CHAPTER

6

In an isolated system, energy is transferred from one object to another whenever work is done.

Key Concepts

In this chapter, you will learn about:

- work, mechanical energy, and power
- the work-energy theorem
- isolated and non-isolated systems
- the law of conservation of energy

Learning Outcomes

When you have finished this chapter, you will be able to:

Knowledge

- use the law of conservation of energy to explain the behaviours of objects within isolated systems
- describe the energy transformations in isolated and non-isolated systems using the work-energy theorem
- calculate power output

Science, Technology, and Society

- explain that models and theories are used to interpret and explain observations
- explain that technology cannot solve all problems
- express opinions on the support found in Canadian society for science and technology measures that work toward a sustainable society

▲ **Figure 6.1**

Tension mounts as the motor pulls you slowly to the top of the first hill. Slowly you glide over the top, then suddenly, you are plunging down the hill at breathtaking speed. Upon reaching the bottom of the hill, you glide to the top of the next hill and the excitement begins all over again. As you race around the roller coaster track, each hill gets a bit lower until, at last, you coast to a gentle stop back at the beginning. You probably realize that because of friction the trolley can never regain the height of the previous hill, unless it is given a boost. It seems obvious to us that as objects move, kinetic energy is always lost.

Energy is the most fundamental concept in physics. Everything that occurs in nature can be traced back to energy. The complicating factor is that there are so many forms of energy it is often very difficult to keep track of what happens to the energy when it is transferred. Energy is a scalar quantity. This chapter concentrates on gravitational potential energy, kinetic energy, and elastic potential energy.

In this chapter you will take the first steps to understanding the role of energy in nature. Specifically, you will learn how energy is given to and taken from objects when they interact with each other.

6-1 QuickLab

Energy Changes of a Roller Coaster

eWEB

This activity uses the roller coaster simulation found at www.pearsoned.ca/school/physicssource.

Problem

How does the energy of a roller coaster vary as it travels on its track?

Materials

computer connected to the Internet
clear plastic ruler

Procedure

1. Click on the start button for the simulation.
2. Observe the motion of the cart.
3. Click on "continue" and note what happens to the motion of the trolley as it moves along the track.
4. Repeat step 3 until the simulation is complete.
5. Reset the simulation.
6. Use a see-through plastic ruler to measure the lengths of the potential energy bar (blue) and the kinetic energy bar (green) before you start the simulation. Record your measurements.
7. Start the simulation.
8. Each time the trolley pauses, measure the length of the potential energy and the kinetic energy bars and record the results in a table similar to Table 6.1.

Questions

1. What assumptions are you making when you measure the lengths of the energy bars?
2. What is the effect on the potential energy of the trolley as it moves upward and downward?
3. What is the effect on the kinetic energy of the trolley as it moves upward and downward? Is this true as the trolley moves upward to the top of the first hill? Explain.
4. From the table, what happens to the energy of the trolley as it moves from the start to position "a"?
5. For each of the positions at which the trolley pauses, how does the sum of lengths of the bars change? What does the sum of these lengths represent?
6. Is there an energy pattern as the trolley moves along the track? Describe the pattern.
7. Do you think that this pattern is representative of nature? Explain.

▼ **Table 6.1**

Position	Length of Potential Energy Bar (mm)	Length of Kinetic Energy Bar (mm)	Sum of Lengths (mm)
start			
a			
b			

Think About It

1. If two cars are identical except for the size of their engines, how will that affect their performance on the highway?
2. What is the "law of conservation of energy"? When does this law apply?
3. When work is done on an object, where does the energy used to do the work go?

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes in your ideas.

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

info **BIT**

Inuit hunters devised unusual ways of storing potential energy in a bow. One way they accomplished this was to tie cords of sinew along the back of the bow (Figure 6.3). The sinew was more heavily braided where strength was needed and less heavily braided where flexibility was important. When the bow was bent, the cords would stretch like a spring to store energy. In the absence of a wood source, bows were often made of antler or bone segments.

e **WEB**

To see photographs and to learn more about the technology of Inuit bows, follow the links at www.pearsoned.ca/school/physicssource.

6.1 Work and Energy

▲ **Figure 6.2** When the string on a bow is pulled back, elastic potential energy is stored in the bow.

▲ **Figure 6.3** Sinew-backed bow of the Inuit Copper people of the Central Arctic.

energy: the ability to do work

▲ **Figure 6.4** During the downhill run, the skier's gravitational potential energy is continually converted into kinetic energy and heat.

An archer is pulling back her bowstring (Figure 6.2). She does work on the bow transforming chemical energy in her muscles into elastic potential energy in the bow. When she releases the string, the bow does work on the arrow. The elastic potential energy of the bow is transformed into the energy of motion of the arrow, called kinetic energy.

As skiers ride up a lift, the lift's motor is transforming chemical energy of the fuel into gravitational potential energy of the individuals. As they go downhill, gravity does work on the skiers transforming their gravitational potential energy into kinetic energy and heat.

In both these examples, work transfers **energy**. In the case of the archer, energy is transformed from chemical energy into elastic potential energy and then into kinetic energy (from the archer to the bow to the arrow). In the case of the skier, energy is transformed from the chemical energy of the motor's fuel into the gravitational potential energy of the skier at the top of the run and then into a combination of changing kinetic energy, gravitational potential energy, and heat of the skier as she speeds downhill. All these processes involve work.

Work Is Done When Force Acts Over a Displacement

When a force ($\vec{F}$) acts on an object resulting in a displacement ($\Delta\vec{d}$), a transfer of energy occurs. This energy transfer is defined as the **work** done by the force. In introductory courses the quantity of work is usually defined by the equation $W = F\Delta d$.

