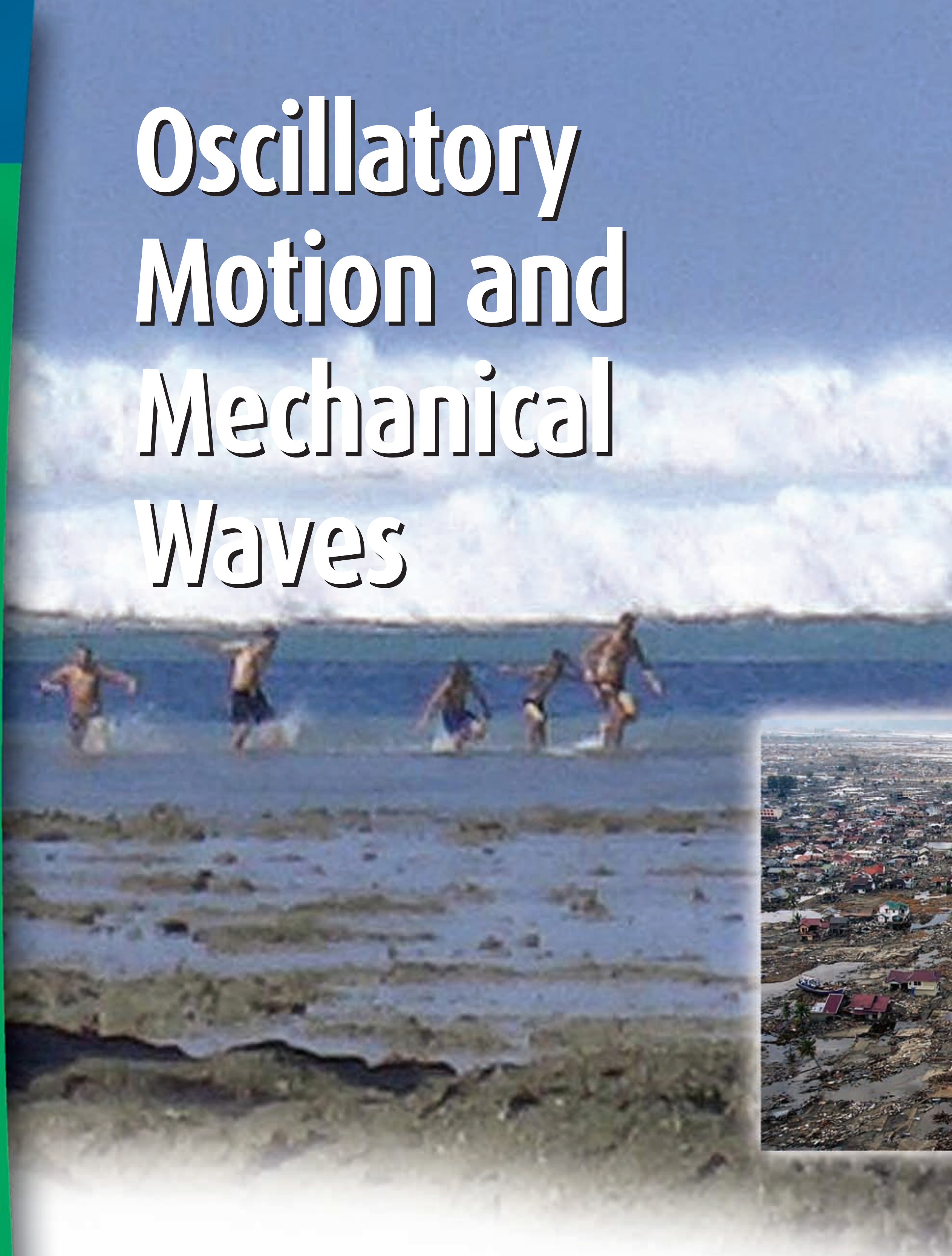
UNIT

IV

Oscillatory Motion and Mechanical Waves



An earthquake more than two thousand kilometres away sent this tsunami speeding across the Indian Ocean. Waves, a form of simple harmonic oscillations, can efficiently transport incredible amounts of energy over great distances. How does a wave move through its medium? How does understanding simple harmonic motion help us understand how waves transport energy?

Unit at a Glance

Unit Themes and Emphases

- Change, Energy, and Matter
- Scientific Inquiry
- Nature of Science

Focussing Questions

As you study this unit, consider these questions:

- Where do we observe oscillatory motion?
- How do mechanical waves transmit energy?
- How can an understanding of the natural world improve how society, technology, and the environment interact?

Unit Project

- By the time you complete this unit, you will have the knowledge and skill to research earthquakes, the nature of earthquake shock waves, and the use of the Richter scale to rate earthquake intensity. On completion of your research, you will demonstrate the operation of a seismograph.

eWEB

To learn more about earthquakes and their environmental effects, follow the links at www.pearsoned.ca/school/physicssource.

CHAPTER

7

Oscillatory motion requires a set of conditions.

Key Concepts

In this chapter, you will learn about:

- oscillatory motion
- simple harmonic motion
- restoring force
- oscillating spring and pendulum
- mechanical resonance

Learning Outcomes

When you have completed this chapter, you will be able to:

Knowledge

- describe oscillatory motion in terms of period and frequency
- define simple harmonic motion as being due to a restoring force that is directly proportional and opposite to the displacement of an object from an equilibrium position
- explain the quantitative relationships among displacement, acceleration, velocity, and time for simple harmonic motion
- define mechanical resonance

Science, Technology, and Society

- explain that the goal of science is knowledge about the natural world

On October 15, 1997, NASA launched the Cassini-Huygens space probe toward Saturn — a distance of 1 500 000 000 km from Earth. The probe's flight path took it by Venus twice, then Earth, and finally past Jupiter on its way to Saturn. This route was planned so that, as the probe approached each planet, it would be accelerated by the planet's gravitational force. Each time, it picked up more speed, allowing it to get to Saturn more quickly (Figure 7.1). Recall from Chapter 4 that increasing the probe's speed this way is referred to as gravity assist.

The entire journey of 3 500 000 000 km took seven years. For this incredible feat to succeed, scientists had to know where the planets would be seven years in the future. How could they do this? They relied on the fact that planets follow predictable paths around the Sun. Nature is full of examples of repetitive, predictable motion. Water waves, a plucked guitar string, the orbits of planets, and even a bumblebee flapping its wings are just a few.

In this chapter, you will examine oscillatory motion. Oscillatory motion is a slightly different form of motion from the circular motion you studied in Chapter 5. You will better understand why bungee jumpers experience the greatest pull of the bungee cord when they are at the bottom of a fall and why trees sway in the wind. You may notice the physics of many objects that move with oscillatory motion and gain a new insight into the wonders of the natural world.

▲ **Figure 7.1** The Cassini-Huygens probe successfully orbits Saturn after a seven-year journey through the solar system.

7-1 QuickLab

Oscillatory Motion of Toys

Problem

What is the time taken by one complete back-and-forth motion of a toy?

Materials

several different wind-up toys, such as:
- swimming frog
- hopping toy
- monkey with cymbals
- walking toy

yo-yo
stopwatch (or wristwatch with a second hand)

▲ **Figure 7.2**

Procedure

1. Fully wind the spring mechanism of one of the toys.
2. Release the winding knob and start your stopwatch.
3. Count the number of complete back-and-forth movements the toy makes in 10 s. These back-and-forth movements are called oscillations.
4. Record the number of oscillations made by the toy.
5. Repeat steps 1 to 4 for each toy. For the yo-yo, first achieve a steady up-and-down rhythm. Then do steps 3 and 4, counting the up-and-down motions. This type of repetitive movement is also an oscillation.

Questions

1. In what ways are the motions of all the toys similar?
2. Divide the number of oscillations of each toy by the time. Be sure to retain the units in your answer. What does this number represent? (Hint: Look at the units of the answer.)
3. Which toy had the most oscillations per second? Which had the least?
4. Divide the time (10 s) by the number of oscillations for each toy. Be sure to retain the units in your answer. How long is the interval of oscillation of each toy?
5. Which toy had the longest time for one oscillation? Which had the shortest time?

Think About It

1. How are the oscillations per second and the time for one oscillation related?
2. What do you think influences the number of oscillations per second of the toys you tested?

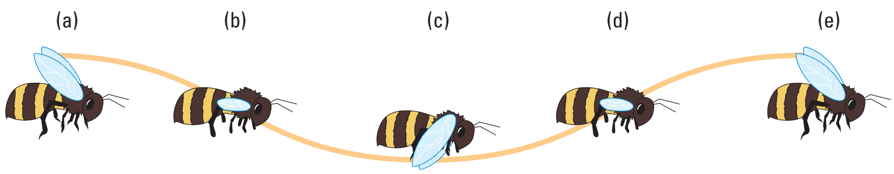
Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes to your ideas.

7.1 Period and Frequency

▲ **Figure 7.3** The wings of a bee in flight make a droning sound because of their motion.

On a warm summer day in your backyard, you can probably hear bees buzzing around, even if they are a few metres away. That distinctive sound is caused by the very fast, repetitive up-and-down motion of the bees' wings (Figure 7.3).

Take a closer look at the bumblebee. The motion of a bee's wings repeats at regular intervals. Imagine that you can examine the bee flying through the air. If you start your observation when its wings are at the highest point (Figure 7.4(a)), you see them move down to their lowest point (Figure 7.4(c)), then back up again. When the wings are in the same position as when they started (Figure 7.4(e)), one complete **oscillation** has occurred. An oscillation is a repetitive back-and-forth motion. One complete oscillation is called a **cycle**.



▲ **Figure 7.4** The bee's wings make one full cycle from (a) to (e). The time for this motion is called the period.

oscillatory motion: motion in which the period of each cycle is constant

The time required for the wings to make one complete oscillation is the period (T). If the period of each cycle remains constant, then the wings are moving up and down with **oscillatory motion**. Recall from Chapter 5 that the number of oscillations per second is the frequency (f), measured in hertz (Hz). The equation that relates frequency and period is:

$$f = \frac{1}{T} \qquad (1)$$

info **BIT**

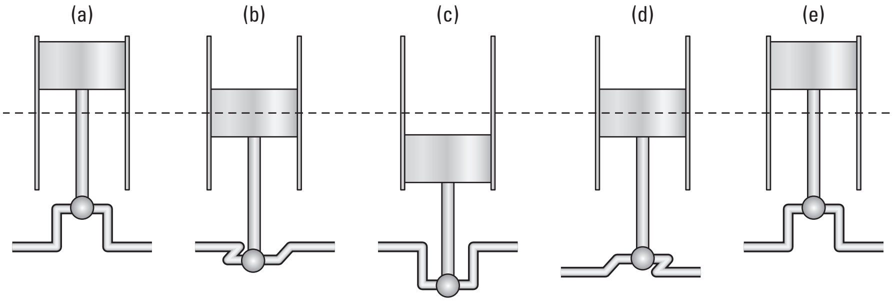
Earthquake waves can have periods of up to several hundred seconds.

Table 7.1 shows the period of a bee's wings as it hovers, along with other examples of periods.

▼ **Table 7.1** Periods of Common Items

Object	Period
Bumblebee wings	0.00500 s
Hummingbird wings	0.0128 s
Medical ultrasound technology	1×10^{-6} to 5×10^{-8} s
Middle C on a piano	0.0040 s
Electrical current in a house	0.0167 s

A piston in the engine of a car also undergoes oscillatory motion if it is moving up and down in equal intervals of time. The piston shown in Figure 7.5 moves from position (a) (its highest point) through (b) to position (c), where it is at its lowest point. It begins moving back up again through (d) to (e), where it returns to its highest position. The range of movement from (a) to (e) is one cycle. A single piston in a Formula 1 racecar can achieve a frequency of 300 cycles/second or 300 Hz (18 000 rpm). The piston makes 300 complete cycles in only 1 s. Conversely, the period of the piston, which is the time for one complete cycle, is the inverse of the frequency. It is a mere 0.003 s or about 100 times faster than the blink of an eye!



▲ **Figure 7.5** The piston makes one complete cycle from positions (a) to (e). The time it takes to do this is its period. The number of times it does this in 1 s is its frequency.

e MATH

Using a graphing calculator, plot period as a function of frequency. Use the following values of frequency: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. If possible, print this graph, label it, and add it to your notes. What is the shape of this graph?

info BIT

A Formula 1 racecar has 10 cylinders but the engine size is limited to 3.0 L, no bigger than many engines in medium-sized cars. The fuel efficiency of F1 cars is approximately 1.3 km/L.

Example 7.1

What is the frequency of an automobile engine in which the pistons oscillate with a period of 0.0625 s?

Analysis and Solution

$$f = \frac{1}{T}$$

$$= \frac{1}{0.0625\ \text{s}}$$

$$= 16.0\ \text{Hz}$$

The frequency of the engine is 16.0 Hz.

Practice Problems

1. Earthquake waves that travel along Earth's surface can have periods of up to 5.00 minutes. What is their frequency?
2. A hummingbird can hover when it flaps its wings with a frequency of 78 Hz. What is the period of the wing's motion?

Answers

1. 3.33×10^{-3} Hz
2. 0.013 s

MINDS ON Examples of Oscillatory Motion

Working with a partner or group, make a list of three or four natural or human-made objects that move with the fastest oscillatory motion that you can think of.

Make a similar list of objects that have very long periods of oscillatory motion. Beside each object estimate its period. The lists must not include the examples already mentioned. Be prepared to share your lists with the class.

7-2 Inquiry Lab

Relating Period and Frequency

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Your teacher may want to do this Inquiry Lab in the gym instead of the science lab.

Question

What is the relationship between the period and the frequency of a bouncing ball?

Hypothesis

Write a hypothesis that relates the period of the ball's bounce to its frequency. Remember to use an "if/then" statement.

Variables

The variables in this lab are the height of the bounce, period, and frequency. Read the procedure, then determine and label the controlled, manipulated, and responding variables.

Materials and Equipment

- basketball
- metre-stick
- tape
- stopwatch
- chair

Procedure

1. Copy Table 7.2 into your notes.

▼ Table 7.2 Bounce, Period, and Frequency

Bounce Height (cm)	Time for 20 Bounces (s)	Period (s/bounce)	Frequency (bounces/s)
50			
75			
100			
125			
150			

2. Find a convenient place to bounce the basketball near the wall. Using the metre-stick, place tape at heights of 50, 75, 100, 125, and 150 cm. Mark the heights on the tape.
3. Using just a flick of the wrist, begin bouncing the ball at the 50-cm mark. The top of the ball should just make it to this height at the top of its bounce. The ball should bounce with a steady rhythm.
4. While one person is bouncing the ball, another person uses the stopwatch to record the time taken for 20 complete bounces. Record this time in Table 7.2.
5. Reset the stopwatch and begin bouncing the ball to the next height up. Record the time for 20 complete bounces as you did in step 4. To achieve the proper height you may have to stand on a chair.

Analysis

1. Using the data for 20 bounces, determine the period and frequency for each height. Record the values in the table.
2. Draw a graph of frequency versus period. What type of relationship is this?
3. What effect did increasing the bounce height have on the period of a bounce?

In 7-2 Inquiry Lab, the ball will make many more bounces in a certain length of time if it travels a shorter distance than a longer one. Its frequency will be high. By necessity, the amount of time it takes to make one bounce (its period) will be small. The next section explores oscillatory motion by going one step further. You will examine a type of oscillatory motion in which the range of motion is related to the applied force.

7.1 Check and Reflect

Knowledge

1. What conditions describe oscillatory motion?
2. Which unit is equivalent to cycles/s?
3. Define period and frequency.
4. How are period and frequency related?
5. Is it possible to increase the period of an oscillatory motion without increasing the frequency? Explain.
6. Give three examples of oscillatory motion that you have observed.

Applications

7. What is the frequency of a swimming water toy that makes 20.0 kicking motions per second?
8. Do the oars on a rowboat move with oscillatory motion? Explain.
9. Determine the frequency of a guitar string that oscillates with a period of 0.004 00 s.
10. A dragonfly beats its wings with the frequency of 38 Hz. What is the period of the wings?
11. A red-capped manakin is a bird that can flap its wings faster than a hummingbird, at 4800 beats per minute. What is the period of its flapping wings?
12. A dog, happy to see its owner, wags its tail 2.50 times a second.
    (a) What is the period of the dog's wagging tail?
    (b) How many wags of its tail will the dog make in 1 min?

Extensions

13. Use your library or the Internet to research the frequency of four to six different types of insect wings. Rank these insect wings from highest to lowest frequency.
14. Which of these motions is oscillatory? Explain.
    (a) a figure skater moving with a constant speed, performing a figure eight
    (b) a racecar racing on an oval track
    (c) your heartbeat
15. Many objects exhibit oscillatory motion. Use your library or the Internet to find the frequency or range of frequencies of the objects below.
    (a) fluorescent light bulbs
    (b) overhead power lines
    (c) human voice range
    (d) FM radio range
    (e) lowest note on a bass guitar

e TEST

To check your understanding of period and frequency, follow the eTest links at www.pearsoned.ca/school/physicssource.

7.2 Simple Harmonic Motion

info BIT
A human eardrum can oscillate back and forth up to 20 000 times a second.

Children on swings can rise to heights that make their parents nervous. But to the children, the sensation of flying is thrilling. At what positions on a swing does a child move fastest? When does the child's motion stop?

Many objects that move with oscillatory motion exhibit the same properties that a child on a swing does. A piston moves up and down in the cylinder of an engine. At the extreme of its motion, it stops for a brief instant as it reverses direction and begins to accelerate downward until it reaches the bottom of its stroke. There it stops again and accelerates back, just as the swing does.

In order for the piston or a child on a swing to experience acceleration, it must experience a non-zero net force. This section explores how the net force affects an object's motion in a special type of oscillatory motion called simple harmonic motion.

7-3 QuickLab

Determining the Stiffness of a Spring

Problem

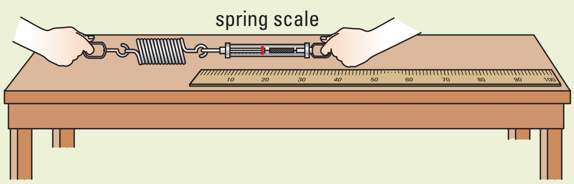
How does the force applied to a spring affect its displacement?

