

**30.** Explain how an inelastic collision does not violate the law of conservation of energy.

**31.** If a system is made up of only one object, show how the law of conservation of momentum can be used to derive Newton's first law.

**32.** A Calgary company, Cerpro, is a world leader in ceramic armour plating for military protection. The ceramic structure of the plate transmits the kinetic energy of an armour-piercing bullet throughout the plate, reducing its penetrating power. Explain if this type of collision is elastic or inelastic.

**33.** A compact car and a heavy van travelling at approximately the same speed perpendicular to each other collide and stick together. Which vehicle will experience the greatest change in its direction of motion just after impact? Why?

**34.** Is it possible for the conservation of momentum to be valid if two objects move faster just before, than just after, collision? Explain, using an example.

**35.** Fighter pilots have reported that immediately after a burst of gunfire from their jet fighter, the speed of their aircraft decreased by 50–65 km/h. Explain the reason for this change in motion.

**36.** A cannon ball explodes into three fragments. One fragment goes north and another fragment goes east. Draw the approximate direction of the third fragment. What scientific law did you use to arrive at your answer?

## Applications

**37.** Calculate the momentum of a 1600-kg car travelling north at 8.5 m/s.

**38.** Draw a momentum vector diagram to represent a 575-g basketball flying at 12.4 m/s [26.0° S of E].

**39.** Calculate the momentum of a 1250-kg car travelling south at 14.8 m/s.

**40.** A bowling ball has a momentum of 28 kg•m/s [E]. If its speed is 4.5 m/s, what is the mass of the ball?

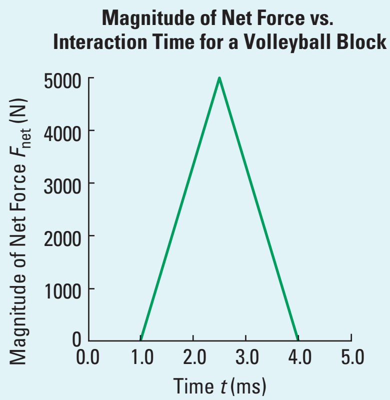
**41.** A curling stone has a momentum of 32 kg•m/s [W]. What would be the momentum if the mass of the stone is decreased to $\frac{7}{8}$ of its original mass and its speed is increased to $\frac{4}{3}$ of its original speed?

**42.** A soccer ball has a momentum of 2.8 kg•m/s [W]. What would be the momentum if its mass decreased to $\frac{3}{4}$ of its original mass and its speed increased to $\frac{9}{8}$ of its original speed?

**43.** The graph below shows the magnitude of the net force as a function of interaction time for a volleyball being blocked. The velocity of the ball changes from 18 m/s [N] to 11 m/s [S].

(a) Using the graph, calculate the magnitude of the impulse on the volleyball.

(b) What is the mass of the ball?




**44.** (a) Calculate the impulse on a soccer ball if a player heads the ball with an average net force of 120 N [210°] for 0.0252 s.

(b) If the mass of the soccer ball is 0.44 kg, calculate the change in velocity of the ball.

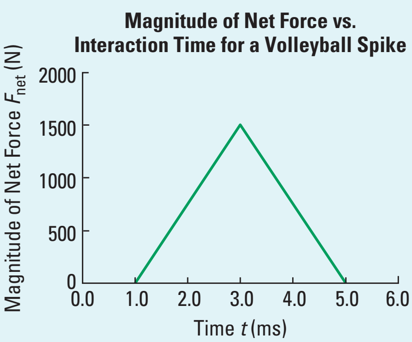
**45.** At a buffalo jump, a 900-kg bison is running at 6.0 m/s toward the drop-off ahead when it senses danger. What horizontal force must the bison exert to stop itself in 2.0 s?

46. (a) What is the minimum impulse needed to give a 275-kg motorcycle and rider a velocity of 20.0 m/s [W] if the motorcycle is initially at rest?
(b) If the wheels exert an average force of 710 N [E] to the road, what is the minimum time needed to reach a velocity of 20.0 m/s [W]?
(c) Explain how the force directed east causes the motorcycle to accelerate westward.
(d) Why is it necessary to specify a minimum impulse and a minimum time?