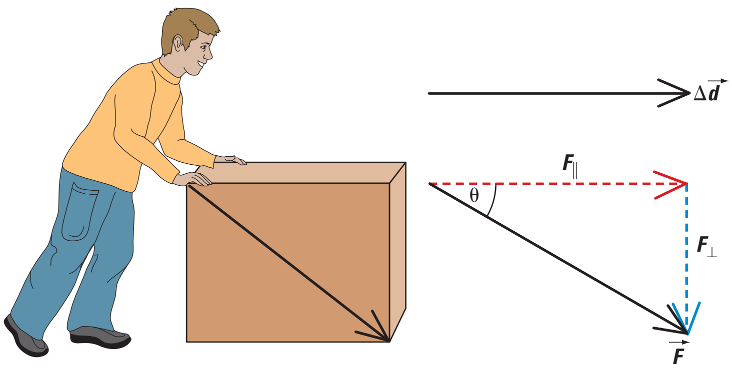
Work is a scalar quantity. However, the relative directions of the vectors $\vec{F}$ and $\Delta\vec{d}$ are important. If the applied force ($\vec{F}$) does not act parallel to the displacement (Figure 6.5), you must resolve the force into components that are parallel ($F_{\parallel}$) and perpendicular ($F_{\perp}$) to the displacement. Only the component of the force parallel to the displacement actually does work. The component of the force acting perpendicular to the displacement does no work.

work: a measure of the amount of energy transferred when a force acts over a given displacement. It is calculated as the product of the magnitude of applied force and the displacement of the object in the direction of that force.

PHYSICS INSIGHT

The unit of work and energy is the joule (J). It is a derived unit.

$$1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$



▲ **Figure 6.5** When a force acts on an object, resulting in a displacement, only the component of the force that acts parallel to the displacement does work. If the box moves horizontally, only the horizontal component, $\vec{F}_{\parallel}$, does work.

Thus, the equation for work is often written as

$$W = F_{\parallel}\Delta d$$

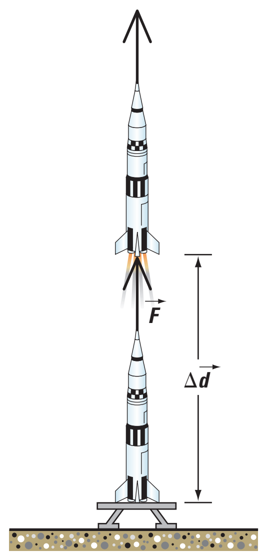
where $F_{\parallel}$ is the magnitude of the component of the force that acts parallel to the displacement. In Figure 6.5, where the angle between the direction of the force and the direction of the displacement is θ, the component of the force parallel to the displacement is given by

$$F_{\parallel} = F\cos\theta$$

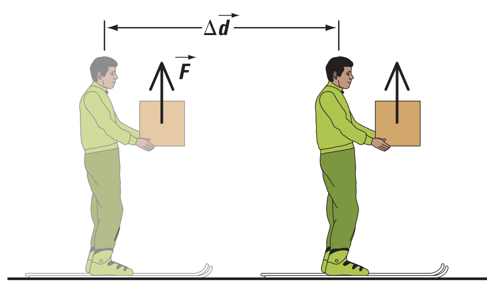
If you replace $F_{\parallel}$ by $F\cos\theta$, the calculation for work becomes

$$W = (F\cos\theta)\Delta d$$

Let's look at two special cases. First, when the force acts parallel to the displacement, the angle $\theta = 0°$ so that $\cos\theta = 1$, making $F_{\parallel} = F$. This results in the maximum value for the work that the force could do over that displacement (Figure 6.6). Second, if the force acts perpendicular to the displacement, there is no parallel component. Mathematically, since $\theta = 90°$ then $\cos\theta = 0$ making $F_{\parallel} = 0$. In this case, the applied force does no work on the object (Figure 6.7).



▲ **Figure 6.6**



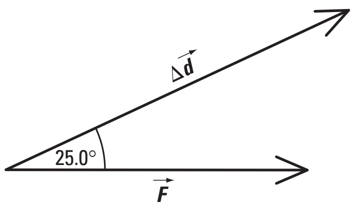
▲ **Figure 6.7**

Concept Check

When a centripetal force acts on an object, the object shows no increase in speed and therefore no increase in kinetic energy. In terms of the work done by the centripetal force, explain why this is true.

Example 6.1

Figure 6.8 shows a force of 150 N [0°] acting on an object that moves over a displacement of 25.0 m [25.0°] while the force acts. What is the work done by this force?



▲ **Figure 6.8**

Given

$\vec{F} = 1.50 \times 10^2$ N [0°]

$\Delta\vec{d} = 25.0$ m [25.0°]

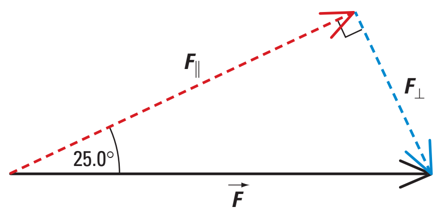
Required

work done by the force (W)

Analysis and Solution

From Figure 6.8, the angle between the force and the displacement is 25.0°. Draw a component diagram (Figure 6.9). The component that does work is F (cos 25.0°). Solve using the equation for work.

$$\begin{aligned} W &= (F\cos\,\theta)\Delta d \\ &= (1.50 \times 10^2\ \text{N})(\cos 25.0°)(25.0\ \text{m}) \\ &= 3.399 \times 10^3\ \text{N·m} \\ &= 3.40 \times 10^3\ \text{J} \end{aligned}$$



▲ **Figure 6.9** Component diagram

Paraphrase and Verify

The work done by the force is 3.40×10^3 J. If the force had acted parallel to the displacement, the maximum amount of work done would have been 3.75×10^3 J. Since cos 25.0° is about 0.91, the answer of 3.40×10^3 J, or 0.91 times the maximum value for W, is reasonable.

Practice Problems

1. You pull a sled along a horizontal surface by applying a force of 620 N at an angle of 42.0° above the horizontal. How much work is done to pull the sled 160 m?
2. A force acts at an angle of 30.0° relative to the direction of the displacement. What force is required to do 9600 J of work over a displacement of 25.0 m?
3. A force of 640 N does 12 500 J of work over a displacement of 24.0 m. What is the angle between the force and the displacement?
4. A bungee jumper with a mass of 60.0 kg leaps off a bridge. He is in free fall for a distance of 20.0 m before the cord begins to stretch. How much work does the force of gravity do on the jumper before the cord begins to stretch?