Materials

spring (with loops at each end)
spring scale
metre-stick
tape

Procedure

1. Make a two-column table in your notebook. Write "Displacement (cm)" as the heading of the left column, and "Force (N)" as the heading of the right column.
2. Determine the maximum length the spring can be pulled without permanently deforming it. If you are unsure, ask your instructor what the maximum displacement of the spring is. Divide this length by 5. This will give you an idea of the even increments through which to pull your spring.
3. Lie the spring flat on the surface of the table so that it lies in a straight line. Leave room for it to be stretched. Do not pull on the spring.
4. Fix one end of the spring to the desk by holding the loop at the end of the spring with your hand. Do not let this end of the spring move.
5. Attach the spring scale to the free end of the spring but do not pull on it yet.
6. Align the 0-cm mark of the metre-stick with the other end of the spring at exactly where the spring scale is attached. Tape the stick to the desk (Figure 7.6).



▲ **Figure 7.6**

7. Pull the spring, using the spring scale, by the incremental distance determined in step 2. Record the values of the displacement and force in your table.
8. Repeat step 7, until you have five values each for displacement and force in your table.
9. Gently release the tension from the spring. Clean up and put away the equipment at your lab station.

Questions

1. Determine the controlled, manipulated, and responding variables.

2. Plot a graph of force versus displacement. Be sure to use a scale that allows you to use the full graph paper. Draw a line of best fit.

3. What kind of relationship does the line of best fit represent?

4. Determine the slope of the line. What are the units of the slope?

5. Extrapolate where the line intercepts the horizontal axis. Why does it intercept there?

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Hooke's Law

Robert Hooke (1635–1703) was a British scientist best remembered for using a microscope to discover plant cells, but his talents extended into many areas (Figure 7.7). He is credited with inventing the universal joint, which is used today on many mechanical devices (including cars); the iris diaphragm used to adjust the amount of light that enters a camera lens; the respirator to help people breathe; and the compound microscope, just to name a few of his inventions. In the field of oscillatory motion, he is acknowledged for his work with elastic materials and the laws that apply to them (Figure 7.8).

▲ **Figure 7.7** Robert Hooke lived at the same time as Sir Isaac Newton.

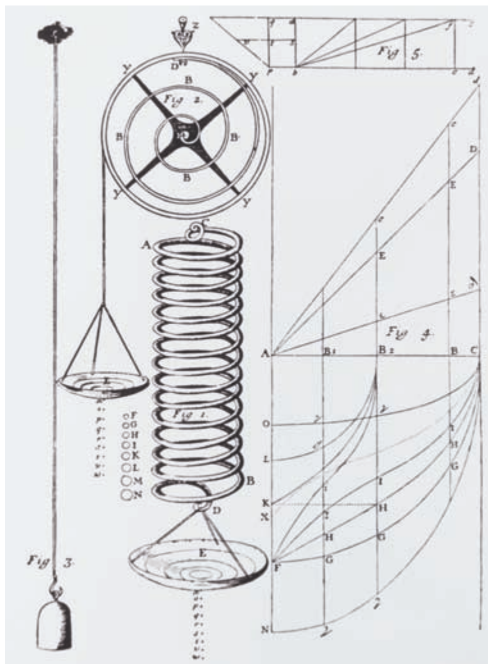
In 1676, Hooke recognized that the more stress (force) is applied to an object, the more strain (deformation) it undergoes. The stress can be applied in many ways. For example, an object can be squeezed, pulled, or twisted. Elastic materials will usually return to their original state after the stress has been removed. This will not occur if too much force is applied or if the force is applied for too long a time. In those cases, the object will become permanently deformed because it was strained beyond the material's ability to withstand the deformation. The point at which the material cannot be stressed farther, without permanent deformation, is called the elastic limit.

A spring is designed to be a very elastic device, and the deformation of a spring (whether it is stretched or compressed) is directly related to the force applied. The deformation of a spring is referred to as its displacement. From his observations, Hooke determined that the deformation (displacement) is proportional to the applied force. This can also be stated as "force varies directly with displacement." It can be written mathematically as:

$$F \propto x$$

This relationship is known as **Hooke's law**, which states:

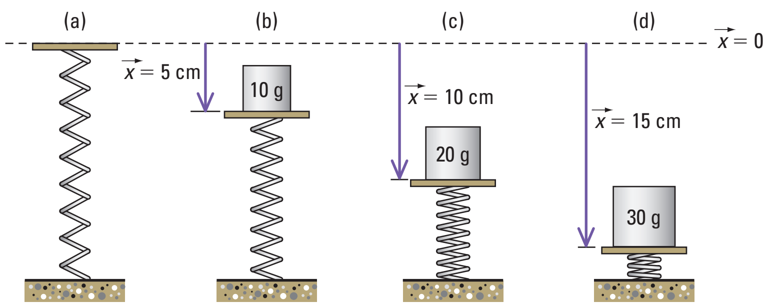
The deformation of an object is proportional to the force causing the deformation.



▲ **Figure 7.8** Hooke's notes show the simple devices he used to derive his law.

Hooke's law: the deformation of an object is proportional to the force causing it

Figure 7.9(a) shows a spring that conforms to Hooke's law. It experiences a displacement that is proportional to the applied force. When no mass is applied to the spring, it is not compressed, and it is in its equilibrium position. As mass is added in increments of 10 g, the displacement (deformation) of the spring increases proportionally in increments of 5 cm, as shown in Figures 7.9(b), (c), and (d).



▲ **Figure 7.9** The spring pictured above conforms to Hooke's law. If the mass is doubled, the displacement will also double, as seen in (b) and (c). If the force (mass) is tripled, the displacement will triple, as seen in (b) and (d).

spring constant: amount of stiffness of a spring

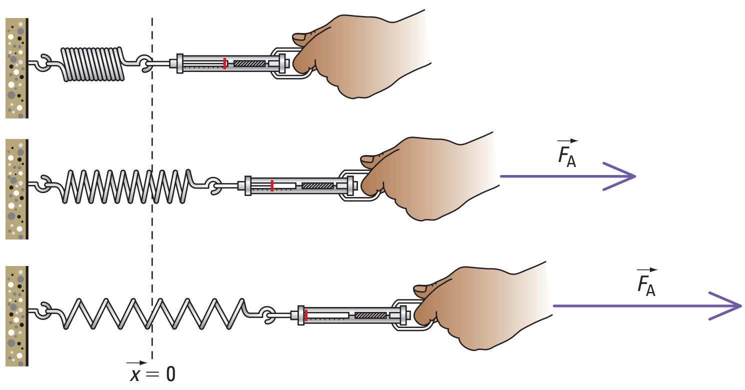
Each spring is different, so the force required to deform it will change from spring to spring. The stiffness of the spring, or **spring constant**, is represented by the letter k. Using k, you can write the equation for Hooke's law as:

$$\vec{F} = k\vec{x}$$

where $\vec{F}$ is the applied force that extends or compresses the spring, k is the spring constant, and $\vec{x}$ is the displacement from the equilibrium position.

Graphing Hooke's Law

Figure 7.10 shows how you can use a force meter to measure the applied force required to pull the spring from its equilibrium position to successively farther displacements. For simplicity, we'll assume that all the springs used in this text are "ideal" springs, meaning that they have no mass.

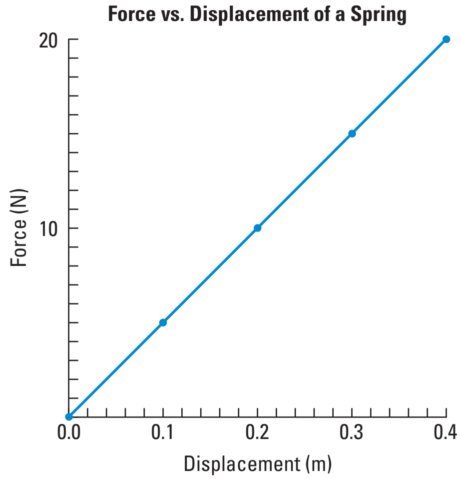


▲ **Figure 7.10** A force meter is attached to a spring that has not been stretched. The spring is then pulled through several displacements. Each time, the force required for the displacement is recorded.

Table 7.3 shows the data collected for this spring and the results plotted on the graph shown in Figure 7.11.

▼ **Table 7.3** Data for Figure 7.11

Displacement (m)	Force (N)
0.00	0.0
0.10	5.0
0.20	10.0
0.30	15.0
0.40	20.0



◄ **Figure 7.11** Graph of data from Table 7.3

Notice that the relationship is linear (a straight line), which means force is proportional to the displacement. The slope of the line can be determined by the following calculations:

$$\text{slope} = \frac{\Delta F}{\Delta x}$$

$$= \frac{(20.0\ \text{N} - 0.0\ \text{N})}{(0.40\ \text{m} - 0.00\ \text{m})}$$

$$= 50\ \text{N/m}$$

This slope represents the spring constant k. The variables F and x are vectors but here we are calculating their scalar quantities so no vector arrows are used. In this example, an applied force of 50 N is needed to stretch (or compress) this spring 1 m. Therefore, the units for the spring constant are newtons per metre (N/m). By plotting the force-displacement graph of a spring and finding its slope, you can determine the spring constant of any ideal spring or spring that obeys Hooke's law.

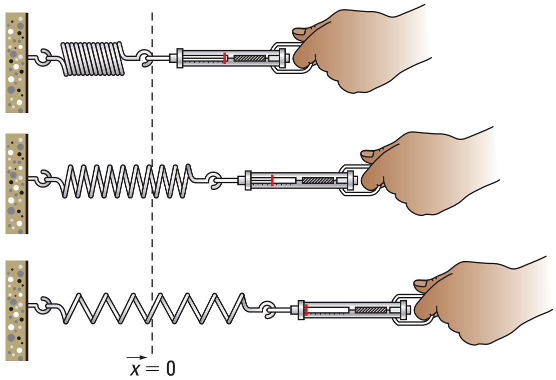
Of the many objects that display elastic properties, springs are arguably the best to examine because they obey Hooke's law over large displacements. Steel cables are also elastic when stretched through relatively small displacements. Even concrete displays elastic properties and obeys Hooke's law through very small displacements.

In simpler terms, the property of elasticity gives a material the ability to absorb stress without breaking. This property is vital to consider when structural engineers and designers build load-bearing structures such as bridges and buildings (Figure 7.12). You will learn more about the factors that must be considered in bridge and building design later in this chapter.

▲ **Figure 7.12** The Jin Mao Tower in Shanghai, China, is 88 storeys high. Skyscrapers are built with elastic materials so they can sway in high winds and withstand the shaking of an earthquake. The main building materials for the Jin Mao Tower are concrete and steel.

Example 7.2

To determine the spring constant of a spring, a student attaches a force meter to one end of the spring, and the other end to a wall as shown in Figure 7.13. She pulls the spring incrementally to successive displacements, and records the values of displacement and force in Table 7.4. Plot the values on a graph of force as a function of displacement. Using a line of best fit, determine the spring constant of the spring.



▲ **Figure 7.13**

Given

▼ **Table 7.4** Data for Figure 7.14

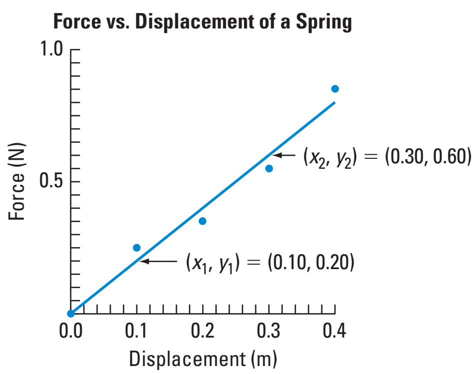
Displacement (m)	Force (N)
0.00	0.00
0.10	0.25
0.20	0.35
0.30	0.55
0.40	0.85

Required

spring constant (k)

Analysis and Solution

Using the values from Table 7.4, plot the graph and draw a line of best fit.



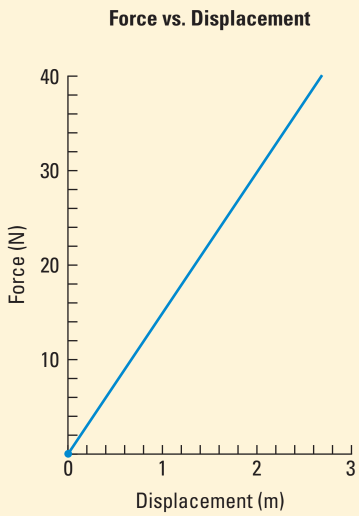
▲ **Figure 7.14**

Practice Problems

1. A spring is stretched through several displacements and the force required is recorded. The data are shown below. Determine the spring constant of this spring by plotting a graph and finding the slope.

Displacement (m)	Force (N)
0.00	0.0
0.10	20.0
0.20	50.0
0.30	80.0
0.40	95.0
0.50	130.0
0.60	150.0

2. Determine the spring constant of a spring that has the force-displacement graph shown in Figure 7.15.



▲ **Figure 7.15**

Answers

1. 2.5×10^2 N/m
2. 15 N/m

The slope of the line gives the spring constant (k). Pick two points from the line and solve for the slope using the equation below. Note the points used in the equation are not data points.

$$\text{slope} = k = \frac{\Delta F}{\Delta x}$$

point 1 = (0.10, 0.20)

point 2 = (0.30, 0.60)

$$k = \frac{(0.60\ \text{N} - 0.20\ \text{N})}{(0.30\ \text{m} - 0.10\ \text{m})}$$

$$= 2.0\ \text{N/m}$$

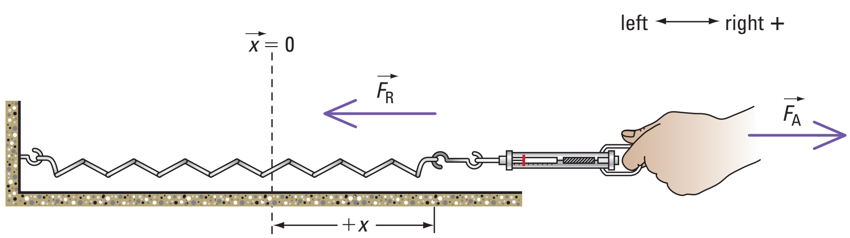
Paraphrase

The spring constant of the spring is 2.0 N/m.

The Restoring Force

Imagine that you have applied a force to pull a spring to a positive displacement ($\vec{x}$) as shown in Figure 7.16. While you hold it there, the spring exerts an equal and opposite force in your hand, as described by Newton's third law in Chapter 3. However, this force is to the left, in the negative direction, and attempts to restore the spring to its equilibrium position. This force is called the **restoring force**.

restoring force: a force acting opposite to the displacement to move the object back to its equilibrium position



▲ **Figure 7.16** The spring system is pulled from its equilibrium position to displacement $\vec{x}$. The displacement is positive, but the restoring force is negative.

The restoring force always acts in a direction opposite to the displacement. Therefore, Hooke's law is properly written with a negative sign when representing the restoring force.

$$\vec{F} = -k\vec{x} \qquad (2)$$

In this case, while the spring is held in this position, the applied force and the restoring force have equal magnitudes but opposite directions, so the net force on the system is zero. In the next section, a mass will be attached to the spring and it will slide on a frictionless horizontal surface. The restoring force will be the only force in the system and will give rise to a repetitive back-and-forth motion called simple harmonic motion.

Example 7.3

A spring has a spring constant of 30.0 N/m. This spring is pulled to a distance of 1.50 m from equilibrium as shown in Figure 7.17. What is the restoring force?

Practice Problems

1. Determine the restoring force of a spring displaced 55.0 cm. The spring constant is 48.0 N/m.
2. A spring requires a force of 100.0 N to compress it a displacement of 4.0 cm. What is its spring constant?

Answers

1. −26.4 N
2. 2.5×10^3 N/m

Analysis and Solution

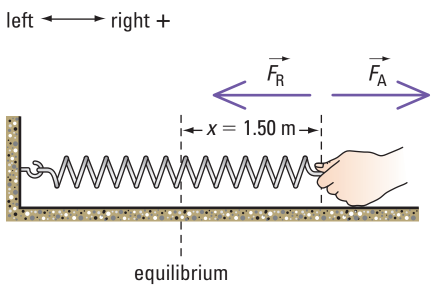
Draw a diagram to represent the stretched spring. Displacement to the right is positive, so the restoring force is negative because it is to the left, according to Hooke's law.

$$\vec{F} = -k\vec{x}$$

$$= \left(-30.0\ \frac{\text{N}}{\cancel{\text{m}}}\right)(1.50\ \cancel{\text{m}})$$

$$= -45.0\ \text{N}$$

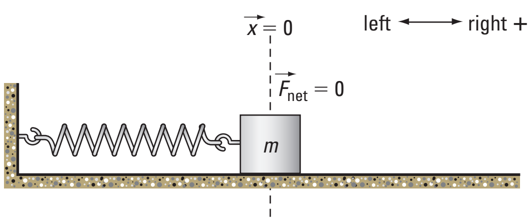
The restoring force is 45.0 N [left].



▲ **Figure 7.17**

Simple Harmonic Motion of Horizontal Mass-spring Systems

Figure 7.18 shows a mass attached to an ideal spring on a horizontal frictionless surface. This simple apparatus can help you understand the relationship between the oscillating motion of an object and the effect the restoring force has on it.

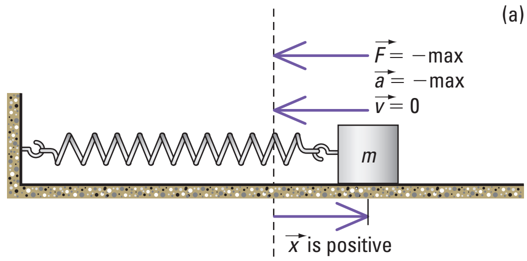


◀ **Figure 7.18** The mass is in its equilibrium position ($\vec{x} = 0$) and is at rest. There is no net force acting on it. Any displacement of the mass to the right is positive, and to the left, negative.

eSIM

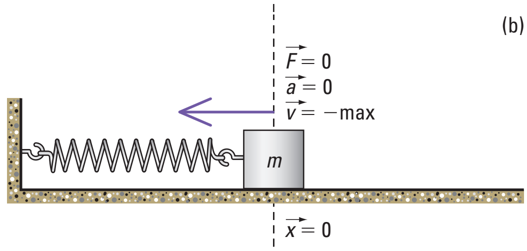
Observe a simulation of simple harmonic motion in a horizontal mass-spring system. Follow the eSIM links at www.pearsoned.ca/school/physicssource.