47. A 1.15-kg peregrine falcon flying at 15.4 m/s [W] captures a 0.423-kg pigeon flying at 4.68 m/s [S]. What will be the velocity of their centre of mass immediately after the interaction?

48. A 275-kg snowmobile carrying a 75-kg driver exerts a net backward force of 508 N on the snow for 15.0 s.
(a) What impulse will the snow provide to the snowmobile and driver?
(b) Calculate the change in velocity of the snowmobile.

49. The graph below shows the magnitude of the net force as a function of time for a 275-g volleyball being spiked. Assume the ball is motionless the instant before it is struck.
(a) Using the graph, calculate the magnitude of the impulse on the volleyball.
(b) What is the speed of the ball when it leaves the player's hand?




50. A Centaur rocket engine expels 520 kg of exhaust gas at $5.0 \times 10^4$ m/s in 0.40 s. What is the magnitude of the net force on the rocket that will be generated?

51. An elevator with passengers has a total mass of 1700 kg. What is the net force on the cable needed to give the elevator a velocity of 4.5 m/s [up] in 8.8 s if it is starting from rest?

52. A 0.146-kg baseball pitched at 40 m/s is hit back toward the pitcher at a speed of 45 m/s.
(a) What is the impulse provided to the ball?
(b) The bat is in contact with the ball for 8.0 ms. What is the average net force that the bat exerts on the ball?

53. An ice dancer and her 80-kg partner are both gliding at 2.5 m/s [225°]. They push apart, giving the 45-kg dancer a velocity of 3.2 m/s [225°]. What will be the velocity of her partner immediately after the interaction?

54. Two students at a barbecue party put on inflatable Sumo-wrestling outfits and take a run at each other. The 87.0-kg student (A) runs at 1.21 m/s [N] and the 73.9-kg student (B) runs at 1.51 m/s [S]. The students are knocked off their feet by the collision. Immediately after impact, student B rebounds at 1.03 m/s [N].
(a) Assuming the collision is completely elastic, calculate the speed of student A immediately after impact using energy considerations.
(b) How different would your answer be if only conservation of momentum were used? Calculate to check.
(c) How valid is your assumption in part (a)?

55. A cannon mounted on wheels has a mass of 1380 kg. It shoots a 5.45-kg projectile at 190 m/s [forward]. What will be the velocity of the cannon immediately after firing the projectile?

56. A 3650-kg space probe travelling at 1272 m/s [0.0°] has a directional thruster rocket exerting a force of $1.80 \times 10^4$ N [90.0°] for 15.6 s. What will be the newly adjusted velocity of the probe?

57. In a movie stunt, a 1.60-kg pistol is struck by a 15-g bullet travelling at 280 m/s [50.0°]. If the bullet moves at 130 m/s [280°] after the interaction, what will be the velocity of the pistol? Assume that no external force acts on the pistol.

58. A 52.5-kg snowboarder, travelling at 1.24 m/s [N] at the end of her run, jumps and kicks off her 4.06-kg snowboard. The snowboard leaves her at 2.63 m/s [62.5° W of N]. What is her velocity just after she kicks off the snowboard?

59. A 1.26-kg brown bocce ball travelling at 1.8 m/s [N] collides with a stationary 0.145-kg white ball, driving it off at 0.485 m/s [84.0° W of N].
(a) What will be the velocity of the brown ball immediately after impact? Ignore friction and rotational effects.
(b) Determine if the collision is elastic.

60. Two people with a combined mass of 128 kg are sliding downhill on a 2.0-kg toboggan at 1.9 m/s. A third person of mass 60 kg inadvertently stands in front and upon impact is swept along with the toboggan. If all three people remain on the toboggan after impact, what will be its velocity after impact?