Answers

1. 7.37×10^4 J
2. 443 N
3. 35.5°
4. 1.18×10^4 J

Gravitational Potential Energy

An object is said to have potential energy if it has the ability to do work by way of its position or state. There are several forms of potential energy. Imagine a ride at the fair where the passengers are lifted vertically before being allowed to drop in free fall, as in Figure 6.10. Ignoring friction, the work done by the machinery to lift the passengers and car to a height, h, is equal to the change in **gravitational potential energy, ΔE_p**. To lift the object straight up at a constant speed, the force applied must be equal but opposite to the force of gravity on the object. The equation for calculating work can be used to develop the equation for change in gravitational potential energy.

$$\begin{aligned}\Delta E_p &= W \\ &= F\Delta d\end{aligned}$$

where F is the magnitude of the force acting *parallel* to the displacement, and Δd is the magnitude of the displacement.

To lift an object of mass m upward at a constant speed, the force is equal in magnitude and parallel, but opposite in direction, to the gravitational force, $\vec{F}_g$. Recall from Unit II that $F_g = mg$ where g is the magnitude of the acceleration due to gravity, which has a constant value of 9.81 m/s^2 near Earth's surface. It follows that

$$F = mg$$

▲ **Figure 6.10** A motor works transferring energy to the ride car. The gravitational potential energy gained produces the exciting free fall.

If the object is moved through a change in height Δh, so that $\Delta d = \Delta h$, the change in potential energy equation becomes

$$\Delta E_p = mg\Delta h$$

So, when an object is moved upward, h increases and Δh is positive, and the potential energy increases (positive change). When an object is moved downward, h decreases and Δh is negative, and the potential energy decreases (negative change).

gravitational potential energy: the energy of an object due to its position relative to the surface of Earth

Concept Check

Does the above equation for change in gravitational potential energy apply to objects that move over very large changes in height (e.g., change as experienced by a rocket)? Explain.

Example 6.2

If the car and its passengers in Figure 6.10 have a mass of 500 kg, what is their change in gravitational potential energy when they are lifted through a height of 48.0 m?

Given

$m = 500$ kg

$g = 9.81$ m/s^2

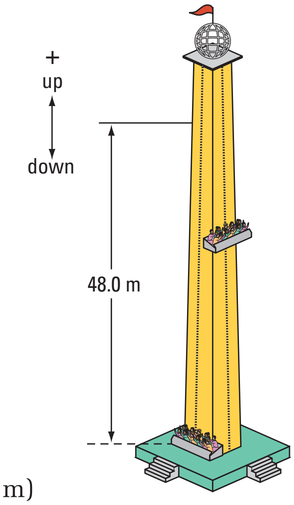
$\Delta h = +48.0$ m

Required

change in gravitational potential energy (ΔE_P)

Analysis and Solution

Sketch the movement of the car as in Figure 6.11 and solve for ΔE_P.



▲ **Figure 6.11**

$$\begin{aligned}\Delta E_P &= mg\Delta h \\ &= (5.00 \times 10^2\ \text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(+48.0\ \text{m}) \\ &= +2.35 \times 10^5 \left(\left(\text{kg} \cdot \frac{\text{m}}{\text{s}^2}\right) \cdot \text{m}\right) \\ &= +2.35 \times 10^5\ (\text{N} \cdot \text{m}) \\ &= +2.35 \times 10^5\ \text{J}\end{aligned}$$

Paraphrase

The change in gravitational potential energy of the car and its passengers is a gain of 2.35×10^5 J. The object moved upward, gaining gravitational potential energy.

Practice Problems

1. An elevator car has a mass of 750 kg. Three passengers of masses 65.0 kg, 30.0 kg, and 48.0 kg, ride from the 8th floor to the ground floor, 21.0 m below. Find the change in gravitational potential energy of the car and its passengers.
2. A book with a mass of 1.45 kg gains 25.0 J of potential energy when it is lifted from the floor to a shelf. How high is the shelf above the floor?
3. The Mars rover lifts a bucket of dirt from the surface of Mars into a compartment on the rover. The mass of the dirt is 0.148 kg and the compartment is 0.750 m above the surface of Mars. If this action requires 0.400 J of energy, what is the gravitational acceleration on Mars?

Answers

1. -1.84×10^5 J
2. 1.76 m
3. 3.60 m/s^2

If E_{p_1} represents the potential energy of an object at height h_1 and E_{p_2} its potential energy when it is lifted to a height h_2, then the change in potential energy is, by definition,

$$\Delta E_p = E_{p_2} - E_{p_1}$$

Since $\Delta E_p = mg\Delta h$

$$E_{p_2} - E_{p_1} = mg(h_2 - h_1)$$

Consider an object at ground level as having zero potential energy. If the object is raised from the ground level, $h_1 = 0$. It follows that

$$E_{p_2} - 0 = mg(h_2 - 0)$$

$$E_{p_2} = mgh_2$$

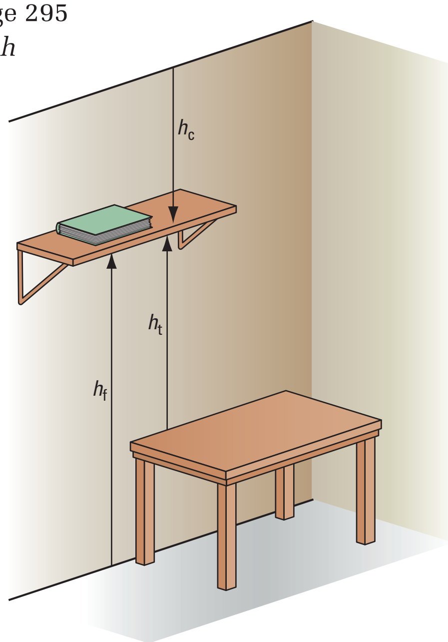
In general, the potential energy of an object at height h, measured from the ground, is

$$E_p = mgh$$

Choosing a Reference Point

You see from the equation for the change in potential energy on page 295 that ΔE depends only on the change in height, Δh. The values of h may be measured from any convenient **reference point**, as long as the reference point is kept the same for all the measurements when solving a problem. The change in height, and therefore the change in gravitational potential energy, is the same regardless of your frame of reference.