The position of the mass is represented by the variable $\vec{x}$ and is measured in metres. In Figure 7.18, there is no tension on the spring nor restoring force acting on the mass, because the mass is in its equilibrium position. Figure 7.19 shows how the restoring force affects the acceleration, displacement, and velocity of the mass when the mass is pulled to a positive displacement and released.



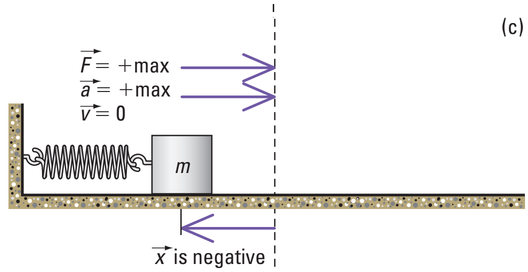
▶ **Figure 7.19(a)** The mass has been pulled to its maximum displacement, called its **amplitude** (symbol A). When the mass is released, it begins oscillating with a displacement that never exceeds this distance. The greater the amplitude, the more energy a system has. In this diagram, $\vec{x} = A$.

At maximum displacement, the restoring force is at a maximum value, and therefore, so is the acceleration of the mass, as explained by Newton's second law ($\vec{F} = m\vec{a}$). When the mass is released, it accelerates from rest ($\vec{v} = 0$) toward its equilibrium position. As the mass approaches this position, its velocity is increasing. But the restoring force is decreasing because the spring is not stretched as much. Remember that force varies directly with displacement.

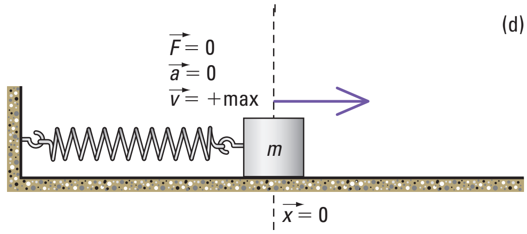


▶ **Figure 7.19(b)** As the mass returns to its equilibrium position ($\vec{x} = 0$), it achieves its maximum velocity. It is moving toward the left (the negative direction), but the restoring force acting on it is zero because its displacement is zero.

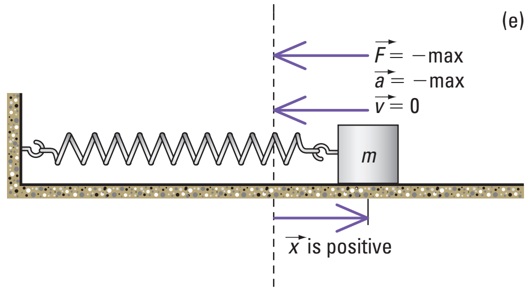
The mass continues to move through the equilibrium position and begins to compress the spring. As it compresses the spring, the restoring force acts on the mass toward the right (the positive direction) to return it to its equilibrium position. This causes the mass to slow down, and its velocity approaches zero.



▶ **Figure 7.19(c)** After passing through the equilibrium position, the mass experiences a restoring force that opposes its motion and brings it to a stop at the point of maximum compression. Its amplitude here is equal, but opposite to its amplitude when it started. At maximum displacement, the velocity is zero. The restoring force has reached its maximum value again. The restoring force is positive, and the displacement is negative. The restoring force again accelerates the mass toward equilibrium.



▶ **Figure 7.19(d)** The mass has accelerated on its way to the equilibrium position where it is now. The restoring force and acceleration are again zero, and the velocity has achieved the maximum value toward the right. At equilibrium, the mass is moving to the right. It has attained the same velocity as in diagram (b), but in the opposite direction.



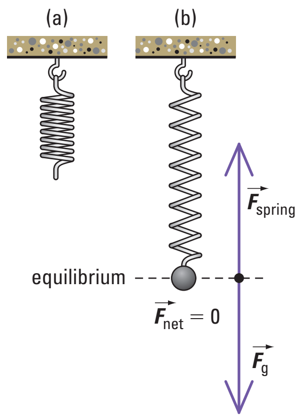
▶ **Figure 7.19(e)** The mass has returned to the exact position where it was released. Again the restoring force and acceleration are negative and the velocity is zero. The oscillation will repeat again as it did in diagram (a).

In Figure 7.19(e), the mass has returned to the position where it started, and one full oscillation has occurred. Throughout its entire motion, the mass-spring system obeys Hooke's law. In other words, at any instant, the restoring force is proportional to the displacement of the mass. Any object that obeys Hooke's law undergoes **simple harmonic motion** (SHM). SHM is oscillatory motion where the restoring force is proportional to the displacement of the mass. An object that moves with SHM is called a **simple harmonic oscillator**.

simple harmonic motion: oscillatory motion where the restoring force is proportional to the displacement of the mass

simple harmonic oscillator: an object that moves with simple harmonic motion

Simple Harmonic Motion of Vertical Mass-spring Systems

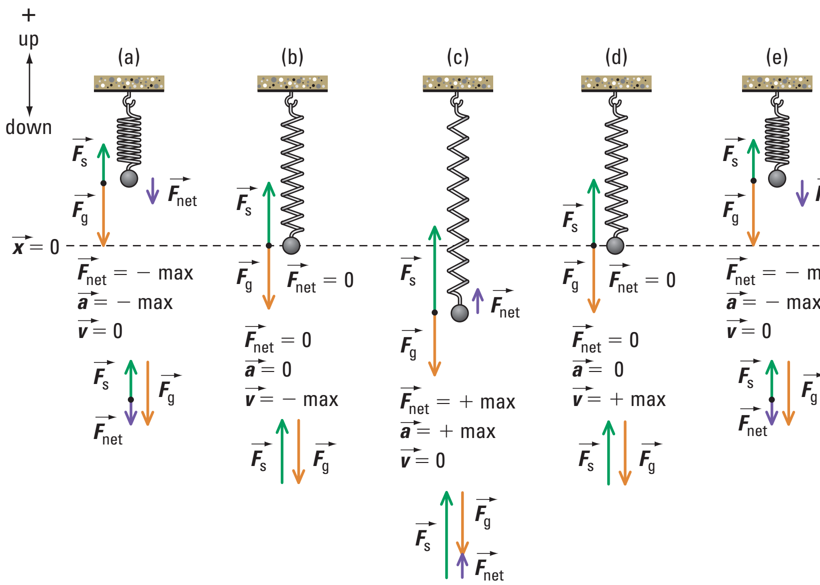


▲ **Figure 7.20** The spring in (a) has no mass attached. In (b), the spring stretches until the force exerted by the mass is equal and opposite to the force of gravity, and equilibrium is reached. The net force (or restoring force) is the vector sum of the force of gravity and the tension of the spring. In this case, it is zero.

Figure 7.20(a) shows a spring without a mass attached, anchored to a ceiling. Assume that the spring itself is massless, so it will not experience any displacement. When a mass is attached, the spring is pulled down and deforms as predicted by Hooke's law. The mass will come to rest when the downward force of gravity is equal to the upward pull (tension) of the spring (Figure 7.20(b)). The displacement of the spring depends on its spring constant. A weak spring has a small spring constant. It will stretch farther than a spring with a large spring constant.

In Figure 7.20(b), the net force (or restoring force) acting on the mass is zero. It is the result of the upward tension exerted by the spring balancing the downward force of gravity. This position is considered the equilibrium position and the displacement is zero.

If the mass is lifted to the position shown in Figure 7.21(a) and released, it will begin oscillating with simple harmonic motion. Its amplitude will equal its initial displacement. Regardless of the position of the mass, the force of gravity remains constant but the tension of the spring varies. In the position shown in Figures 7.21(b) and (d), the net (restoring) force is zero. This is where the spring's tension is equal and opposite to the force of gravity. In the position shown in Figure 7.21(c), the displacement of the spring is equal to the amplitude, and the tension exerted by the spring is at its maximum. The mass experiences the greatest restoring force, which acts upward.



▶ **Figure 7.21** The net force (the restoring force) is the vector sum of the upward force exerted by the spring and the downward force of gravity. The force of gravity is always negative and constant, but the force exerted by the spring varies according to the displacement, so the net force changes as the position of the mass changes from (a) to (e). The values of $\vec{F}_{net}$, $\vec{a}$, and $\vec{v}$ are identical to the horizontal mass-spring system.

When the mass is below the equilibrium position, the upward force exerted by the tension of the spring is greater than the gravitational force. So the net force — and therefore the restoring force — is upward. Above the equilibrium position, the downward force of gravity exceeds the upward tension of the spring, and the restoring force is downward. The values of velocity, acceleration, and restoring force change in exactly the same way that they do in a horizontal mass-spring system.

PHYSICS INSIGHT

For any frictionless simple harmonic motion, the restoring force is equal to the net force.

Example 7.4

A spring is hung from a hook on a ceiling. When a mass of 510.0 g is attached to the spring, the spring stretches a distance of 0.500 m. What is the spring constant?

Given

$x = 0.500$ m

$m = 510.0$ g $= 0.5100$ kg

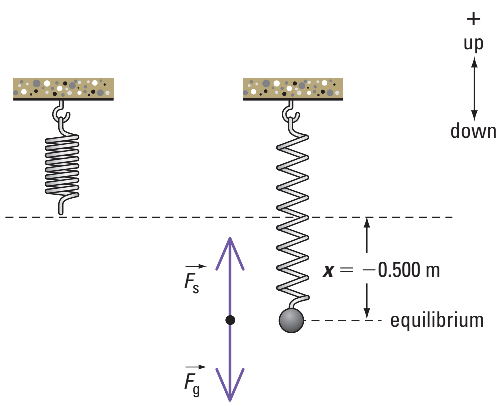
$g = 9.81$ m/s^2

Required

spring constant (k)

Analysis and Solution

Draw a diagram to show the mass-spring system and the forces acting on the mass.



▲ **Figure 7.22**

The mass is not moving so the net force on the mass is zero. $\vec{F}_s$ and $\vec{F}_g$ are therefore equal in magnitude.

$$kx = mg$$

$$k = \frac{mg}{x}$$

$$= \frac{(0.5100 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{0.500 \text{ m}}$$

$$= 10.0 \text{ N/m}$$

Paraphrase

The spring constant is 10.0 N/m.

Practice Problems

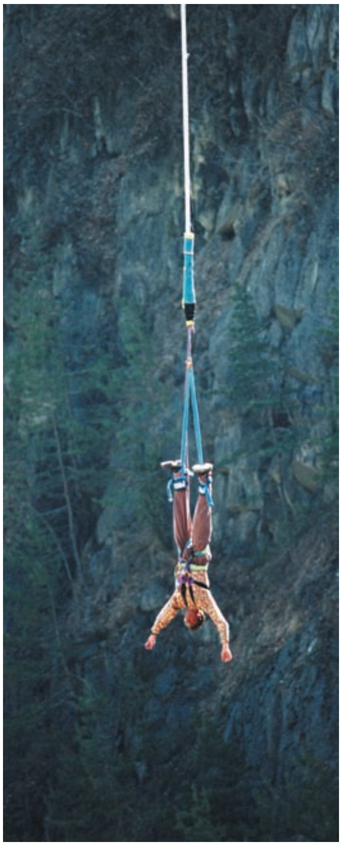
1. Five people with a combined mass of 275.0 kg get into a car. The car's four springs are each compressed a distance of 5.00 cm. Determine the spring constant of the springs. Assume the mass is distributed evenly to each spring.
2. Two springs are hooked together and one end is attached to a ceiling. Spring A has a spring constant (k) of 25 N/m, and spring B has a spring constant (k) of 60 N/m. A mass weighing 40.0 N is attached to the free end of the spring system to pull it downward from the ceiling. What is the total displacement of the mass?

Answers

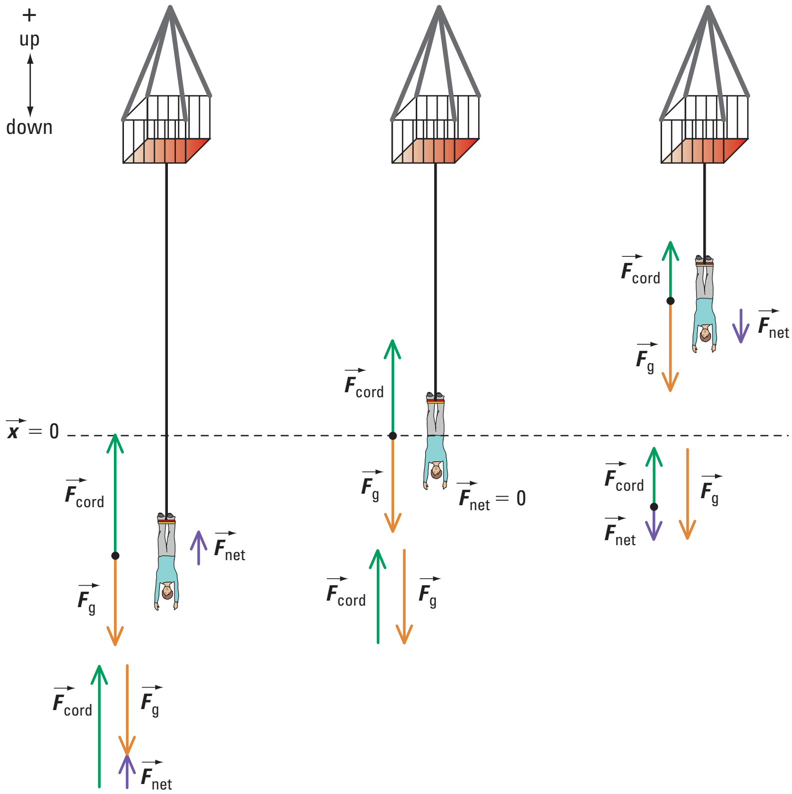
1. 1.35×10^4 N/m
2. -2.3 m

Examples of Simple Harmonic Motion

Provided the cord doesn't go slack, a person making a bungee jump will bob up and down with SHM, as shown in Figures 7.23 and 7.24. The cord acts as the spring and the person is the mass.



▲ **Figure 7.23** A bungee jumper experiences SHM as long as the cord does not go slack.



▲ **Figure 7.24** The bungee jumper bouncing up and down on the cord after a jump in (a) is a vertical mass-spring system. The cord acts as a spring and the jumper is the mass. The restoring (net) force acting on the bungee jumper is the same as it was for the vertical mass-spring system. When the oscillating finally stops, the jumper will come to a stop in the equilibrium position.

The reeds of woodwind instruments, such as the clarinet, behave as simple harmonic oscillators. As the musician blows through the mouthpiece, the reed vibrates as predicted by the laws of SHM.

Once a simple harmonic oscillator is set in motion, it will slowly come to rest because of friction *unless* a force is continually applied. We will examine these conditions in section 7.4 on resonance.

SHM is repetitive and predictable, so we can state the following:

- The restoring force acts in the opposite direction to the displacement.
- At the extremes of SHM, the displacement is at its maximum and is referred to as the amplitude. At this point, force and acceleration are also at their maximum, and the velocity of the object is zero.
- At the equilibrium position, the force and acceleration are zero, and the velocity of the object is at its maximum.

e WEB

After the first few oscillations following the jump, the bungee jumper oscillates with simple harmonic motion. To learn more about simple harmonic motion in the vertical direction, follow the links at www.pearsoned.ca/school/physicssource.

Concept Check

1. Must the line on a graph of force versus displacement for a spring always intercept the origin? Explain.
2. In what situation might the line on a graph of force as a function of displacement for a spring become non-linear?
3. A student wants to take a picture of a vertical mass-spring system as it oscillates up and down. At what point in the mass's motion would you suggest that she press the button to take the clearest picture?
4. Instead of plotting a force-displacement graph for a spring, a student plots a restoring force-displacement graph. Sketch what this graph might look like.
5. How would you write the equation for Hooke's law to reflect the shape of the graph above?

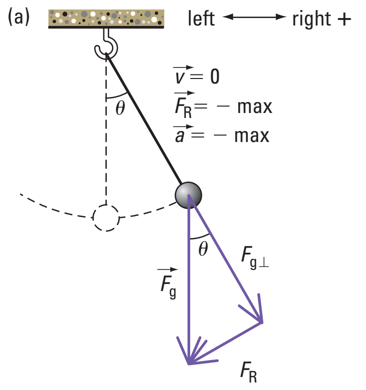
Simple Harmonic Motion of a Pendulum

The Cassini-Huygens space probe featured at the beginning of this chapter is named in honour of two distinguished scientists. Among many other notable accomplishments, the Italian astronomer Giovanni Cassini (1625–1712) observed the planets Mars and Jupiter and measured their periods of rotation. Christiaan Huygens (1629–1695), a Dutch mathematician and astronomer, invented the first accurate clock. It used a swinging pendulum and was a revolution in clock making (Figure 7.25).

For small displacements, a swinging pendulum exhibits SHM. Since SHM is oscillatory, a clock mechanism that uses a pendulum to keep time could be very accurate. Up until Huygens's time, clocks were very inaccurate. Even the best clocks could be out by as much as 15 minutes a day. They used a series of special gears and weights that didn't always produce a uniform rate of rotation — a necessity for an accurate mechanical clock. Huygens recognized that if he could take advantage of the uniform oscillations of a pendulum, he could produce a much better clock. When completed, his pendulum clock was accurate to within one minute a day. This may not be very accurate by today's standards, but was easily the best of its time. Pendulum clocks became the standard in time keeping for the next 300 years.

Let's examine cases where an ideal pendulum swings through a small angle, as explained in Figure 7.26 (a) to (e). In this book, all pendulums are considered ideal. That is, we assume that the system is frictionless, and the entire mass of the pendulum is concentrated in the weight. While this is not possible in reality, it is reasonable to make these assumptions here because they provide reasonably accurate results.