61. An aerosol paint can is accidentally put in a fire pit. After the fire is lit, the can is heated and explodes into two fragments. A 0.0958-kg fragment (A) flies off at 8.46 m/s [E]. The other fragment (B) has a mass of 0.0627 kg. The 0.0562-kg of gas inside bursts out at 9.76 m/s [N]. What will be the velocity of fragment B immediately after the explosion? Assume that no mass is lost during the explosion, and that the motion of the fragments lies in a plane.

62. A 0.185-kg golf club head travelling horizontally at 28.5 m/s hits a 0.046-kg golf ball, driving it straight off at 45.7 m/s.
    (a) Suppose the golfer does not exert an external force on the golf club after initial contact with the ball. If the collision between the golf club and the ball is elastic, what will be the speed of the club head immediately after impact?
    (b) Show that the law of conservation of momentum is valid in this interaction.

63. A student on a skateboard is travelling at 4.84 m/s [0°], carrying a 0.600-kg basketball. The combined mass of the student and skateboard is 50.2 kg. He throws the basketball to a friend at a velocity of 14.2 m/s [270°]. What is the resulting velocity of the centre of mass of the student-skateboard combination immediately after the throw? Ignore frictional effects.

64. An oxygen molecule of mass $5.31 \times 10^{-26}$ kg with a velocity of 4.30 m/s [0.0°] collides head-on with a $7.31 \times 10^{-26}$-kg carbon dioxide molecule which has a velocity of 3.64 m/s [180.0°]. After collision the oxygen molecule has a velocity of 4.898 m/s [180.0°].
    (a) Calculate the velocity of the carbon dioxide molecule immediately after collision.
    (b) Determine by calculation whether or not the collision is elastic.

65. An isolated stationary neutron is transformed into a $9.11 \times 10^{-31}$-kg electron travelling at $4.35 \times 10^{5}$ m/s [E] and a $1.67 \times 10^{-27}$-kg proton travelling at 14.8 m/s [E]. What is the momentum of the neutrino that is released?

66. An 8.95-kg bowling ball moving at 3.62 m/s [N] hits a 0.856-kg bowling pin, sending it off at 3.50 m/s [58.6° E of N].
    (a) What will be the velocity of the bowling ball immediately after collision?
    (b) Determine if the collision is elastic.

67. A wooden crate sitting in the back of a pickup truck travelling at 50.4 km/h [S] has a momentum of magnitude 560 kg•m/s.
    (a) What is the mass of the crate?
    (b) What impulse would the driver have to apply with the brakes to stop the vehicle in 5.25 s at an amber traffic light? Use $m_T$ for the total mass of the truck.
    (c) If the coefficient of friction between the crate and the truck bed is 0.30, will the crate slide forward as the truck stops? Justify your answer with calculations.

68. A firecracker bursts into three fragments. An 8.5-g fragment (A) flies away at 25 m/s [S]. A 5.6-g fragment (B) goes east at 12 m/s. Calculate the velocity of the 6.7-g fragment (C). Assume that no mass is lost during the explosion, and that the motion of the fragments lies in a plane.

69. A spherical molecule with carbon atoms arranged like a geodesic dome is called a buckyball. A 60-atom buckyball (A) of mass $1.2 \times 10^{-24}$ kg travelling at 0.92 m/s [E] collides with a 70-atom buckyball (B) of mass $1.4 \times 10^{-24}$ kg with a velocity of 0.85 m/s [N] in a laboratory container. Buckyball (A) bounces away at a velocity of 1.24 m/s [65° N of E].
    (a) Calculate the speed of buckyball (B) after the collision assuming that this is an elastic collision.
    (b) Use the conservation of momentum to find the direction of buckyball (B) after the collision.

70. A moose carcass on a sled is being pulled by a tow rope behind a hunter's snowmobile on a horizontal snowy surface. The sled and moose have a combined mass of 650 kg and a momentum of $3.87 \times 10^{3}$ kg•m/s [E].
    (a) Calculate the velocity of the moose and sled.
    (b) The magnitude of the force of friction between the sled and the snow is 1400 N. As the hunter uniformly slows the snowmobile, what minimum length of time is needed for him to stop and keep the sled from running into the snowmobile (i.e., keep the same distance between the sled and the snowmobile)?