Look at the book resting on the shelf in Figure 6.12. The value of h for the shelf can be defined relative to the floor (h_f), relative to the table (h_t), or even relative to the ceiling above the shelf (h_c), in which case h_c will have a negative value. Usually, you choose the frame of reference that most simplifies your measurements and calculations for Δh. For example, if you were trying to determine how much gravitational potential energy the book would lose as it fell from the shelf to the tabletop, then it would be logical to use the tabletop as your reference point. If you used another position as a reference point, your calculations might be slightly more complex, but the final answer for the amount of gravitational potential energy the book loses would be the same.

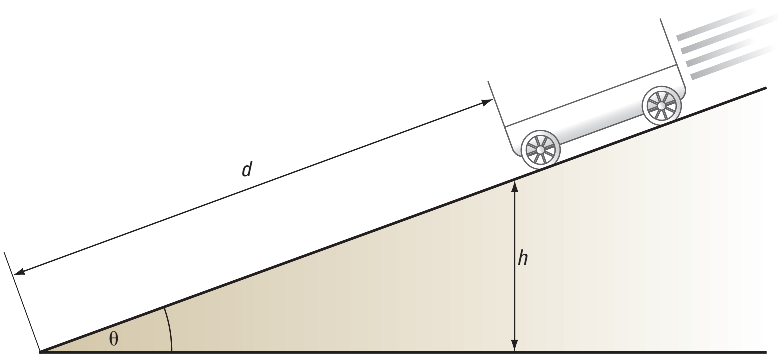


▲ **Figure 6.12** The book has gravitational potential energy due to its position on the shelf.

Change in gravitational potential energy depends only on change in vertical height.

The change in gravitational potential energy of an object depends only on the change in height. For example, the change in gravitational potential energy of a cart rolling down a frictionless ramp as in Figure 6.13 depends only on the vertical measurement, h. The actual distance an object travels, while it moves through a given change in height, does not affect its change in gravitational potential energy.

reference point: an arbitrarily chosen point from which distances are measured



▲ **Figure 6.13** As the cart rolls down the ramp, only the change in height h affects its change in gravitational potential energy.

PHYSICS INSIGHT

The calculation of h from Figure 6.13 involves the use of the trigonometric ratio $\sin\theta = \dfrac{h}{d}$.

Example 6.3

Figure 6.14 shows a toy car track set up on a tabletop.

(a) What is the gravitational potential energy of the car, which has a mass of 0.0250 kg, relative to the floor?

(b) Calculate the change in gravitational potential energy of the car when it arrives at the bottom of the hill.

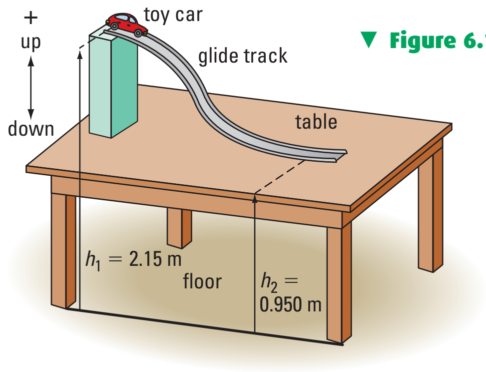
Given

$m = 0.0250$ kg

$g = 9.81$ m/s^2

$h_1 = 2.15$ m

$h_2 = 0.950$ m



▼ **Figure 6.14**

Required

(a) gravitational potential energy at the top of the hill relative to the floor (E_{p_1})

(b) change in the gravitational potential energy as the car moves from the top to the bottom of the hill (ΔE_p)

Analysis and Solution

(a) To find gravitational potential energy relative to the floor, use that surface to define $h = 0$ and make all height measurements from there.

$$\begin{aligned} E_{p_1} &= mgh_1 \\ &= (0.0250 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(2.15 \text{ m}) \\ &= +0.527 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\ &= 0.527 \text{ J} \end{aligned}$$

(b) To find the change in gravitational potential energy, use the data and Figure 6.14 to calculate the change in height ($\Delta h = h_2 - h_1$).

$$\begin{aligned} \Delta h &= h_2 - h_1 \\ &= 0.950 \text{ m} - 2.15 \text{ m} \\ &= -1.20 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta E_p &= mg(\Delta h) \\ &= (0.0250 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(-1.20 \text{ m}) \\ &= -0.294 \text{ J} \end{aligned}$$

Paraphrase and Verify

(a) The gravitational potential energy relative to the floor is 0.527 J.

(b) The change in gravitational potential energy is −0.294 J. As the car rolls down the hill it loses 0.294 J of gravitational potential energy.

Note: You could calculate E_{p_2} first and then use $\Delta E_p = E_{p_2} - E_{p_1}$

Practice Problems

1. A pile driver drops a mass of 550 kg from a height of 12.5 m above the ground onto the top of a pile that is 2.30 m above the ground. Relative to ground level, what is the gravitational potential energy of the mass
   (a) at its highest point?
   (b) at its lowest point?
   (c) What is the change in the gravitational potential energy of the mass as it is lifted from the top of the pile to its highest point?
2. A roller coaster trolley begins its journey 5.25 m above the ground. As the motor tows it to the top of the first hill, it gains 4.20×10^5 J of gravitational potential energy. If the mass of the trolley and its passengers is 875 kg, how far is the top of the hill above the ground?
3. A winch pulls a 250-kg block up a 20.0-m-long inclined plane that is tilted at an angle of 35.0° to the horizontal. What change in gravitational potential energy does the block undergo?