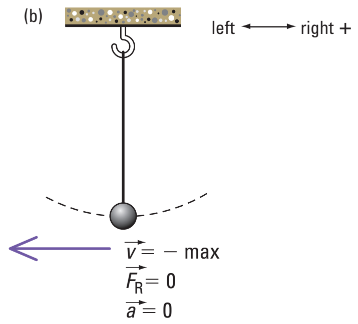
▲ **Figure 7.25** A replica of Huygens's pendulum clock



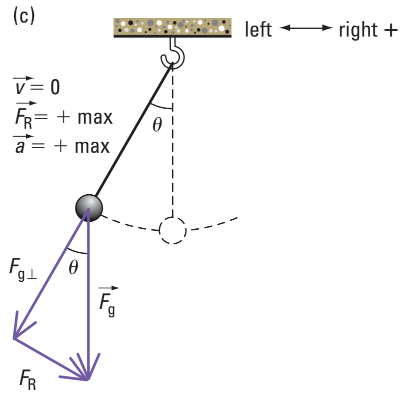
◀ **Figure 7.26(a)** The mass (called a "bob") is attached to the string and has been pulled from its equilibrium (rest) position through a displacement angle of θ. It has a mass m. When the bob's displacement is farthest to the right, the restoring force is a maximum negative value and velocity is zero.

When the pendulum is released, gravity becomes the restoring force. Given the direction of the force of gravity, the acceleration due to gravity is straight down. However, the motion of the pendulum is an arc. A component of gravity acts along this arc to pull the bob back toward equilibrium and is, by definition, the restoring force ($\vec{F}_R$).

We can express F_R in terms of F_g with the following equation: $F_R = F_g(\sin\theta)$

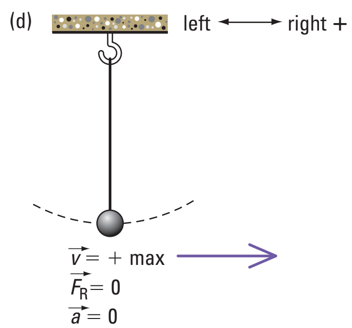


◀ **Figure 7.26(b)** As the bob accelerates downward, its velocity begins to increase and the restoring force ($\vec{F}_R$) becomes less and less. When it reaches the equilibrium position, no component of gravity is acting parallel to the motion of the bob, so the restoring force is zero, but the velocity has reached its maximum value.



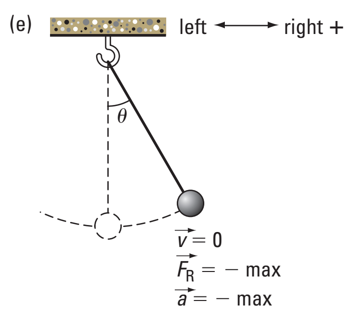
◀ **Figure 7.26(c)** The bob has reached its maximum displacement to the left. The restoring force has also reached a maximum value but it acts toward the right. The bob's velocity is zero again.

The bob passes through the equilibrium position and begins to move upward. As it does so, the restoring force becomes larger as the displacement of the bob increases. But just like the mass-spring system, the restoring force is acting in a direction opposite to the bob's displacement. At the other extreme of the bob's displacement, the restoring force has slowed the bob to an instantaneous stop. In this position, the displacement and restoring force are a maximum, and the bob's velocity is zero.



◀ **Figure 7.26(d)** The bob's displacement is again zero, and so is the restoring force. The bob has achieved its maximum velocity, but this time it is to the right.

On its back swing, the bob moves through the equilibrium position again, as shown here. The velocity is a maximum value, just as it was in Figure 7.26(b), but now it's in the opposite direction.



◀ **Figure 7.26(e)** The restoring force once more brings the bob's motion to a stop for an instant at the position farthest to the right. The restoring force is a maximum negative value, and the bob's velocity is zero. The pendulum has made one complete oscillation as shown in diagrams (a) to (e).

Note that the motion of the pendulum bob and the mass-spring systems are similar. Figure 7.19(a–e) on page 355 and Figure 7.26(a–e) on this page are comparable because both systems undergo the same changes to force, velocity, and acceleration at the same displacements.

Motion with Large Amplitudes

Earlier in this chapter, you read that a pendulum acts as a simple harmonic oscillator for small angles. Why is that? How is its motion different from SHM at larger angles?

The best way to answer these questions is to plot a graph of force versus displacement like those done for springs earlier in this chapter (e.g., Figure 7.11 on page 351). The displacement of the pendulum can be measured by its angle from the vertical. If the graph is linear, then the restoring force is proportional to the displacement, and the pendulum has moved in SHM, as described by Hooke's law. To create this graph, use the equation for restoring force that you saw in the explanation of Figure 7.26(a) on the previous page:

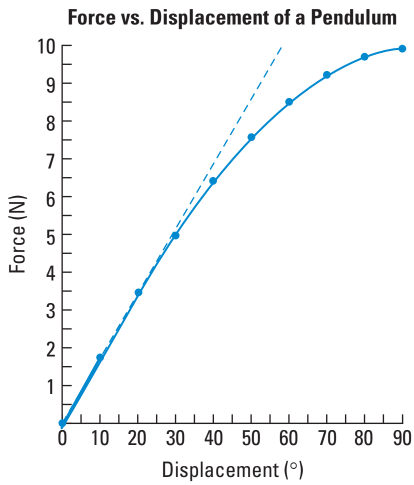
$$F_R = F_g(\sin \theta) \qquad (3)$$

From this equation, we can plot the values for angles up to 90° for a bob with a mass of 1.0 kg.

As the graph in Figure 7.27 shows, the line is not linear, so the restoring force does not vary proportionally with the displacement. Strictly speaking, a pendulum is not a true simple harmonic oscillator. However, the line is almost linear up to about 20°. At angles of less than 15°, the deviation from a straight line is so small that, for all practical purposes, it is linear.

▼ **Table 7.5** Data for Figure 7.27

Angle (°)	Restoring Force (N)
0	0
10	1.70
20	3.36
30	4.91
40	6.31
50	7.51
60	8.50
70	9.22
80	9.66
90	9.81



▲ **Figure 7.27** For the pendulum to be a true simple harmonic oscillator, its graph of restoring force versus displacement should be linear, as the dotted line suggests. After 15°, its line departs from the straight line, and its motion can no longer be considered SHM.

Example 7.5

Determine the magnitude of the restoring force for a pendulum bob of mass 100.0 g that has been pulled to an angle of 10.0° from the vertical.

Practice Problems

1. Determine the restoring force of a pendulum that is pulled to an angle of 12.0° left of the vertical. The mass of the bob is 300.0 g.
2. At what angle must a pendulum be displaced to create a restoring force of 4.00 N [left] on a bob with a mass of 500.0 g?

Answers

1. 0.612 N [right]
2. 54.6°

Given

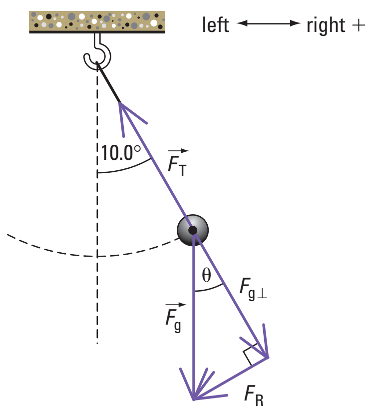
$g = 9.81\ \text{m/s}^2$
$m = 100.0\ \text{g} = 0.1000\ \text{kg}$

Required

restoring force (F_R)

Analysis and Solution

Draw a diagram of the pendulum in its displaced position to show the forces acting on the bob.



▲ **Figure 7.28**

The restoring force $\vec{F}_R$ is the component of $\vec{F}_g$ that is tangential to the arc path of the pendulum.

$$\begin{aligned} F_R &= F_g(\sin\theta) \\ &= mg(\sin\theta) \\ &= (0.1000\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\sin 10.0°) \\ &= 0.170\ \text{N} \end{aligned}$$

Paraphrase

The magnitude of the restoring force acting on the pendulum is 0.170 N.

When Christiaan Huygens designed the first pendulum clock, his primary concern was to have the clock operate with a very consistent period. For a uniform period, he could use gear ratios in the mechanism to translate the motion of the pendulum to meaningful units of time, such as minutes and hours.

Which factors influence the period of a pendulum, and which do not? To discover how a pendulum's mass, amplitude, and length influence its period, do 7-4 Inquiry Lab.

7-4 Inquiry Lab

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

A Pendulum and Simple Harmonic Motion

Question

What is the relationship between the period of a pendulum and its mass, amplitude, and length?

Materials and Equipment

thermometer clamp
retort stand
1.00-m thread (e.g., dental floss)
4 masses: 50 g, 100 g, 150 g, 200 g
ruler (30 cm) or metre-stick
protractor
stopwatch or watch with a second hand

Hypothesis

Before you begin parts A, B, and C, state a suitable hypothesis for each part of the lab. Remember to write your hypotheses as "if/then" statements.

Variables

The variables are the length of the pendulum, the mass of the pendulum, elapsed time, and the amplitude of the pendulum. Read the procedure for each part and identify the controlled, manipulated, and responding variables each time.

Part A: Mass and Period

Procedure

1. Copy Table 7.6 into your notebook.

▼ Table 7.6 Mass and Period

Length of Pendulum =			
Mass (g)	**No. of Cycles/20 s**	**Frequency (Hz)**	**Period (s)**
50			
100			
150			
200			

2. Attach the thermometer clamp to the top of the retort stand. Attach the thread to the thermometer clamp. Make sure the thread is a little shorter than the height of the clamp from the table.
3. Squeeze one end of the thread in the clamp and use a slip knot on the other end to attach the first mass. The mass should hang freely above the table.
4. Measure the length of the thread from the clamp to the middle of the mass. Record this as the length of the pendulum at the top of Table 7.6.
5. Pull the mass on the thread back until it makes an angle of 15° with the vertical, as measured with the protractor.
6. Remove the protractor, and release the mass as you start to time it. Count the number of complete oscillations it makes in 20 s. Record this number in your table.
7. Remove the mass and replace it with the next mass. Loosen the clamp and adjust the length of the thread so that it is the same length as for the previous mass. (Remember to measure length to the middle of the mass.) Repeat steps 5 to 7 until all the masses are used.

Analysis

1. Determine the frequency and period of each mass. Record the numbers in your table.
2. Plot a graph of period versus mass. Remember to place the manipulated variable on the horizontal axis.
3. What conclusion can you draw about the relationship between the mass and the period of a pendulum? Explain your answer and show any relevant calculations.

Part B: Amplitude and Period

Procedure

1. Copy Table 7.7 into your notebook.

▼ **Table 7.7** Amplitude and Period

Length of Pendulum =			
Amplitude (°)	**No. of Cycles/20 s**	**Frequency (Hz)**	**Period (s)**
5			
10			
15			
20			

2. Use the same apparatus as in part A.
3. Attach a 200-g mass to the free end of the thread.
4. Measure the length of the thread from the clamp to the middle of the mass. Record this length at the top of Table 7.7.
5. Pull the mass on the thread back until it makes an angle of 5° with the vertical, as measured with the protractor.
6. Remove the protractor, and release the mass as you start to time it. Count the number of complete oscillations it makes in 20 s. Record this number in Table 7.7.
7. Repeat steps 5 and 6, each time increasing the amplitude by 5°.

Analysis

1. Determine the frequency and period for each amplitude and record the numbers in the appropriate columns in the table.
2. Plot a graph of period versus amplitude. Remember to place the manipulated variable on the horizontal axis.
3. What conclusion can you draw about the relationship between the amplitude and the period of a pendulum? Show any relevant calculations.

Part C: Length and Period

Procedure

1. Copy Table 7.8 into your notebook.

▼ **Table 7.8** Length and Period

Length (m)	No. of Cycles/20 s	Frequency (Hz)	Period (s)

2. Use the same apparatus as in part A. Start with a pendulum length of 1.00 m.
3. Attach a 200-g mass to the free end of the thread.
4. Measure the length of the thread from the clamp to the middle of the mass. Record this length in Table 7.8.
5. Pull the mass on the thread back until it makes an angle of 15° with the vertical, as measured with the protractor.
6. Remove the protractor, and release the mass as you start to time it. Count the number of complete oscillations it makes in 20 s. Record this number in Table 7.8.
7. Repeat steps 4 to 6, but each time decrease the length of the pendulum by half.

Analysis

1. Determine the frequency and period for each length and record the values in the appropriate column in the table.
2. Plot a graph of period versus length. Remember to place the manipulated variable on the horizontal axis.
3. What conclusion can you draw about the relationship between the length and the period of a pendulum? Show any relevant calculations.

***e*LAB**

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Pendulums and mass-spring systems are not the only simple harmonic oscillators. There are many other examples: a plucked guitar string, molecules vibrating within a solid, and water waves are just a few. In section 7.4, you will explore some human-made examples of SHM and learn about an interesting property called resonance.

7.2 Check and Reflect

Knowledge

1. The restoring force of a vertical mass-spring system is determined by the mass attached to the spring and the spring constant k. What two factors determine the restoring force of a pendulum?

2. Copy the following tables into your notes. Then fill in the blanks by using the words, "zero" or "maximum."

Pendulum System	Displacement	Acceleration	Velocity	Restoring Force
max $\vec{x}$				
max $\vec{a}$				
max $\vec{v}$				
min $\vec{F}$				

Mass-spring System	Displacement	Acceleration	Velocity	Restoring Force
max $\vec{x}$				
max $\vec{a}$				
max $\vec{v}$				
min $\vec{F}$				

3. Explain why a pendulum is not a true simple harmonic oscillator.

Applications

4. A mass of 2.0 kg is attached to a spring with a spring constant of 40.0 N/m on a horizontal frictionless surface. Determine the restoring force acting on the mass when the spring is compressed to a displacement of −0.15 m.

5. A spring hangs vertically from a ceiling and has a spring constant of 25.0 N/m. How far will the spring be stretched when a 4.0-kg mass is attached to its free end?

6. An applied force of −25.0 N is required to compress a spring −0.20 m. What force will pull it to a displacement of +0.15 m?

7. Two students are given the task of determining the spring constant of a spring as accurately as possible. To do this, they attach a force meter to a spring that is lying on a desk and is anchored at the other end. One student pulls the spring through several displacements, while the other records the force applied, as shown in the table below. Using this table, plot a graph of force versus displacement. Find the spring constant by determining the slope of the line of best fit.

Displacement (m)	Force (N)
0.00	0.00
0.10	0.15
0.20	0.33
0.30	0.42
0.40	0.60

8. Determine the restoring force for a pendulum bob with a mass of 0.400 kg that is pulled to an angle of 5.0° from the vertical.

9. A toy car, with a wind-up spring motor, on a horizontal table is pulled back to a displacement of 20.0 cm to the left and released. If the 10.0-g car initially accelerates at 0.55 m/s^2 to the right, what is the spring constant of the car's spring? (Hint: The restoring force is $F = ma$.)

Extension

10. Obtain three different types of rulers: plastic, metal, and wooden. Fix one end of each ruler to the side of a desk so the ruler juts out horizontally a distance of 25 cm from the edge. Hang enough weight on the end that sticks out to make the ruler bend downward by 2 to 3 cm. Record the deflection of the ruler and the mass used in each case. (Note: The deflection does not have to be the same for each ruler.) Use these data to determine the spring constant for each ruler. Rank the rulers from highest spring constant to lowest.

e TEST

To check your understanding of simple harmonic motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

7.3 Position, Velocity, Acceleration, and Time Relationships

info **BIT**

An accelerometer is a device used to measure acceleration. It is designed like a mass-spring system. The force exerted by the accelerating object causes the mass to compress the spring. The displacement of the mass is used to determine the positive or negative acceleration of the object. Accelerometers are commonly used in airbag systems in cars (see Chapter 3). If the car slows down too quickly, the displacement of the mass is large and it triggers the airbag to deploy.

One way for ball players to practise their timing is by attempting to throw a ball through a tire swinging on a rope. Someone just beginning this kind of practice might throw too early or too late, missing the tire altogether. Part of the difficulty has to do with the continually changing velocity of the tire.

Choosing the best time to throw the ball is an exercise in physics. With practice, the human brain can learn to calculate the proper time to throw without even being aware that it is doing so.

Throwing the ball through the tire is much more difficult than it sounds because the tire is a simple harmonic oscillator for small amplitudes. Not only is the velocity continually changing, but so is the restoring force and the acceleration. The only constant for a swinging tire is its period.

In this section, you will mathematically analyze acceleration, velocity, and period for SHM in a mass-spring system, and then determine the period of a pendulum.

Both mass-spring systems and pendulums are simple harmonic oscillators, as described in section 7.2, but they are different from each other. The mass-spring system has a spring constant k, but the pendulum does not. For this reason, we will look at each separately, starting with the mass-spring system.

Acceleration of a Mass-spring System

In section 7.2, you learned that two equations can be used to describe force in the mass-spring system: Newton's second law and Hooke's law. They can be written mathematically as:

- Newton's second law: $\vec{F}_{\text{net}} = m\vec{a}$
- Hooke's law: $\vec{F} = -k\vec{x}$ (2)

PHYSICS INSIGHT

An object **does not** have to be moving to experience acceleration.

Since both equations refer to the restoring force, you can equate them:

$$\vec{F}_{\text{net}} = \vec{F}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{a} = -\frac{k\vec{x}}{m} \qquad (4)$$

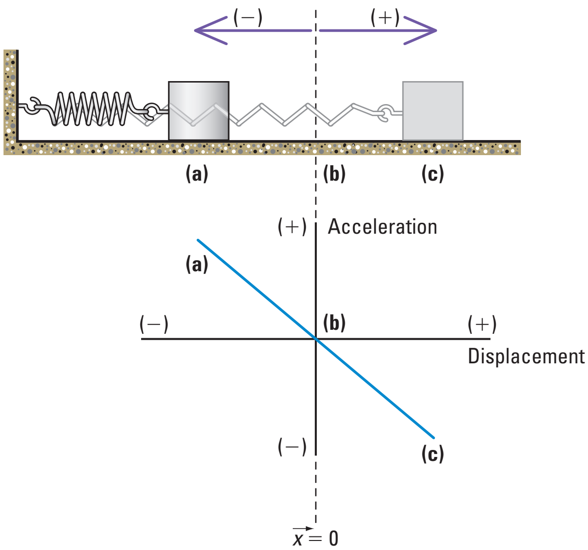
where $\vec{a}$ is the acceleration in metres per second squared; k is the spring constant in newtons per metre; $\vec{x}$ is the displacement in metres; and m is the mass of the oscillator in kilograms.