71. A 940-kg car is travelling at 15 m/s [W] when it is struck by a 1680-kg van moving at 20 m/s [50.0° N of E]. If both vehicles join together after impact, what will be the velocity of their centre of mass immediately after impact?

72. A 0.450-kg soccer ball is kicked parallel to the floor at 3.24 m/s [E]. It strikes a basketball sitting on a bench, driving it at 2.177 m/s [30.0° S of E]. The soccer ball goes off at 1.62 m/s [60.0° N of E]. What is the mass of the basketball?

73. A cue ball moving at 2.00 m/s [0.0°] hits a stationary three-ball, sending it away at 1.58 m/s [36.0°]. The cue ball and three-ball each have a mass of 0.160 kg. Calculate the velocity of the cue ball immediately after collision. Ignore friction and rotational effects.

74. A hunter claims to have shot a charging bear through the heart and "dropped him in his tracks." To immediately stop the bear, the momentum of the bullet would have to be as great as the momentum of the charging bear. Suppose the hunter was shooting one of the largest hunting rifles ever sold, a 0.50 caliber Sharps rifle, which delivers a $2.27 \times 10^{-2}$ kg bullet at 376 m/s. Evaluate the hunter's claim by calculating the velocity of a 250-kg bear after impact if he was initially moving directly toward the hunter at a slow 0.675 m/s [S].

75. An object explodes into three fragments (A, B, and C) of equal mass. What will be the approximate direction of fragment C if
   (a) both fragments A and B move north?
   (b) fragment A moves east and fragment B moves south?
   (c) fragment A moves [15.0°] and fragment B moves [121°]?

## Extensions

76. Research the physics principles behind the design of a Pelton wheel. Explain why it is more efficient than a standard water wheel. Begin your search at www.pearsoned.ca/school/physicssource.

77. A fireworks bundle is moving upward at 2.80 m/s when it bursts into three fragments. A 0.210-kg fragment (A) moves at 4.52 m/s [E]. A 0.195-kg fragment (B) flies at 4.63 m/s [N]. What will be the velocity of the third fragment (C) immediately after the explosion if its mass is 0.205 kg? Assume that no mass is lost during the explosion.

78. Research the types of rockets being used in NASA's current launchings. What is their thrust and time of firing? If possible, obtain data to calculate the impulse on the rocket. How much does the mass of a rocket change over time as it accelerates?

79. Two billiard balls collide off centre and move at right angles to each other after collision. In what directions did the impulsive forces involved in the collision act? Include a diagram in your answer.

80. A 2200-kg car travelling west is struck by a 2500-kg truck travelling north. The vehicles stick together upon impact and skid for 20 m [48.0° N of W]. The coefficient of friction for the tires on the road surface is 0.78. Both drivers claim to have been travelling at 90 km/h before the crash. Determine the truth of their statements.

81. Research the developments in running shoes that help prevent injuries. Interview running consultants, and consult sales literature and the Internet. How does overpronation or underpronation affect your body's ability to soften the road shock on your knees and other joints? Write a brief report of your findings. Begin your search at www.pearsoned.ca/school/physicssource.

82. A 3.5-kg block of wood is at rest on a 1.75-m high fencepost. When a 12-g bullet is fired horizontally into the block, the block topples off the post and lands 1.25 m away. What was the speed of the bullet immediately before collision?

## Consolidate Your Understanding

83. Write a paragraph describing the differences between momentum and impulse. Include an example for each concept.

84. Write a paragraph describing how momentum and energy concepts can be used to analyze the motion of colliding objects. Include two examples: One is a one-dimensional collision and the other is a two-dimensional collision. Include appropriate diagrams.

### Think About It

Review your answers to the Think About It questions on page 447. How would you answer each question now?

### eTEST

To check your understanding of momentum and impulse, follow the eTest links at www.pearsoned.ca/school/physicssource.