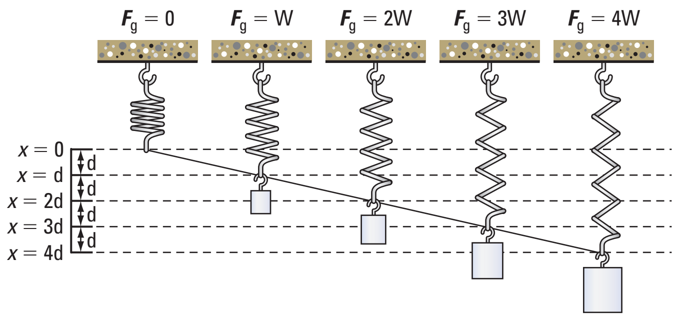
Answers

1. (a) 6.74×10^4 J
   (b) 1.24×10^4 J
   (c) 5.50×10^4 J
2. 54.2 m
3. 2.81×10^4 J

Hooke's Law

In 1676, Robert Hooke, an English physicist, showed that the stretch produced by a force applied to a spring was proportional to the magnitude of the force. This relationship is known as Hooke's Law and applies to any elastic substance when a force is exerted upon it. Thus, if a mass is suspended from a spring (Figure 6.15) the position (x) of the mass changes in proportion to the force (the weight (F_g) of the mass) exerted on the spring.

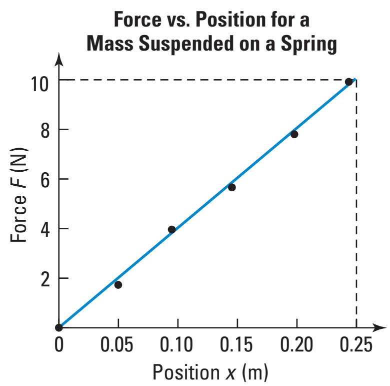
In an experiment to test this prediction, students suspended a series of masses from a spring and measured the position for each mass. Their data are shown in Table 6.2.



▲ **Figure 6.15** The stretch produced by a force applied on a spring is proportional to the magnitude of the force.

▼ **Table 6.2** Students' experimental data

Mass m (g)	Weight F_g (N)	Position x (m)
0	0	0
200	1.96	0.050
400	3.92	0.099
600	5.87	0.146
800	7.85	0.197
1000	9.81	0.245



◄ **Figure 6.16** Graph of data from Table 6.2

The students then plotted a graph of the magnitude of the applied force (F_g) as a function of the position (x) of the spring. The resulting line is a straight line with a constant slope (Figure 6.16). The equation of this line is $F = kx$ where k is the slope of the line. The slope of the line is determined by the properties of the spring and is defined as the **elastic** or **spring constant** (k). This constant tells us how hard it is to stretch/compress the spring from the equilibrium position at $x = 0$.

For the graph in Figure 6.16, the slope is found as shown below:

$$
\begin{aligned}
k &= \frac{\Delta F}{\Delta x} \\
&= \frac{F_f - F_i}{x_f - x_i} \\
&= \frac{10.0 \text{ N} - 3.0 \text{ N}}{0.250 \text{ m} - 0.075 \text{ m}} \\
&= 40 \frac{\text{N}}{\text{m}}
\end{aligned}
$$

This force-position graph is characteristic for all springs whether the force stretches or compresses the spring. When a heavier or lighter spring is used, the slope of the line changes but the line is still straight.

You will deal with Hooke's Law in greater depth when you study simple harmonic motion in Chapter 7.

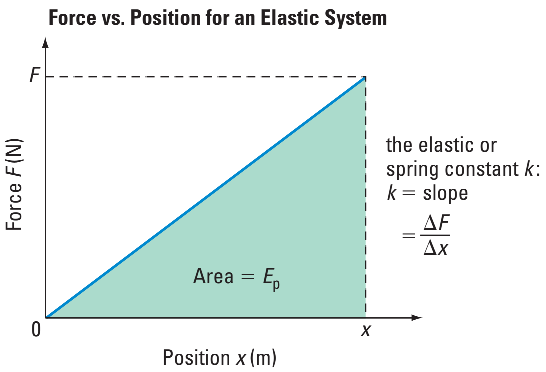
PHYSICS INSIGHT

A spring becomes non-elastic at a certain critical stretch value called the *elastic limit*. If the force applied does not exceed the elastic limit, the material will return to its original shape. If a spring is stretched beyond its elastic limit, its shape will be permanently distorted or the spring may break.

Elastic Potential Energy

elastic potential energy: the energy resulting from an object being altered from its standard shape, without permanent deformation

When the archer in Figure 6.2 draws her bow she stores another form of potential energy, **elastic potential energy**, in the bow. Both gravitational potential energy and elastic potential energy form part of mechanical energy. The study of elastic potential energy requires the use of Hooke's law.



▲ **Figure 6.17** The area under the force-position curve is equal to the work done by the force to stretch the spring to that position.

The amount of energy stored in a spring is equal to the work done to stretch (or compress) the spring, without causing any permanent deformation. The force is not constant, so the equation for work used earlier, ($W = F\Delta d$) does not apply, because that equation requires a constant force acting over the displacement. However, when force-position graphs are used, work is equivalent to the area under the curve. The units for this area are N·m, equivalent to joules, the unit for work. You can therefore determine the amount of work done to stretch the spring from its equilibrium position to the position x by calculating the area of the shaded portion of Figure 6.17.

Calculation of Elastic Potential Energy

The area under the curve in Figure 6.17 is the shaded triangle whose area is calculated by $A = \frac{1}{2}hb$. The base (b) is equal to the magnitude of the position (x), and the height (h) is equal to the magnitude of the force (F) at that position. Thus, in terms of force and position, the equation for the area under the curve is

$$W = \frac{1}{2}Fx$$

From Hooke's law, the magnitude of the force (F) is equal to $F = kx$, so the work done to stretch the spring can be written as:

$$\begin{aligned} W &= \frac{1}{2}(kx)(x) \\ &= \frac{1}{2}kx^2 \end{aligned}$$

The work done to stretch (or compress) a spring from its equilibrium position to any position (x) results in storing elastic potential energy (E_p) in the spring. Therefore, the equation for the elastic potential energy stored in the spring is given by

$$E_p = \frac{1}{2}kx^2$$

PHYSICS INSIGHT

ΔE_p, the change in elastic potential energy depends on the square of the stretch in the spring. That is:

$$\begin{aligned} \Delta E_p &= \frac{1}{2}k(x_2^2 - x_1^2) \\ &\neq \frac{1}{2}k(\Delta x)^2 \end{aligned}$$

Concept Check

Explain why it is incorrect to try to find the change in elastic potential energy of a stretched spring from the measurement of the change in the stretch.