The acceleration of a horizontal mass-spring simple harmonic oscillator can be determined by its spring constant, displacement, and mass. It's logical that the acceleration of the mass depends on how stiff the spring is and how far it is stretched from its equilibrium position. It is also reasonable to assume that, if the mass is large, then the acceleration will be small. This assumption is based on Newton's second law.

The acceleration depends on the displacement of the mass, so the acceleration changes throughout the entire motion as shown in Figure 7.29. Since acceleration of a simple harmonic oscillator is not uniform, only the instantaneous acceleration of the mass can be determined by equation 4.

e MATH

To see how the spring constant, mass, position and acceleration are related graphically, visit www.pearsoned.ca/school/physicssource.

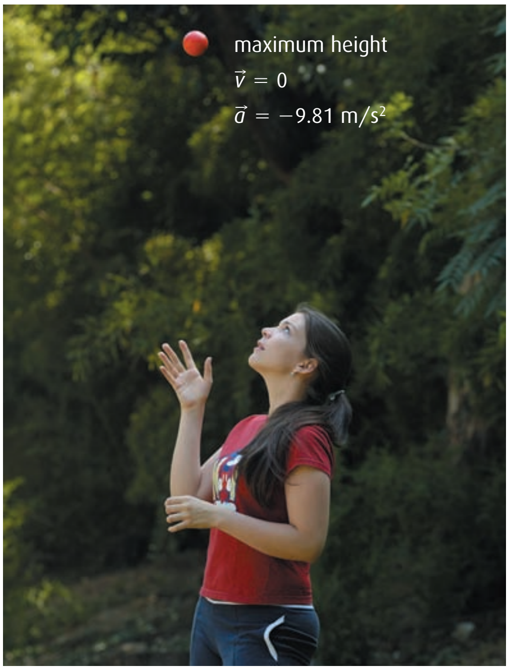


◀ **Figure 7.29** The acceleration of a simple harmonic oscillator depends on its position. In position (a), the oscillator moves from its maximum displacement and maximum positive acceleration through to position (b), where the displacement and acceleration are zero. It then moves to position (c), where the oscillator again experiences a maximum acceleration and displacement in the other direction.

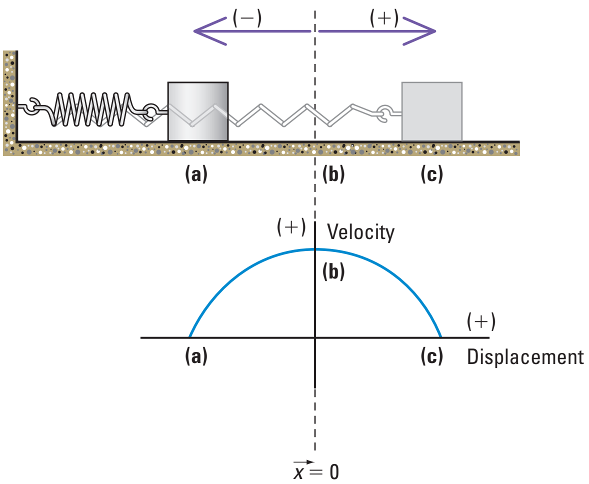
The Relationship Between Acceleration and Velocity of a Mass-spring System

The acceleration of a simple harmonic oscillator is continually changing, so it should come as no surprise that the velocity changes too. As we have just seen, the maximum acceleration occurs when the oscillator is at its maximum displacement.

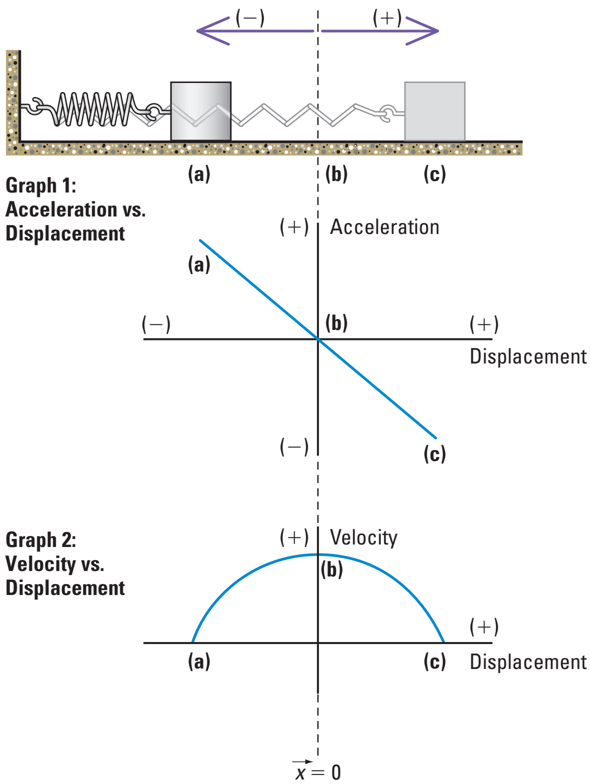
At this position, it is tempting to think that the velocity will be at its maximum as well, but we know that this is not the case. Remember, the acceleration is at its greatest magnitude at the extremes of the motion, yet the oscillator has actually stopped in these positions! In some ways, a ball thrown vertically into the air is similar (Figure 7.30). The acceleration of gravity acts on the ball to return it to the ground. When the ball reaches its maximum height, it comes to a stop for an instant, just like the mass-spring system you studied earlier in this chapter.



▶ **Figure 7.30** At its maximum height, the ball stops for a brief instant, yet the acceleration of gravity acting on it is not zero. This is similar to the mass-spring system.



▲ **Figure 7.31** The velocity of a simple harmonic oscillator is not uniform. The mass experiences its greatest acceleration at the extremes of its motion where the velocity is zero. Only after the mass accelerates from position (a) to (b) does its velocity reach its maximum value. The mass then decelerates from (b) to (c) where it comes to a stop again.



▲ **Figure 7.32** The mass-spring system experiences a changing acceleration and velocity as it makes one-half of a full oscillation from position (a) to position (c).

Since the acceleration of the oscillator decreases as it approaches the equilibrium position (Figure 7.31(a)), the velocity does not increase at a uniform rate. The velocity-displacement graph looks like Figure 7.31(b).

Figure 7.32 shows a diagram of a simple harmonic oscillator (a mass-spring system) as it moves through one-half of a complete oscillation from (a) to (b) to (c). Below the diagram are the acceleration-displacement and velocity-displacement graphs. The diagram of the oscillator and the graphs are vertically aligned so the graphs show you what is happening as the mass-spring system moves.

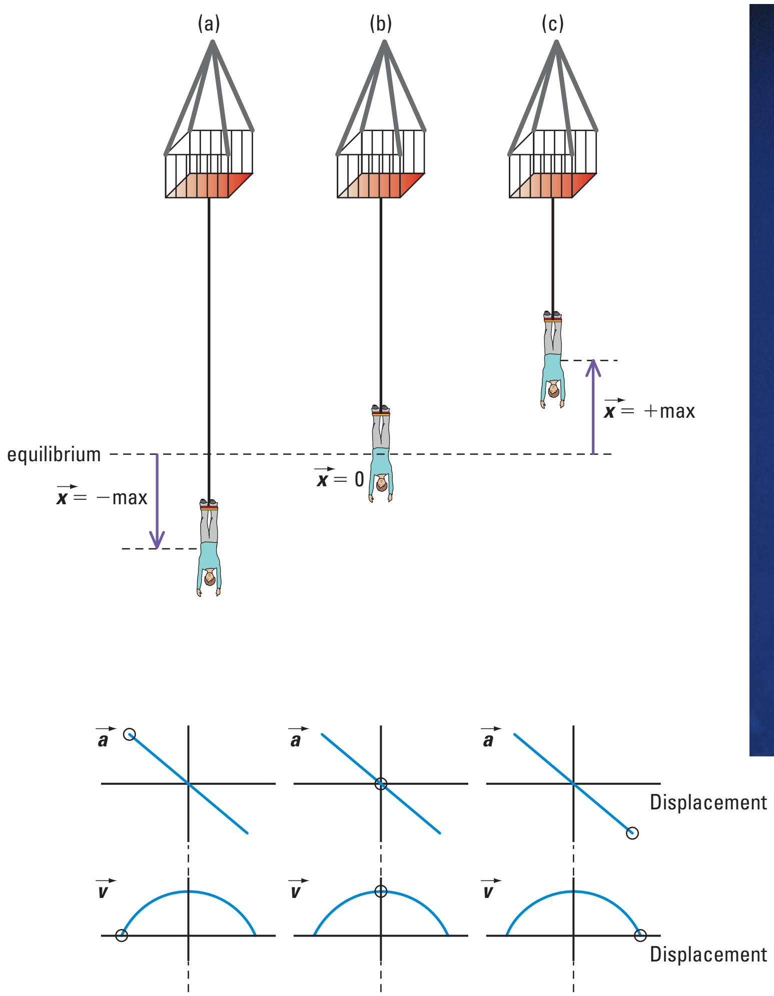
In the diagram at the top of Figure 7.32, you can see the oscillator in position (a). It is at its farthest displacement to the left and the spring is compressed. The velocity-displacement graph shows that the oscillator's velocity in this position is zero (graph 2). You can also see from the acceleration-displacement graph that the acceleration at that moment is positive and a maximum, but the displacement is negative (graph 1).

Acceleration and Displacement — Always Opposite

In fact, if you look closely at graph 1, you might notice how the acceleration and displacement are always opposite to one another, regardless of the position of the mass. The acceleration is positive while the displacement is negative, and vice versa. This isn't surprising, however, because it is what the negative sign in the equation for Hooke's law illustrates: $\vec{F} = -k\vec{x}$.

Look again at Figure 7.32 and follow the mass as it moves from position (a) to positions (b) and (c). As the oscillator accelerates from position (a) to the right, it picks up speed. The velocity-displacement graph (graph 2) shows that the velocity is positive and increasing as the oscillator approaches position (b), yet the acceleration is decreasing, as shown in graph 1. The oscillator goes through the equilibrium position with a maximum positive velocity, but now the acceleration becomes negative as the spring tries to pull the oscillator back (graph 1). This is why the oscillator slows down and the velocity-displacement graph returns to zero in position (c).

Consider a vertical mass-spring system. A bungee jumper will experience a positive acceleration when she is below the equilibrium position and a negative acceleration when above it (Figures 7.33 and 7.34).



▲ **Figure 7.33**

▲ **Figure 7.34** After a jump, the bungee jumper is shown in three positions: At the lowest point (a), in the equilibrium position (b), and at her maximum displacement (c). In each case the circled region on the graphs indicates her acceleration and velocity.

Maximum Speed of a Mass-spring System

Now you know that a simple harmonic oscillator will experience its greatest speed at the equilibrium position. What factors influence this speed, and how can we calculate it?

In our examples, the mass-spring system is frictionless, and no external forces act on it. This is referred to as an isolated system and the law of conservation of energy applies. We will use this concept to derive the equation for the maximum speed.

Recall from Chapter 6 that the total mechanical energy in an isolated system remains constant. That means that the kinetic and potential energy of the system may vary, but their sum is always the same.

In other words, at any position in the motion of a mass-spring system, the sum of kinetic and potential energies must be equal to the total energy of the system. Recall that kinetic energy is expressed as:

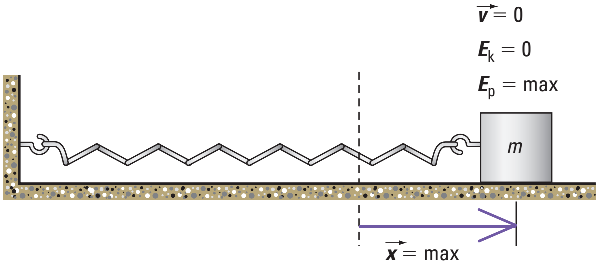
$$E_k = \frac{1}{2}mv^2$$

Recall that elastic potential energy is expressed as:

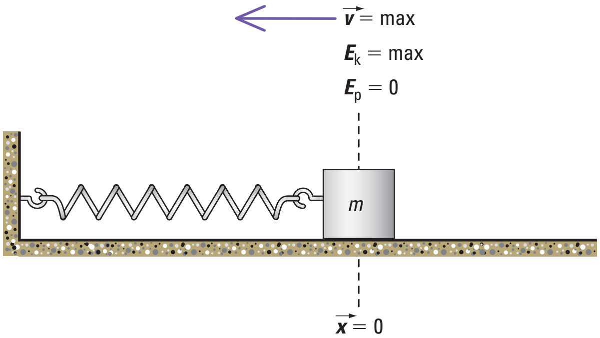
$$E_p = \frac{1}{2}kx^2$$

Let's begin by looking at the energy of the system in two positions:

- When the mass is at the maximum displacement (Figure 7.35(a)).
- When the mass is at the minimum displacement (Figure 7.35(b)).



▲ **Figure 7.35(a)** The oscillator at its maximum displacement. Potential energy has reached a maximum value ($E_{p_{max}}$) because the oscillator's displacement is a maximum ($\vec{x} = A$). The kinetic energy is zero ($\vec{v} = 0$).



▲ **Figure 7.35(b)** The oscillator at its minimum displacement ($\vec{x} = 0$). Kinetic energy has reached a maximum value ($E_{k_{max}}$) because the oscillator has a maximum velocity. The potential energy is zero ($\vec{x} = 0$).

Remember that the total energy of the system remains constant regardless of the oscillator's position. The equation for the total energy is:

$$E_T = E_p + E_k$$

The kinetic energy of the oscillator at its maximum displacement is zero so the total energy of the oscillator at that position must be:

$$E_T = E_{p_{max}}$$

The potential energy of the oscillator at its minimum displacement is zero so the total energy of the oscillator at that position must be:

$$E_T = E_{k_{max}}$$

Because the total energy is always the same, we can write:

$$E_{k_{max}} = E_{p_{max}}$$

or

$$\frac{1}{2}mv^2_{max} = \frac{1}{2}kx^2_{max}$$

If we use A to represent x_{max}, we can write:

$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$$

We can then simplify this equation to solve for v_{max}:

$$\frac{1}{\cancel{2}}mv^2_{max} = \frac{1}{\cancel{2}}kA^2$$

$$mv^2_{max} = kA^2$$

$$v^2_{max} = \frac{kA^2}{m}$$

Then we take the square root of each side:

$$v_{max} = \sqrt{\frac{kA^2}{m}}$$

or

$$v_{max} = A\sqrt{\frac{k}{m}} \qquad (5)$$

Factors That Influence the Maximum Speed of a Mass-spring System

Three factors influence the maximum speed of a mass-spring system:

- the amplitude of the oscillations: If the oscillator moves through a large amplitude, the restoring force increases in proportion to the amplitude. As the restoring force increases, so does the acceleration, and the oscillator will achieve a greater velocity by the time it reaches the equilibrium position.
- the stiffness of the spring: A stiffer spring with a higher spring constant exerts a stronger restoring force and creates a greater maximum velocity for the same reasons that increasing the amplitude does.
- the mass of the oscillator: Changing the mass of an oscillator has a different effect. If the mass increases, the velocity of the oscillations decreases. This is because the oscillator has more inertia. A larger mass is harder to accelerate so it won't achieve as great a speed as a similar mass-spring system with less mass.

eWEB

To find out how these factors are taken into account in bungee jumping, follow the links at www.pearsoned.ca/school/physicssource.

Concept Check

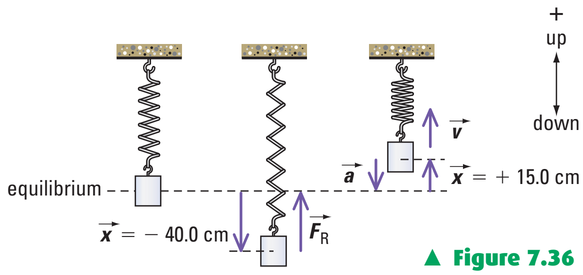
1. When acceleration is negative, displacement is positive and vice versa. Why?
2. Why is the velocity-time graph of a simple harmonic oscillator a curved line?
3. The acceleration-displacement graph and velocity-displacement graph are shown in Figure 7.32 on page 368 for half of an oscillation only. Sketch three more acceleration-displacement and velocity-displacement graphs for the second half of the oscillation.
4. Suppose the amplitude of an object's oscillation is doubled. How would this affect the object's maximum velocity?

PHYSICS INSIGHT

When a mass is hanging from a vertical spring at rest in the equilibrium position, the downward force of gravity $\vec{F}_g$ is equal and opposite to the upward force exerted by the spring $\vec{F}_s$, so the restoring force is zero. The force of gravity acting on the mass doesn't change. If the spring is displaced from the equilibrium position, the restoring force will just be the force of the spring due to its displacement $\vec{x}$. This is expressed as $\vec{F}_R = -k\vec{x}$.

Example 7.6

A 100.0-g mass hangs motionless from a spring attached to the ceiling. The spring constant (k) is 1.014 N/m. The instructor pulls the mass through a displacement of 40.0 cm [down] and releases it. Determine: (a) the acceleration when the mass is at a displacement of 15.0 cm [up], and (b) the maximum speed of the mass.