Example 6.4

A spring is stretched to a position 35.0 cm from its equilibrium position. At that point the force exerted on the spring is 10.5 N.

(a) What is the elastic potential energy stored in the spring?

(b) If the stretch in the spring is allowed to reduce to 20.0 cm, what is the change in the elastic potential energy?

Given

$x_1 = 35.0 \text{ cm} = 0.350 \text{ m}$

$F_1 = 10.5 \text{ N}$

$x_2 = 20.0 \text{ cm} = 0.200 \text{ m}$

Required

(a) elastic potential energy in the spring stretched to 0.350 m (E_{p_1})

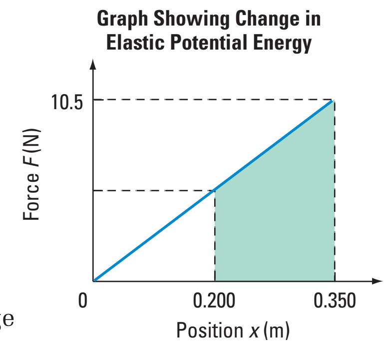
(b) change in the elastic potential energy when the stretch is reduced from 0.350 m to 0.200 m (ΔE_p)

Analysis and Solution

(a) Calculate the value for k, the elastic constant for the spring, using Hooke's law.

$$F_1 = kx_1$$

$$k = \frac{F_1}{x_1} = \frac{10.5 \text{ N}}{0.350 \text{ m}} = 30.0 \ \frac{\text{N}}{\text{m}}$$



▲ **Figure 6.18**

From the data given, plot the graph of change in elastic potential energy, Figure 6.18. Next, use $E_p = \frac{1}{2}kx^2$, to find the elastic potential energy for a stretch of 0.350 m. This is equivalent to finding the area of the large triangle in Figure 6.18.

$$\begin{aligned} E_{p_1} &= \frac{1}{2}kx_1^2 \\ &= \frac{1}{2}\left(30.0 \ \frac{\text{N}}{\text{m}}\right)(0.350 \text{ m})^2 \\ &= 1.8375 \left(\frac{\text{N}}{\text{m}} \cdot \text{m}^2\right) \\ &= 1.84 \text{ N} \cdot \text{m} \\ &= 1.84 \text{ J} \end{aligned}$$

(b) To find the change in the elastic potential energy, first find the elastic potential energy at a stretch of 0.200 m and then subtract from that value the answer to part (a). This is equivalent to finding the shaded area of the graph in Figure 6.18.

Practice Problems

1. A force of 125 N causes a spring to stretch to a length of 0.250 m beyond its equilibrium position.
   (a) What is the elastic potential energy stored in the spring?
   (b) If the spring contracts to a stretch of 0.150 m, what is the change in elastic potential energy?
2. An engineer is designing the suspension system for a car. He decides that the coil spring used in this car should compress 4.00 cm when a force of 1000 N is applied to it.
   (a) What is the spring constant of the spring?
   (b) If the spring is compressed a distance of 14.0 cm, what force must have been exerted on it?
3. The elastic constant for a spring is 750 N/m.
   (a) How far must you stretch a spring from its equilibrium position in order to store 45.0 J of elastic potential energy in it?
   (b) If you wanted to double the elastic potential energy stored in the spring, how much farther would you need to stretch it?
4. A spring has an elastic constant of 4.40×10^4 N/m. What is the change in elastic potential energy stored in the spring when its stretch is increased from 12.5 cm to 15.0 cm?
5. When a spring is stretched by 0.400 m from its equilibrium position, its elastic potential energy is 5.00×10^2 J.
   (a) What is the magnitude of the force required to produce this amount of stretch?
   (b) If the force causing the stretch is changed to 1000 N, how much change in elastic potential energy results?

The elastic potential energy for a stretch of 0.200 m is:

$$
\begin{aligned}
E_{\mathrm{P}_2} &= \frac{1}{2}kx_2^2 \\
&= \frac{1}{2}\left(30.0\ \frac{\mathrm{N}}{\mathrm{m}}\right)(0.200\ \mathrm{m})^2 \\
&= 0.600\ \mathrm{N{\cdot}m} \\
&= 0.600\ \mathrm{J}
\end{aligned}
$$

The change in the elastic potential energy is:

$$
\begin{aligned}
\Delta E_{\mathrm{P}} &= E_{\mathrm{P}_2} - E_{\mathrm{P}_1} \\
&= 0.600\ \mathrm{J} - 1.84\ \mathrm{J} \\
&= -1.24\ \mathrm{J}
\end{aligned}
$$

Paraphrase

(a) The energy stored in the spring at the initial stretch is 1.84 J.

(b) When the stretch is reduced from 0.350 m to 0.200 m, the elastic potential energy stored in the spring reduced by 1.24 J to 0.600 J.

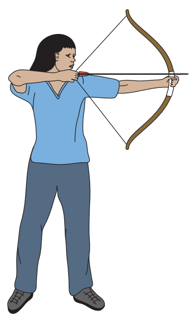
Answers

1. (a) 15.6 J
   (b) −10.0 J
2. (a) 2.50×10^4 N/m
   (b) 3.50×10^3 N
3. (a) 0.346 m
   (b) 0.143 m
4. 1.51×10^2 J
5. (a) 2.50×10^3 N
   (b) −420 J

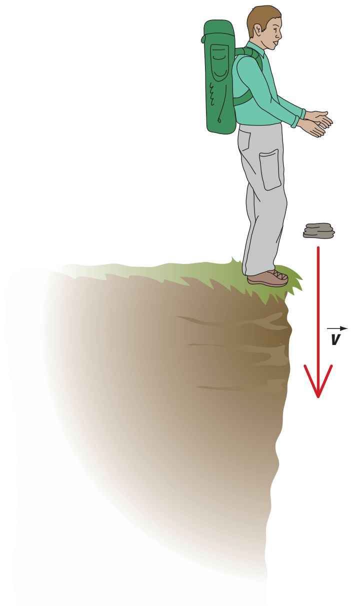
Kinetic Energy

Examine Figure 6.19. When the archer releases the arrow, the bowstring exerts a non-zero force on the arrow, which accelerates the arrow toward its target. As the arrow gains speed, it gains **kinetic energy (E_k)**.

kinetic energy: the energy due to the motion of an object



▲ **Figure 6.19**



▲ **Figure 6.20** When an object is in free fall, gravity is working to increase its kinetic energy.

When the hiker in Figure 6.20 drops the rock off the cliff, the force of gravity accelerates the rock downward increasing its speed and thus its kinetic energy. Kinetic energy is a scalar quantity. The kinetic energy of an object is calculated using the equation

$$E_k = \frac{1}{2}mv^2$$

PHYSICS INSIGHT

There are two kinds of kinetic energy. The kinetic energy studied here is more correctly referred to as translational kinetic energy, since the objects are moving along a line. Earth has both translational kinetic energy (because it orbits the Sun) and rotational kinetic energy (because it spins on its axis).

Concept Check

If the kinetic energy of an object doubles, by what factor does its speed increase?

Example 6.5

On a highway, a car of mass 1.2×10^3 kg, travelling at 20 m/s, has kinetic energy equal to a loaded van of mass 4.8×10^3 kg. What is the speed of the van?

Given

$m_1 = 1.2 \times 10^3$ kg; $m_2 = 4.8 \times 10^3$ kg;

$v_1 = 20$ m/s

$E_{k_{car}} = E_{k_{van}}$

Required

the speed of the van (v_2)

Analysis and Solution

The two vehicles have equal kinetic energy $\left(\frac{1}{2}mv^2\right)$.

Find the kinetic energy of the car and then use that value to solve for the speed of the van.

$$\begin{aligned} E_{k_{car}} &= \frac{1}{2}mv_{car}^2 \\ &= \frac{1}{2}(1.2 \times 10^3 \text{ kg})\left(20\frac{\text{m}}{\text{s}}\right)^2 \\ &= 2.4 \times 10^5 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\ &= 2.4 \times 10^5 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_{van}} &= 2.4 \times 10^5 \text{ J} \\ &= \frac{1}{2}mv_{van}^2 \\ v_{van} &= \sqrt{\frac{(2.4 \times 10^5 \text{ J})(2)}{4.8 \times 10^3 \text{ kg}}} \\ &= 10 \text{ m/s} \end{aligned}$$

Paraphrase

The van is travelling at 10 m/s.

Practice Problems

1. A 45.0-kg girl pedals a 16.0-kg bicycle at a speed of 2.50 m/s. What is the kinetic energy of the system?
2. A car travelling at 80.0 km/h on a highway has kinetic energy of 4.2×10^5 J. What is the mass of the car?
3. A skateboarder with a mass of 65.0 kg increases his speed from 1.75 m/s to 4.20 m/s as he rolls down a ramp. What is the increase in his kinetic energy?

Answers

1. 1.91×10^2 J
2. 1.7×10^3 kg
3. 474 J

Project LINK

How will the concept of the kinetic energy of a moving vehicle relate to the design of your persuader apparatus?

MINDS ON Energy of Impact

There is evidence that many meteors have hit Earth's surface. The vast quantity of kinetic energy that these meteors have at the time of impact is revealed by the size of the craters that they create (Figure 6.21).

1. What types of measurements would scientists need to make in order to estimate the kinetic energy of the meteor at the instant of impact?
2. What types of experiments could be done to verify the scientists' assumptions?
3. Investigate the incidence of meteor collisions in Canada. Where is the meteor impact crater that is closest to where you live? Approximately how many meteors have landed in Alberta? What was the greatest kinetic energy for a meteor that landed (a) in Alberta (b) in Canada?

▶ **Figure 6.21** Meteor impact craters are found in all regions of Earth. This one, called the Barringer crater, is in Arizona.

Example 6.6

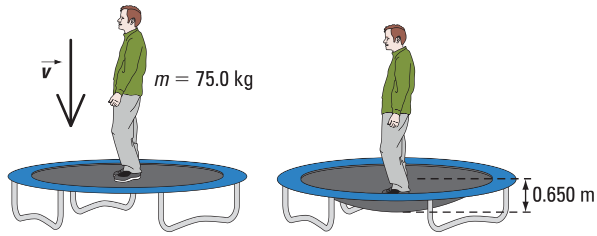
A man on a trampoline has a mass of 75.0 kg. At the instant he first touches the surface of the trampoline (at its rest position) he is descending with a speed of 8.00 m/s. At his lowest point, the man is 0.650 m below the trampoline's rest position. (a) What is the kinetic energy of the man when he first contacts the trampoline? (b) If you assume that, at his lowest point, all of the man's kinetic energy is transformed into elastic potential energy, what is the elastic constant for the trampoline?

Given

$m = 75.0$ kg

$v = 8.00$ m/s

$x = 0.650$ m



▲ **Figure 6.22**

Required

(a) kinetic energy of the man (E_k)

(b) the elastic constant of the trampoline (k)

Analysis and Solution

(a) Find the initial kinetic energy, by using $E_k = \frac{1}{2}mv^2$

$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(75.0\text{ kg})\left(8.00\ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 2.40 \times 10^3\text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$
$$= 2.40 \times 10^3\text{ J}$$

(b) Assume that the elastic potential energy at 0.650 m is 2.40×10^3 J and solve for the elastic constant.

$$E_p = \frac{1}{2}kx^2$$

Solve for k.

$$k = \frac{2E_p}{x^2}$$
$$= \frac{2(2.40 \times 10^3\text{ J})}{(0.650\text{ m})^2}$$
$$= 1.136 \times 10^4\ \frac{\text{N}}{\text{m}}$$
$$= 1.14 \times 10^4\ \frac{\text{N}}{\text{m}}$$

Paraphrase

(a) The kinetic energy of the man is 2.40×10^3 J.