▲ **Figure 7.36**

Given

$m = 100.0 \text{ g} = 0.1000 \text{ kg}$
$k = 1.014 \text{ N/m}$
$\vec{x} = 40.0 \text{ cm [down]} = 0.400 \text{ m [down]}$

Required

(a) acceleration ($\vec{a}$) when $\vec{x} = 15.0$ cm [up] = 0.150 m [up]
(b) maximum speed (v_{max})

Analysis and Solution

(a) The mass will begin to oscillate when released. Acceleration is a vector quantity so direction is important.

$$\vec{F}_{net} = \vec{F}_R$$
$$m\vec{a} = -k\vec{x}$$
$$\vec{a} = \frac{-k\vec{x}}{m}$$
$$= \frac{\left(-1.014 \frac{\text{N}}{\cancel{\text{m}}}\right)(+0.150 \cancel{\text{m}})}{0.1000 \text{ kg}}$$
$$= -1.52 \text{ m/s}^2$$

(b) The maximum speed occurs when the mass is in the equilibrium position, whether it is moving up or down. The displacement of the mass before it is released is the amplitude (A) of the mass's oscillation.

$$v_{max} = A\sqrt{\frac{k}{m}}$$
$$= 0.400 \text{ m}\sqrt{\frac{1.014 \frac{\text{N}}{\text{m}}}{0.1000 \text{ kg}}}$$
$$= 1.27 \text{ m/s}$$

Paraphrase

(a) The mass has an acceleration of 1.52 m/s² [down] when it is 15.0 cm above the equilibrium position.
(b) The maximum speed of the mass is 1.27 m/s.

Practice Problems

1. A 0.724-kg mass is oscillating on a horizontal frictionless surface attached to a spring (k = 8.21 N/m). What is the mass's displacement when its instantaneous acceleration is 4.11 m/s² [left]?
2. A 50.0-g mass is attached to a spring with a spring constant (k) of 4.00 N/m. The mass oscillates with an amplitude of 1.12 m. What is its maximum speed?
3. An instructor sets up an oscillating vertical mass-spring system (k = 6.05 N/m). The maximum displacement is 81.7 cm and the maximum speed is 2.05 m/s. What is the mass of the oscillator?

Answers

1. 0.362 m [right]
2. 10.0 m/s
3. 0.961 kg

Period of a Mass-spring System

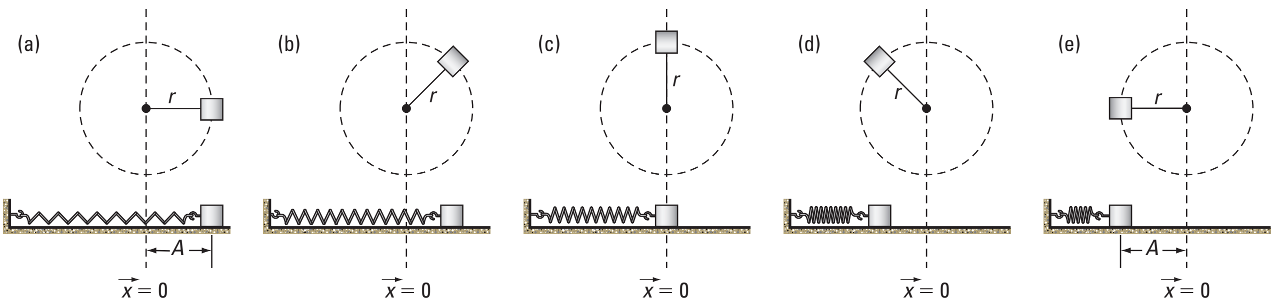
The next time you are travelling in a vehicle at night, watch for bicycles moving in the same direction as your vehicle. Notice the peculiar motion of the pedals as they reflect the light from your headlights (Figure 7.37). From a distance, these reflectors don't appear to be moving in a circular path, but seem to be moving up and down. The apparent up-and-down motion of the pedals is the same kind of motion as a mass-spring system oscillating back and forth, so it is simple harmonic motion. This observation proves useful because it is an example of how circular motion can be used to describe simple harmonic motion. The next few pages show how to derive equations for the period and maximum speed of a simple harmonic oscillator.

▲ **Figure 7.37** From a distance, the reflectors on the bicycle pedals would appear to be moving up and down instead of in a circle.

Two conditions are necessary if circular motion is to be used to replicate simple harmonic motion:

1. The period of both the circular motion and the simple harmonic motion must be the same.
2. The radius of the circular motion must match the amplitude of the oscillator.

For example, look at Figure 7.38, where a mass moving in a circular path with a radius *r* is synchronized with a mass-spring simple harmonic oscillator. This illustration demonstrates how circular motion can be used to describe SHM.



▲ **Figure 7.38** A mass moving in a circle is a simple harmonic oscillator that corresponds to the mass-spring oscillator shown below it. One-half of a complete cycle is shown here.

For our purposes, the following conditions are true:

- The radius of the circular motion is equal to the amplitude of the oscillator ($r = A$, as shown in Figure 7.38(a)).
- The mass in circular motion moves at a constant speed.
- The periods of the mass in circular motion and the oscillator in the mass-spring system are the same.

▲ **Figure 7.39** The strings of a piano all have different masses. Even if they vibrate with the same amplitude they will have a different period of vibration because each string has a different mass. A heavy string will vibrate with a longer period (and lower frequency) than a lighter string.

Deriving the Equation for the Period of a Mass-spring System

Recall that the maximum velocity of a simple harmonic oscillator occurs when it is in its equilibrium position, which is position (c) in Figure 7.38. At the exact moment that the mass in circular motion is in position (c), its velocity is in the same direction as the velocity of the mass-spring system, and they are both moving at the same speed. But if the mass moving in a circular path is moving at a constant speed, then it must always be moving at the maximum speed of the mass-spring oscillator! Therefore, the maximum speed (v_{max}) of the mass-spring system is equal to the speed (v) of the circular mass system.

The speed of an object moving in a circular path was derived in Chapter 5. It is:

$$v = \frac{2\pi r}{T} \qquad (6)$$

The speed of the circular motion (v) matches the maximum speed on the mass-spring oscillator (v_{max}), and the radius of the circle matches its amplitude. Therefore, we can customize the equation for the mass-spring oscillator:

$$v_{max} = \frac{2\pi A}{T} \qquad (7)$$

If we equate equation 5 and equation 7, we get:

$$A\sqrt{\frac{k}{m}} = \frac{2\pi A}{T}$$

We can then solve for T:

$$\cancel{A}\sqrt{\frac{k}{m}} = \frac{2\pi \cancel{A}}{T}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (8)$$

PHYSICS INSIGHT

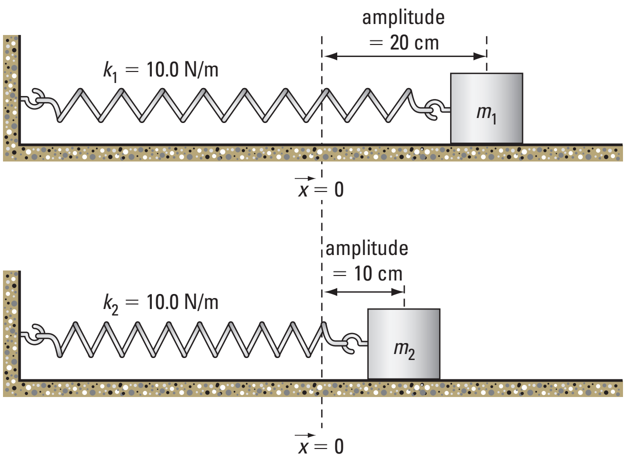
The period of a simple harmonic oscillator does not depend on displacement.

This equation describes the period of a simple harmonic oscillator, where T is the period of the oscillator in seconds; k is the spring constant in newtons per metre; and m is the mass of the oscillator in kilograms. Figure 7.39 is an example of an application of this equation.

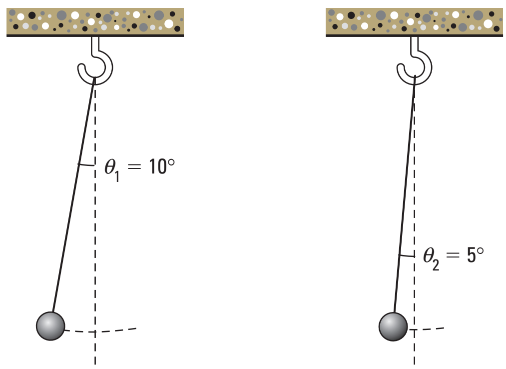
Factors Affecting the Period of an Oscillating Mass

The larger the oscillating mass is, the longer its period of oscillation is. This seems reasonable since a large mass takes longer to speed up or slow down. It would also seem reasonable that the period should be inversely related to the spring constant, as the equation suggests. After all, the stiffer the spring, the more force it exerts over smaller displacements. Therefore, you could expect the mass to oscillate more quickly and have a smaller period.

What is interesting about this equation is not what influences the period but what does not. It may seem odd that the displacement of the mass has no influence on the period of oscillation. This means that if you were to pull a mass-spring system to a displacement $\vec{x}$ and then let go, it would have the same period of oscillation as it would if you pulled it to a displacement of $2\vec{x}$ and released it! The two identical mass-spring systems in Figure 7.40 have different amplitudes but the same period.



▲ **Figure 7.40** Two identical mass-spring systems have the same spring constant and mass, but different amplitudes. Which has the longest period? They have the same period because displacement doesn't affect period.



▲ **Figure 7.41** Two identical pendulums have the same mass and length, but different amplitudes. Which one has the longest period? They have the same period because displacement doesn't affect the period of a simple harmonic oscillator.

This relationship is true for any simple harmonic oscillator, including a pendulum with a small amplitude. It is easy enough to test. Take two pendulums with the same mass and length (Figure 7.41). Pull both bobs back to different displacements. Remember to keep the displacements small so the pendulums oscillate with SHM. Release them at the same time. You will discover that both make one full oscillation in unison. This means they return to the point of release at exactly the same time. The pendulum that begins with the larger displacement has farther to travel but experiences a larger restoring force that compensates for this.

Example 7.7

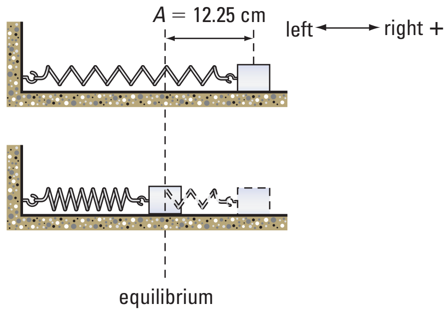
What is the period of oscillation of a mass-spring system that is oscillating with an amplitude of 12.25 cm and has a maximum speed of 5.13 m/s? The spring constant (k) is 5.03 N/m.

Given

$A = 12.25 \text{ cm} = 0.1225 \text{ m}$

$k = 5.03 \text{ N/m}$

$v_{max} = 5.13 \text{ m/s}$



▲ **Figure 7.42**

Required

period of the oscillations (T)

Analysis and Solution

To determine the period of the oscillator, you need to know the oscillator's mass. Use the maximum speed equation (equation 5) to find the mass:

$$E_{k_{max}} = E_{p_{max}}$$

$$\frac{mv_{max}^2}{\cancel{2}} = \frac{kA^2}{\cancel{2}}$$

$$mv_{max}^2 = kA^2$$

$$m = \frac{kA^2}{v_{max}^2}$$

$$= \frac{\left(5.03\ \dfrac{\text{N}}{\text{m}}\right)(0.1225\ \cancel{\text{m}})^2}{\left(5.13\ \dfrac{\cancel{\text{m}}}{\text{s}}\right)^2}$$

$$= 2.868 \times 10^{-3} \text{ kg}$$

Then use equation 8 to determine the period:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{2.868 \times 10^{-3} \text{ kg}}{5.03\ \dfrac{\text{N}}{\text{m}}}}$$

$$= 0.150 \text{ s}$$

Paraphrase

The period of the mass-spring oscillator is 0.150 s.

Practice Problems

1. A mass of 2.50 kg is attached to a horizontal spring and oscillates with an amplitude of 0.800 m. The spring constant is 40.0 N/m. Determine:
   a) the acceleration of the mass when it is at a displacement of 0.300 m
   b) the maximum speed
   c) the period
2. A 2.60-g mass experiences an acceleration of 20.0 m/s^2 at a displacement of 0.700 m on a spring. What is k for this spring?
3. What is the mass of a vertical mass-spring system if it oscillates with a period of 2.0 s and has a spring constant of 20.0 N/m?
4. What is the period of a vertical mass-spring system that has an amplitude of 71.3 cm and maximum speed of 7.02 m/s? The spring constant is 12.07 N/m.

Answers

1. a) −4.80 m/s^2
   b) 3.20 m/s
   c) 1.57 s
2. 0.0743 N/m
3. 2.0 kg
4. 0.638 s

Concept Check

1. What effect does doubling the displacement have on the period of oscillation of a simple harmonic oscillator? Explain your answer.
2. In order to compare circular motion to the motion of a simple harmonic oscillator, what two conditions must be satisfied?
3. If the mass and spring constant of a mass-spring oscillator were doubled, what effect would this have on the period of the oscillations?
4. Two mass-spring systems with identical masses are set oscillating side by side. Compare the spring constants of the two systems if the period of one system is twice the other.

The Period of a Pendulum

Christiaan Huygens recognized that a pendulum was ideally suited for measuring time because its period isn't affected by as many of the factors that influence a mass-spring system. A pendulum doesn't have a spring constant, k, like the mass-spring system does, and unlike the mass-spring system, the mass of the pendulum does not affect its period. Because of these factors, a new equation for a pendulum's period must be derived. In doing so, you will discover why its mass is irrelevant and what factors play a role in its period of oscillation.

Take a closer look at the pendulum when it is at a small displacement of 15° or less, as shown in Figure 7.43. For a small angle (θ), the displacement of the bob can be taken as x. The sine of angle θ is expressed as:

$$\sin\theta = \frac{x}{l}$$

Recall that the restoring force for a pendulum is $F_R = F_g \sin\theta$. Use the above expression for $\sin\theta$ in this equation:

$$F_R = F_g\left(\frac{x}{l}\right)$$

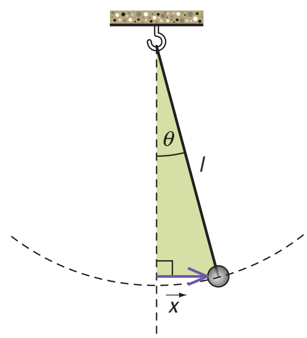
Recall also that in a mass-spring system, the restoring force is $F = -kx$. We want to solve for the period (T), which is a scalar quantity, so the negative sign in Hooke's law can be omitted. The two equations for restoring force can then be equated:

$$kx = F_g\left(\frac{x}{l}\right)$$

$$F_g = mg$$

$$k\cancel{x} = (mg)\frac{\cancel{x}}{l}$$

$$k = \frac{mg}{l}$$



▲ **Figure 7.43** For a pendulum with a small displacement of 15° or less, the displacement is x.

None of the values of mass, gravitational field strength, or length change for a pendulum. They are constants, which are represented by k. Substitute them into equation 8 (page 374):

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{\cancel{m}}{\left(\frac{(\cancel{m}g)}{l}\right)}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \qquad (9)$$

where l is the length of the pendulum string in metres; and g is the gravitational field strength in newtons per kilogram.

Recall that the length of the pendulum is always measured from the point where it is attached at the top, to the centre of mass of the bob, *not* the point at which the string or wire is attached to the bob. Also recall that the period of the pendulum's swing does not depend on the mass of the pendulum bob. This may not seem logical but it is indeed the case — just as the acceleration of an object in free fall doesn't depend on the mass of the object.

PHYSICS INSIGHT

The period of a pendulum does not depend on its mass or amplitude.

The Pendulum and Gravitational Field Strength

Equation 9 is useful when it is manipulated to solve for g, the gravitational field strength. As you learned in section 4.3 of Chapter 4, the gravitational field strength varies with altitude and latitude.

The magnitude of the gravitational field is 9.81 N/kg at any place on Earth's surface that corresponds to the average radius of Earth. However, very few places on the surface of Earth are at exactly the average radius. To determine the exact value of g at any point, you can use a pendulum. If you manipulate equation 9 and solve for g, you get:

$$g = \frac{4\pi^2 l}{T^2} \qquad (10)$$

eSIM

Learn more about the motion of a pendulum and the factors that affect it. Follow the eSIM links at www.pearsoned.ca/school/physicssource.

Due to the changing nature of Earth's gravity, Christiaan Huygens's pendulum clock (introduced in section 7.2) was only accurate if it was manufactured for a specific place. For example, pendulum clocks designed to operate in London could not be sold in Paris because the accuracy could not be maintained. The difference in gravitational field strength between London and Paris meant that the period of oscillation would be slightly different.

The difference in g between two locations could be quite small, but the cumulative effect on a pendulum clock would be significant. An extreme example of the varying value of g at different geographic locations can be illustrated by using a pendulum to determine the gravitational field strength at the top of Mount Everest.

Example 7.8

What is the gravitational field strength at the top of Mount Everest at an altitude of 8954.0 m, if a pendulum with a length of 1.00 m has a period of 2.01 s?

Analysis and Solution

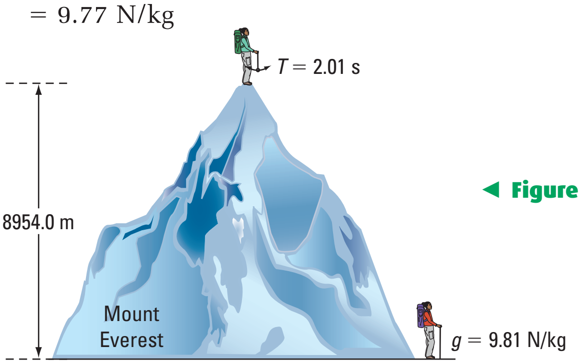
Use equation 10 to determine g. Note that no vector arrow is required with the symbol g because you are calculating a scalar quantity.

$$g = \frac{4\pi^2 l}{T^2}$$

$$= \frac{(4\pi^2)(1.00 \text{ m})}{(2.01 \text{ s})^2}$$

$$= 9.77 \text{ m/s}^2$$

$$= 9.77 \text{ N/kg}$$



◄ **Figure 7.44**

The gravitational field strength at the top of Mount Everest is 9.77 N/kg, which is very close to the accepted value of 9.81 N/kg. The extra height of Mount Everest adds very little to the radius of Earth.

Practice Problems

1. What is the gravitational field strength on Mercury, if a 0.500-m pendulum swings with a period of 2.30 s?
2. A pendulum swings with a period of 5.00 s on the Moon, where the gravitational field strength is 1.62 N/kg [down]. What is the pendulum's length?
3. What period would a 30.0-cm pendulum have on Mars, where the gravitational field strength is 3.71 N/kg [down]?

Answers

1. 3.73 N/kg [down]
2. 1.03 m
3. 1.79 s

At the top of Mount Everest, a pendulum will swing with a slightly different period than at sea level. So a pendulum clock on Mount Everest, oscillating with a longer period than one at sea level, will report a different time.

Huygens's clocks also suffered from another problem: the pendulum arm would expand or contract in hot or cold weather. Since the length of the arm also determines the period of oscillation, these clocks would speed up or slow down depending on the ambient temperature.

Given their limitations, pendulum clocks were not considered the final solution to accurate timekeeping. Further innovations followed that you may want to research on your own.

e WEB

To learn more about pendulum clocks and the evolution of timekeeping, create a timeline of the evolution of clock design. In your timeline include what the innovation was, who invented it, and the year it was introduced. Begin your search at www.pearsoned.ca/school/physicssource.

Concept Check

1. An archer is doing target practice with his bow and arrow. He ties an apple to a string and sets it oscillating left to right, down range. In what position of the apple should he aim so that he increases his chances of hitting it? Explain your answer.
2. What factors affect the accuracy of pendulum clocks? Why?

7.3 Check and Reflect

Knowledge

1. Explain the effect that changing each of the following factors has on the period of a mass-spring system:
   (a) amplitude
   (b) spring constant
   (c) mass
2. Explain what effect changing each of the following factors has on the period of a pendulum:
   (a) amplitude
   (b) gravitational field strength
   (c) mass
3. Describe the positions that a mass-spring system and pendulum are in when:
   (a) acceleration is a maximum
   (b) velocity is a maximum
   (c) restoring force is maximum
4. Why is the acceleration of a simple harmonic oscillator not uniform?
5. A mass-spring system has a negative displacement and a positive restoring force. What is the direction of acceleration?

Applications

6. What length of pendulum would oscillate with a period of 4.0 s on the surface of Mars ($g = 3.71$ N/kg)?
7. A mass of 3.08 kg oscillates on the end of a horizontal spring with a period of 0.323 s. What acceleration does the mass experience when its displacement is 2.85 m to the right?
8. A 50.0-kg girl bounces up and down on a pogo stick. The girl has an instantaneous acceleration of 2.0 m/s^2 when the displacement is -8.0 cm. What is the spring constant of the pogo stick's spring?
9. A pendulum bob ($m = 250.0$ g) experiences a restoring force of 0.468 N. Through what angle is it displaced?
10. A 50.0-cm pendulum is placed on the Moon, where g is 1.62 N/kg. What is the period of the pendulum?

Extensions

11. A horizontal mass-spring system oscillates with an amplitude of 1.50 m. The spring constant is 10.00 N/m. Another mass moving in a circular path with a radius of 1.50 m at a constant speed of 5.00 m/s is synchronized with the mass-spring system. Determine the mass-spring system's:
    (a) period
    (b) mass
    (c) maximum acceleration
12. A quartz crystal ($m = 0.200$ g) oscillates with simple harmonic motion at a frequency of 10.0 kHz and has an amplitude of 0.0500 mm. What is its maximum speed?
13. A horizontal mass-spring system has a mass of 0.200 kg, a maximum speed of 0.803 m/s, and an amplitude of 0.120 m. What is the mass's position when its acceleration is 3.58 m/s^2 to the west?
14. Suppose an inquisitive student brings a pendulum aboard a jet plane. The plane is in level flight at an altitude of 12.31 km. What period do you expect for a 20.0-cm pendulum? (Hint: First determine the gravitational field strength as shown in Chapter 4.)

e TEST

To check your understanding of position, velocity, acceleration, and time relationships in mass-spring systems and pendulums, follow the eTest links at www.pearsoned.ca/school/physicssource.

7.4 Applications of Simple Harmonic Motion

People's arms swing as they walk. An annoying rattle can develop in a car when it reaches a certain speed. A child can make large waves in the bathtub by sliding back and forth. Many things can be made to vibrate, and when they do, they seem to do it with a period of motion that is unique to them. After all, how often do you think about your arms swinging as you walk? You don't — they seem to swing of their own accord and at their own frequency. The water in the bathtub will form very large waves when the child makes a back-and-forth motion at just the right rate. Any other rate won't create the waves that splash over the edge and soak the floor, which, of course, is the goal.

In all these cases, the object vibrates at a natural frequency. **Resonant frequency** is the natural frequency of vibration of an object. In other words, objects that are caused to vibrate do so at a natural frequency that depends on the physical properties of the object. All objects that can vibrate have a resonant frequency, including a pendulum.

info **BIT**

When you walk with a drink in your hand at the right speed, your motion creates resonance in the liquid. This makes waves that splash over the edge of the cup. To prevent this, people walk slowly so resonance doesn't occur, often without knowing why this works.

resonant frequency: the natural frequency of vibration of an object

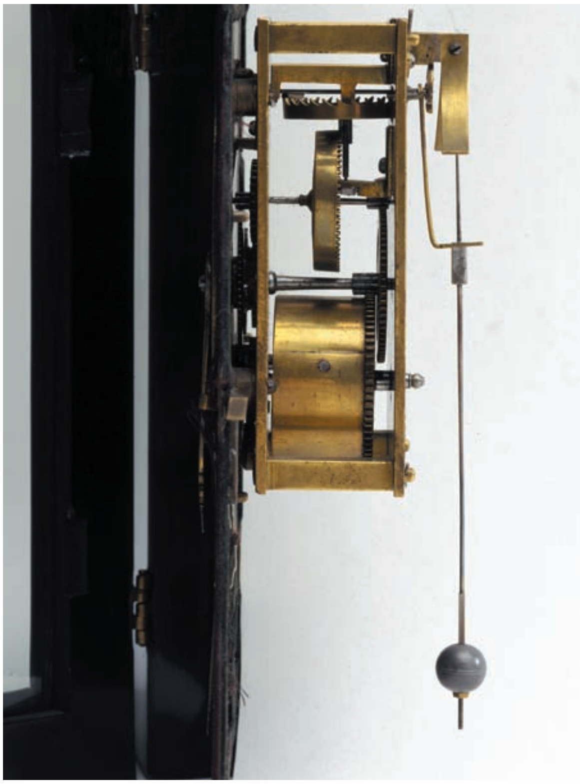
Maintaining a Pendulum's Resonant Frequency

A pendulum swings back and forth at its resonant frequency. Since the acceleration of gravity does not change if we stay in the same place, the only factor that affects the resonant frequency is the pendulum's length. All pendulums of the same length oscillate with the same natural (resonant) frequency.

Huygens made use of this fact when he designed his pendulum clock (Figure 7.45). He knew that all pendulum clocks would keep the same time as long as the length of the pendulum arms was the same. Their resonant frequencies would be identical.

However, Huygens faced some challenges in making a pendulum clock. The arm of the pendulum would expand or contract with temperature, affecting its period. But this was a relatively minor issue compared to another difficulty that had to be overcome — friction. Unless something was done, friction would very quickly stop the pendulum from swinging.

To compensate for the effects of friction, he designed his clocks so that the pendulum was given a small push at just the right moment in its swing. The timing of these pushes coincided with the resonant frequency of the pendulum. By doing this, Huygens could make the pendulum swing for as long as the periodic force was applied.



▲ **Figure 7.45** The interior of Huygens's clock

Forced Frequency

To visualize how this works, imagine a child on a swing. A swing is an example of a pendulum, with the child as the bob. The swing moves back and forth at its natural frequency, which depends only on its length. To keep the swing going with the same amplitude, all the parent has to do is push at just the right moment. The timing of the pushes must match the frequency of the swing.

As anyone who has pushed a swing can attest, it takes very little energy to keep a swing swinging to the same height. The frequency at which the force is applied to keep the swing moving is called the **forced frequency**. If the forced frequency matches or is close to the resonant frequency of the object, then very little force is required to keep the object moving. The resonant frequency won't change though, because it depends only on the length of the pendulum. If the parent decides to push a little harder each time the swing returns, then the swing's **amplitude** will increase, but not its frequency. A larger force than is needed to overcome friction will create a larger amplitude of motion. If the forced frequency isn't close to the resonant frequency, then the object will not vibrate very much and will have a small amplitude.

forced frequency: the frequency at which an external force is applied to an oscillating object

PHYSICS INSIGHT

The forced frequency that is the same as the resonant frequency will increase the amplitude of the SHM, but will not change the resonant frequency.

Imagine trying to increase the frequency of a pendulum by increasing the forced frequency. Much of the force won't be transferred to the pendulum because the pendulum won't be in the right position when the force is applied. The pendulum will bounce around but there will be no increase in its amplitude of vibration, and its motion will become harder to predict. The flowchart in Figure 7.46 on the next page summarizes the relationship between forced frequency and resonant frequency.

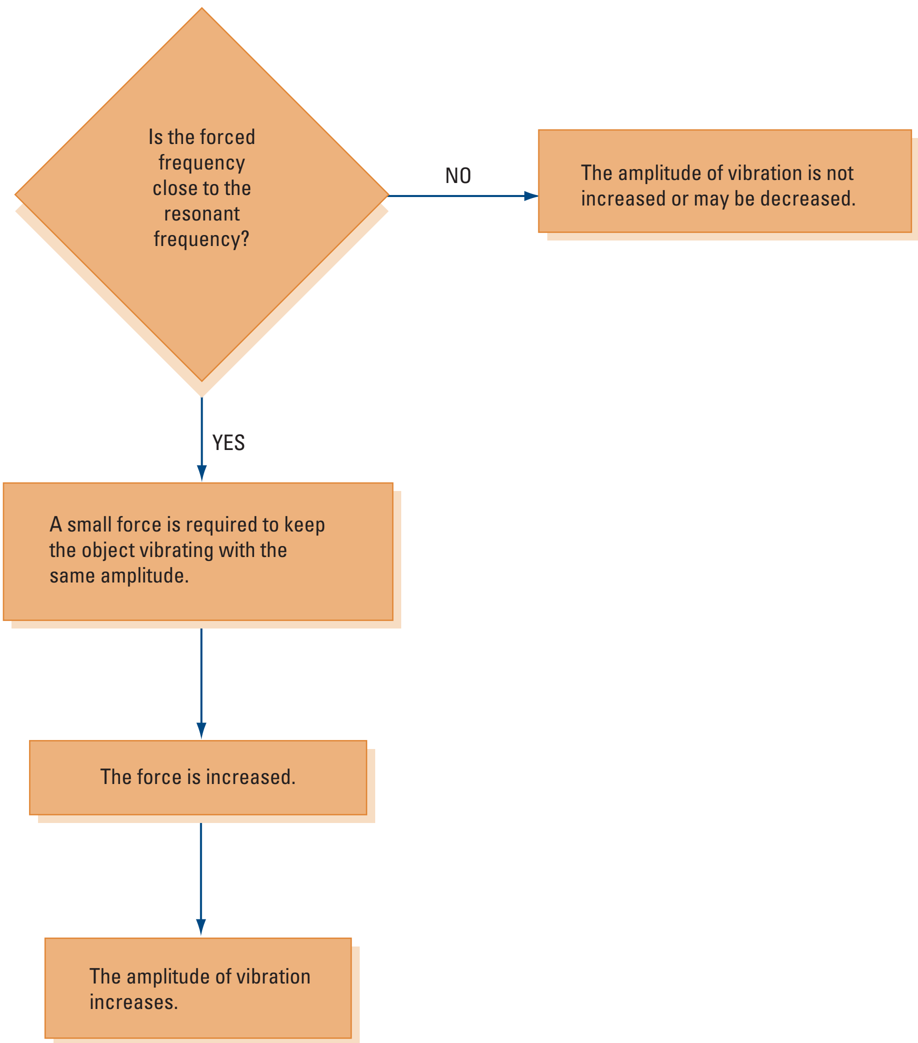
Mechanical Resonance

A forced frequency that matches the resonant frequency is capable of creating very large amplitudes of oscillation. This is referred to as **mechanical resonance**. This can be a good or bad thing. The larger the amplitude, the more energy the system has. Huygens's pendulum clock didn't need to have large oscillations, so a very small force could keep the pendulum swinging. A small weight-driven mechanism was used to provide the force needed. The force simply had to be applied with the same frequency as the pendulum. Huygens managed to do this without much difficulty.

mechanical resonance: the increase in amplitude of oscillation of a system as a result of a periodic force whose frequency is equal or very close to the resonant frequency of the system

His pendulum clocks were a great success but weren't completely practical since they had to be placed on solid ground. A pendulum clock would not work aboard a ship because sailing ships of the time were buffeted by the waves more than today's large ocean-going vessels are. The motion of the ship on the waves would disturb a pendulum's SHM, so sailors could not take advantage of the increased accuracy these clocks provided.

The key to successfully navigating across an ocean (where there are no landmarks) was to use an accurate clock on the ship. This clock could be synchronized to a clock in Greenwich, England, which is situated on the prime meridian of 0° longitude. As the ship travelled east or west, the sailors could compare their local time, using the Sun and a sundial, to the ship's clock, which was still synchronized to the time on the prime meridian. The difference in time between the two clocks could be used to compute their longitudinal position. However, it wasn't until the 1700s that a brilliant clockmaker, John Harrison, successfully made a marine chronometer (ship's clock) that was immune to the buffeting of waves and temperature. It contained several ingenious innovations and, for better or worse, made the pendulum obsolete in navigation.



▲ **Figure 7.46** Flowchart of the effect of forced frequency on resonant amplitude

7-5 QuickLab

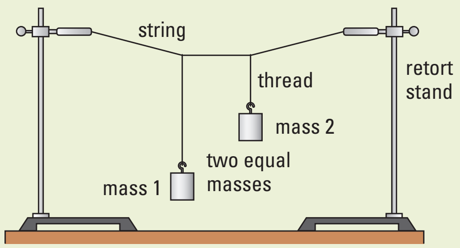
Investigating Mechanical Resonance

Problem

How can we cause a pendulum to begin oscillating at its resonant frequency using a forced frequency?

Materials

retort stands
string
thread
2 identical masses (200 g each)



▲ **Figure 7.47**

Procedure

Part A

1. Read the questions in the next column before doing the lab.
2. Set up the two retort stands about 75 cm apart.
3. Tie the string to both retort stands at the same height (50.0 cm) on each stand, as shown in Figure 7.47. Clamp the ends of the string to the retort stands so they don't slip. The string should be taut.
4. Pull the two retort stands farther apart if you need to remove slack from the string.
5. Attach the thread to one mass and tie the other end to the string so that the distance from the mass to the string is 30 cm. This is mass 1.
6. Repeat step 5 for the second mass (mass 2) so that it has a length of 10 cm and is attached to the string about 15 cm from the first mass.
7. Make sure neither mass is moving, then pull mass 2 back a small distance and release it. Observe the motion of mass 1 as mass 2 oscillates. Make a note of the maximum amplitude that mass 1 achieves.
8. Repeat step 7 three more times. Each time, lengthen the thread of mass 2 by 10 cm. For your last trial, the thread of mass 2 should be 40.0 cm long.

Part B

9. Adjust the thread length of mass 2 so that it is as close to the thread length of mass 1 as possible.
10. Make sure both masses are motionless. Pull back and release mass 2. Note the amplitude of vibration that mass 1 achieves.
11. Pull the retort stands farther apart and hold them there so the tension in the string is increased, and the string is almost horizontal.
12. Make sure both masses are motionless, then pull back mass 2 and release it. Note the amplitude of vibration that mass 1 achieves.

Questions

Part A

1. At what thread length did mass 2 create the maximum oscillation of mass 1? Explain why this happened, in terms of frequency.

2. At what thread length did mass 2 create the minimum oscillation of mass 1? Explain why this happened, in terms of frequency.

3. Why did mass 1 have a large amplitude of vibration in only one case?

Part B

4. What effect did increasing the tension on the string have on the amplitude achieved by mass 1?

5. Why did increasing the tension alter the maximum oscillation of mass 1?

6. Write a sentence describing the effect that increasing the tension had on the resonant amplitude of mass 1. Use the terms *forced frequency* and *resonant amplitude* in your answer.

eLAB

For a probeware lab, go to www.pearsoned.ca/school/physicssource.

Resonance Effects on Buildings and Bridges

A forced frequency that matches the resonant frequency can create problems for designers of bridges and skyscrapers. A bridge has a resonant frequency that can be amplified by the effect of wind. Air flows around the top and bottom of a bridge and can cause it to vibrate. The bridge will vibrate at its resonant frequency, with a large amplitude, even though the force applied by the wind may be relatively small. As the bridge vibrates, it may flex more than it is designed to and could conceivably vibrate to pieces.

A skyscraper is also susceptible to forced vibrations caused by the wind. Most skyscrapers have a huge surface area and catch a lot of wind. Even though a building is a rigid structure, the force of the wind can make it sway back and forth. The wind causes a phenomenon called "vortex shedding." It can create a forced vibration that matches the natural frequency of the building's back-and-forth vibration. The unfortunate result is to increase the sway (amplitude) of the building. The occupants on the top floors of the skyscraper will feel the effects the most. Over time, the continual large sway could weaken the building's structural supports and reduce its lifespan.

Reducing Resonance Effects

To counter resonance effects on bridges and buildings, engineers build them in such a way as to reduce the amplitude of resonance. Bridge designers make bridges more streamlined so that the wind passes over without imparting much energy. They also make bridges stiff, so a larger force is needed to create a large amplitude. The second-largest bridge in the world, the Great Belt East Bridge of Denmark is built with a smooth underside, like an airplane wing, that greatly increases its streamlined shape (Figure 7.48). It is not likely that a forced vibration would cause it to resonate.

Skyscraper designers employ many strategies to lessen resonant vibration. One very effective approach is to use a large mass at the top of the building, called a "tuned mass damper," which is free to oscillate back and forth (Figure 7.49). Controlled by computers, it can be made to vibrate at the resonant frequency of the building. When the building sways left, the mass moves right, and when the building sways right, the mass moves left. This has the effect of cancelling the vibration of the building. Any process that lessens the amplitude of an object's oscillations is referred to as "damping."

▲ **Figure 7.48** The Great Belt East Bridge of Denmark is 6.8 km long and is constructed with a smooth underside. This allows air to flow by without inducing a resonant frequency.

▲ **Figure 7.49** The Taipei 101 building in Taiwan was completed in 2004 and stands 101 stories high. The inset shows a tuned mass damper in the building designed by Motioneering Inc. of Guelph, Ontario. It has a huge mass and vibrates opposite to the direction of the building, cancelling much of the amplitude of the resonant vibration.

THEN, NOW, AND FUTURE — Stressed-out Airplanes

Ask any mechanical engineers, and they will tell you the importance of designing equipment to minimize vibration. Vibration causes excess wear on parts and stress on materials. Nowhere is this more evident than on an airplane. Yancey Corden knows this better than most people. Yancey is an aircraft maintenance engineer, and one of his jobs is to inspect aircraft for excess metal fatigue.

Yancey was born on the Pavilion Reserve in south central British Columbia but grew up north of Williams Lake. His father maintained their car, boat, and other equipment around the home. Yancey watched and helped his father, and during this time, his interest in mechanics grew.

Not long after finishing high school, Yancey enrolled in the aircraft maintenance engineer program at the British Columbia Institute of Technology located in Burnaby. He is now qualified with an M1 certification, which allows him to work on planes under 12 500 kg, and an M2 certification, which allows him to work on larger planes.

He received specialized training in structural maintenance. This involves an in-depth knowledge of the skin and frame of the plane.

▲ **Figure 7.50** Yancey Corden

In 2003 Yancey moved to Alberta where he works in Red Deer for a company called Air Spray. Air Spray maintains and repairs Lockheed L-188 airplanes. These planes were originally manufactured as passenger craft in the 1950s, but because of their rugged design, they have been converted to firefighting aircraft today. They carry over 10 000 kg of water and fire retardant and are capable of dumping the entire amount in three seconds.

When a dump of water and fire retardant occurs, the wings and airframe of the plane undergo a huge shift of force. This happens because the airplane springs upward due to the lighter load, and as a result, the wings tend to flutter up and down. This vibration causes stress fractures on the wing, and it is Yancey's job to find them. If a problem is found, Yancey designs the solution. This could involve fabricating a new part or simply fixing the existing one.

He enjoys his job because each day is different and brings new challenges.

He is thankful that he had the foresight to maintain good marks when he went to high school because the physics and science courses he took directly applied to his training. He is very proud of his heritage but he also believes it is important to focus on who you are and where you are going.

Questions

1. What factors contribute to metal fatigue on a firefighting airplane?
2. What steps must be taken to gain a licence as an aircraft maintenance engineer?
3. To what factors does Yancey attribute his success?

Quartz Clocks

The technology of clock design and manufacture has taken huge leaps since the 1600s when Huygens built his first pendulum clock. Today, quartz clocks are the most accurate timepieces commercially available. They have an accuracy of about 1/2000 of a second a day.

A quartz clock works on the principle of resonance. Inside each quartz clock is a tiny crystal of quartz. Quartz is a mineral that naturally forms into crystals. It also has a property unique to just a handful of materials: it will bend when a voltage is applied to it. If a pulse of voltage is applied to it, the crystal will begin to vibrate at its resonant frequency, just as a cymbal vibrates when hit by a drumstick. Once the quartz crystal is set vibrating, the circuitry of the clock times successive voltage pulses to synchronize with the frequency of the crystal. The synchronized voltage provides the forced frequency to keep the crystal oscillating just as the pendulums of Huygens's clocks needed a synchronized forced frequency to keep them from running down.

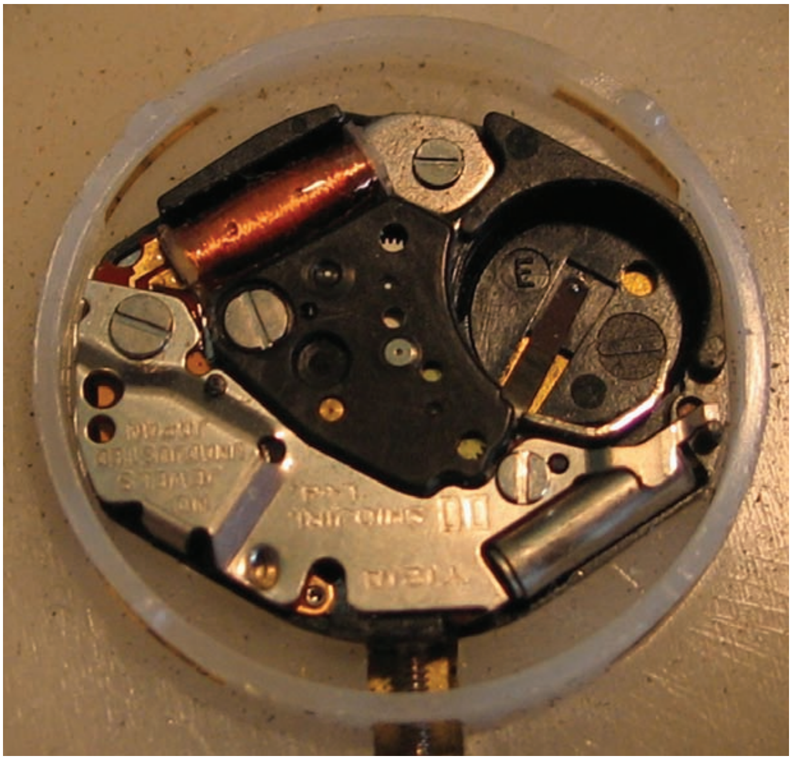
The difference is that the pendulum clock receives the forced frequency through mechanical means, while the quartz crystal clocks get the forced vibration from electrical means.

Resonant Frequency of a Quartz Crystal

The crystal's resonant frequency depends on its size and shape and is not affected significantly by temperature. This makes it ideal for keeping time. As the crystal gets larger, more voltage is required to make it oscillate, and its resonant frequency decreases. A piece of quartz could be cut to oscillate once every second, but it would be far too large for a wristwatch and would require a large voltage to operate. If the crystal size is decreased, less voltage is required to make it oscillate.

Quartz crystals are cut to a size and shape small enough to fit into a watch and use a small voltage (Figure 7.51). In most of today's quartz watches, the crystal vibrates with a resonant frequency of about 30 kHz and operates at 1.5 V. A small microprocessor in the watch combines these oscillations to make one oscillation per second so the watch can display time in a meaningful way.

The topic of resonant frequencies is large and can't possibly be fully covered in this unit. You will learn more about resonance in musical instruments in Chapter 8.



▶ **Figure 7.51** The quartz crystal in a wristwatch is enclosed in the small metal cylinder (lower right).

info BIT

A substance that deforms with an applied voltage is called a piezoelectric material. A piezoelectric material will also create a voltage if stressed. Some butane lighters use a piezoelectric material to create a flame. As the lighter trigger is pressed, butane gas is released and the piezoelectric material undergoes stress. The piezoelectric material creates a voltage that causes a spark to jump a very small gap at the end of the lighter, igniting the butane.

e WEB

Atomic clocks keep time extremely precisely. Do they use a principle of resonance to keep such accurate time? Begin your search at www.pearsoned.ca/school/physicssource.

7.4 Check and Reflect

Knowledge

1. What provides the force necessary to start a building or bridge oscillating?
2. What is forced frequency?
3. Explain what engineers use to reduce resonant vibrations of buildings and how these devices or structures work.
4. Explain the effect of applying a force to a vibrating object with the same frequency.
5. Identify two limitations of Huygens's pendulum clock.
6. Can a pendulum clock built to operate at the equator have the same accuracy at the North Pole? Explain.
7. What is damping? Use an example in your explanation.

Applications

8. How could a person walking across a rope bridge prevent resonant vibration from building up in the bridge?
9. An opera singer can shatter a champagne glass by sustaining the right musical note. Explain how this happens.
10. Tuning forks are Y-shaped metal bars not much bigger than a regular fork. They can be made to vibrate at a specific frequency when struck with a rubber hammer. A piano tuner uses tuning forks to tune a piano. Explain, in terms of resonance, how this might be done.
11. Students are asked to find ways to dampen or change the resonant frequency of a pendulum. Here is a list of their suggestions. Identify the ones that would work and those that would not. In each case, justify your answer.
    (a) Apply a forced frequency that is different from the resonant frequency.
    (b) Place the pendulum in water.
    (c) Increase the mass of the pendulum bob.
    (d) Move the pendulum to a higher altitude.
12. What factors affect the resonance of a quartz crystal?
13. (a) What are two advantages of a quartz clock over a pendulum clock?
    (b) Are there any disadvantages of a quartz clock compared with a pendulum clock?

Extensions

14. Use the knowledge you have gained about the design of a pendulum clock and the equation for its period in section 7.3 to answer the following question. What would the length of the pendulum's arm have to be so that it would oscillate with a resonant frequency of 1.00 Hz in Alberta (g = 9.81 N/kg)? Under what conditions would it be most accurate?
15. Use your local library or the Internet to find out what automobile manufacturers do to reduce resonant frequencies in cars.
16. Investigate other methods not mentioned in the text that bridge designers use to lessen resonant vibrations.
17. Tuned mass dampers are not just used on buildings; cruise ships also have them. Explain why a cruise ship might have them and how they would be used.
18. Use your local library or the Internet to explore orbital resonance. In one or two paragraphs, explain how it applies to Saturn's rings.

e TEST

To check your understanding of applications of simple harmonic motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

Key Terms and Concepts

period
frequency
oscillation
cycle
oscillatory motion
Hooke's law
spring constant
restoring force
simple harmonic motion
simple harmonic oscillator
resonant frequency
amplitude
forced frequency
mechanical resonance

Key Equations

$\vec{F} = -k\vec{x}$

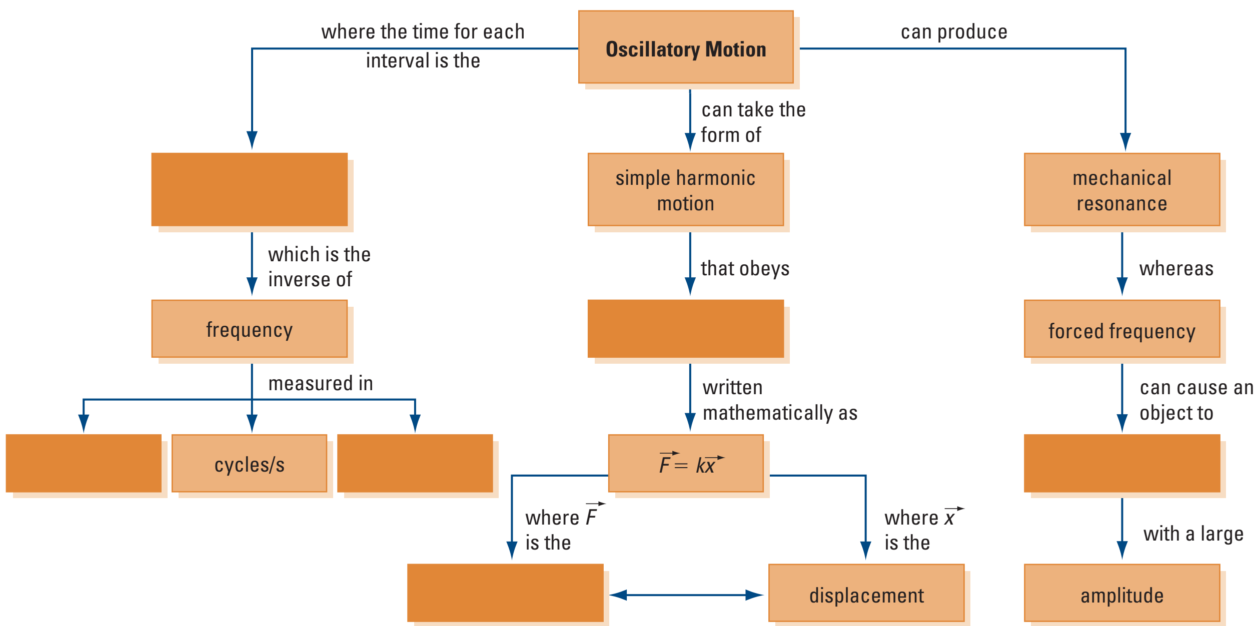
$v_{max} = A\sqrt{\frac{k}{m}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.



▲ **Figure 7.52**

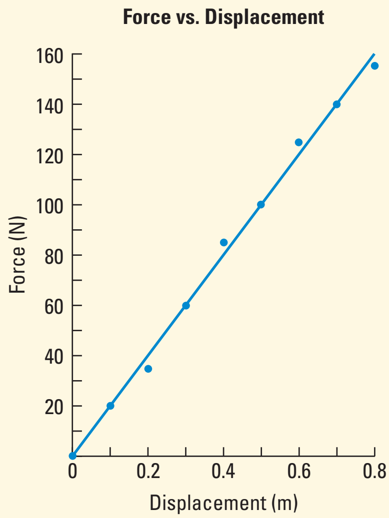
CHAPTER 7 REVIEW

Knowledge

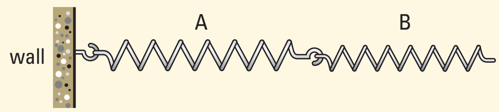
1. (7.1) What is oscillatory motion? Use an example in your answer.
2. (7.1) Under what conditions must a ball be bounced so it has oscillatory motion?
3. (7.2) What is the defining property of an elastic material?
4. (7.2) What force, or forces, act on an isolated, frictionless simple harmonic oscillator?
5. (7.2) State the directional relationship that exists between the restoring force and displacement of a simple harmonic oscillator.
6. (7.2) What quantity does the slope of a force-displacement graph represent?
7. (7.2) What can be said about a pendulum's position if the restoring force is a non-zero value?
8. (7.3) Why isn't acceleration uniform for a simple harmonic oscillator?
9. (7.3) Why is it acceptable to consider a pendulum a simple harmonic oscillator for small displacements, but not for large displacements?
10. (7.4) If the forced frequency and the resonant frequency are similar, what effect does this have on an oscillator?

Applications

11. Determine the restoring force acting on a 1.0-kg pendulum bob when it is displaced:
    (a) 15°
    (b) 5°
12. Determine the frequency of a guitar string that oscillates with a period of 0.0040 s.
13. What is the period of a ball with a frequency of 0.67 Hz?
14. After a diver jumps off, a diving board vibrates with a period of 0.100 s. What is its frequency?
15. What is the restoring force on a 2.0-kg pendulum bob displaced 15.0°?
16. Determine the spring constant from the following graph:



17. A spring hangs from the ceiling in a physics lab. The bottom of the spring is 1.80 m from the floor. When the teacher hangs a mass of 100 g from the bottom of the spring, the spring stretches 50.0 cm.
    (a) What is its spring constant?
    (b) What force must a person apply to pull the 100.0-g mass on the bottom of the spring down through a displacement of 20.0 cm?
    (c) The 100.0-g mass is removed and a 300.0-g mass is attached. What is the distance of the mass above the floor?
18. Two different springs, A and B, are attached together at one end. Spring A is fixed to the wall as shown. The spring constant of A is 100.0 N/m and B is 50.0 N/m. What is the combined stretch of the two springs when a force of 25.0 N [right] is applied to the free end of spring B?



19. Students stretch an elastic band attached to a force meter through several displacements and gather the following data. Use a graphing calculator or another acceptable method to plot the graph of this data and determine if the elastic band moves as predicted by Hooke's law.

Displacement (cm)	Force (N)
0.00	0.00
10.0	3.80
20.0	15.2
30.0	34.2
40.0	60.8
50.0	95.0

20. How long must the arm of a pendulum clock be to swing with a period of 1.00 s, where the gravitational field strength is 9.81 N/kg?

21. What is the period of a 10.0-kg mass attached to a spring with a spring constant of 44.0 N/m?

22. Determine the maximum velocity of a 2.00-t crate suspended from a steel cable (k = 2000.0 N/m) that is oscillating up and down with an amplitude of 12.0 cm.

23. A 0.480-g mass is oscillating vertically on the end of a thread with a maximum displacement of 0.040 m and a maximum speed of 0.100 m/s. What acceleration does the mass have if it is displaced 0.0200 m upwards from the equilibrium position?

24. Determine the period of oscillation of a pendulum that has a length of 25.85 cm.

25. An astronaut who has just landed on Pluto wants to determine the gravitational field strength. She uses a pendulum that is 0.50 m long and discovers it has a frequency of vibration of 0.182 Hz. What value will she determine for Pluto's gravity?

26. A student is given the relationship for a pendulum: $T = 2\pi\sqrt{X}$

(a) What does X represent?

(b) The student records the period of the pendulum and finds it is 1.79 s. What is the pendulum's length?

Extensions

27. A spring (k = 10.0 N/m) is suspended from the ceiling and a mass of 250.0 g is hanging from the end at rest. The mass is pulled to a displacement of 20.0 cm and released.

(a) What is the maximum velocity of the mass?

(b) What is the period of oscillation of the mass if it is displaced 15.0 cm and released?

28. A horizontal mass-spring system has a mass M attached to a spring that oscillates back and forth at a frequency of 0.800 Hz. Determine the frequency in the following cases.

(a) The mass is doubled.

(b) The amplitude is tripled.

29. Identify which of the following examples is SHM and which is not. Explain.

(a) a bouncing ball

(b) a hockey player moving a puck back and forth with his stick

(c) a plucked guitar string

Consolidate Your Understanding

Create your own summary of oscillatory motion, simple harmonic motion, restoring force, and mechanical resonance by answering the questions below. If you want to use a graphic organizer, refer to Student References 4: Using Graphic Organizers on pp. 869–871. Use the Key Terms and Concepts listed above and the Learning Outcomes on page 342.

1. Prepare a quick lesson that you could use to explain Hooke's law to a peer using the following terms: restoring force, displacement, linear relationship.
2. Construct a two-column table with the title "Mass-spring System." The first column has the heading, "Factors Affecting Period" and the second column has the heading, "Factors Not Affecting Period." Categorize the following factors into the two columns: mass, spring constant, amplitude, restoring force, velocity.

Think About It

Review your answers to the Think About It questions on page 343. How would you answer each question now?

***e*TEST**

To check your understanding of oscillatory motion, follow the eTest links at www.pearsoned.ca/school/physicssource.