(b) The elastic constant of a spring that stores 2.40×10^3 J of elastic potential energy when it is stretched 0.650 m is 1.14×10^4 N/m.

Practice Problems

1. A bow that has an elastic constant of 2500 N/m is stretched to a position of 0.540 m from its rest position.
   (a) What is the elastic potential energy stored in the bow?
   (b) If all of the elastic potential energy of the bow were to be transformed into kinetic energy of a 95.0-g arrow, what would be the speed of the arrow?
2. Cannon A fires a 1.5-kg ball with a muzzle velocity of 550 m/s, while cannon B fires cannon balls with one-third the mass but at twice the muzzle velocity. Which of these two cannons would be more effective in damaging a fortification? Explain why.
3. It is estimated that the meteor that created the crater shown in Figure 6.21 on the previous page had a radius of 40 m, a mass of approximately 2.6×10^8 kg, and struck Earth at a speed of nearly 7.20×10^4 km/h.
   (a) What was the kinetic energy of the meteor at the instant of impact?
   (b) When one tonne (t) of TNT explodes, it releases about 4.6×10^9 J of energy. In terms of tonnes of TNT, how much energy did the meteor have at impact?

Answers

1. (a) 365 J (b) 87.6 m/s
2. $E_{k_A} : E_{k_B} = 3 : 4$. Ball B will do more damage.
3. (a) 5.2×10^{16} J (b) 1.1×10^7 t

6.1 Check and Reflect

1. If a force does not act parallel to the resulting displacement, what is the effect on the work done by the force?

2. Describe how a non-zero force can act on an object over a displacement and yet do no work.

3. Explain why the frame of reference affects the calculated value of an object's gravitational potential energy but not the change in its gravitational potential energy.

4. What is meant by elastic potential energy?

5. A force of 1500 N [up] acts to lift an object of 50.0-kg mass to a height of 24.0 m above its original position.
   (a) How much work did the force do on the object?
   (b) What was the gain in the object's gravitational potential energy?
   (c) What might account for the difference in the two answers?

6. A force of 850 N [30°] acts on an object while it undergoes a displacement of 65.0 m [330°]. What is the work the force does on the object?

7. You are working on the 5^{th} floor of a building at a height of 18.0 m above the sidewalk. A construction crane lifts a mass of 350 kg from street level to the 12^{th} floor of the building, 22.0 m above you. Relative to your position, what is the gravitational potential energy of the mass
   (a) at street level?
   (b) when it is on the 12^{th} floor?
   (c) What is its change in gravitational potential energy as it is raised?

8. A spring has an elastic constant of 650 N/m. Initially, the spring is compressed to a length of 0.100 m from its equilibrium position.
   (a) What is the elastic potential energy stored in the spring?
   (b) How much further must the spring be compressed if its potential energy is to be tripled?

9. Two cars (A and B) each have a mass of 1.20×10^3 kg. The initial velocity of car A is 12.0 m/s [180°] while that of car B is 24.0 m/s [180°]. Both cars increase their velocity by 10.0 m/s [180°].
   (a) Calculate the gain in kinetic energy of each car.
   (b) If both cars gain the same amount of velocity, why do they gain different amounts of kinetic energy?

10. A cart with a mass of 3.00 kg rolls from the top of an inclined plane that is 7.50 m long with its upper end at a height of 3.75 m above the ground. The force of friction acting on the cart as it rolls is 4.50 N in magnitude.
    (a) What is the change in gravitational potential energy when the cart moves from the top of the inclined plane to ground level?
    (b) What is the work done by friction?

11. An ideal spring with an elastic constant of 2000 N/m is compressed a distance of 0.400 m.
    (a) How much elastic potential energy does this compression store in the spring?
    (b) If this spring transfers all of its elastic potential energy into the kinetic energy of a 2.00-kg mass, what speed would that mass have? Assume the initial speed of the mass is zero.

e TEST

To check your understanding of potential and kinetic energy, follow the eTest links at www.pearsoned.ca/school/physicssource.

6.2 Mechanical Energy

mechanics: the study of kinematics, statics, and dynamics

mechanical energy: the sum of potential and kinetic energies

In physics, the study of **mechanics** includes kinematics (the study of motion), statics (the study of forces in equilibrium), and dynamics (the study of non-zero forces and the motion that results from them). Gravitational potential energy, elastic potential energy, and kinetic energy form what is called **mechanical energy**. When work is done on a system, there may be changes in the potential and kinetic energies of the system. This relationship is expressed as the work-energy theorem.

The Work-Energy Theorem

info **BIT**

Doubling the speed of a vehicle means quadrupling the necessary stopping distance. This relationship between stopping distance and speed is based on the physics of work and kinetic energy.

A 1.50×10^5-kg jet plane waits at the end of the runway. When the air-traffic controller tells the pilot to take off, the powerful engines can each produce more than 2.5×10^5 N of thrust to accelerate the plane along the runway. In order to produce the speed of about 250 km/h required for takeoff, the engines would need to convert more than 3×10^8 J of chemical energy from the fuel supply into kinetic energy.

▶ **Figure 6.23** The work done by the jet's engines must convert enough chemical energy into kinetic energy to produce a velocity sufficient for takeoff.

e **LAB**

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

As explained by Newton's Laws of Motion, the non-zero net force causes the jet plane to accelerate along the runway. Since a change in kinetic energy must involve a change in speed, kinetic energy changes are always the result of the acceleration, which in turn is caused by a non-zero net force. In terms of work and energy, this means that changes in kinetic energy (ΔE_k) are always the result of the work done by a non-zero net force (W_{net}).

$$\begin{aligned}\Delta E_k &= W_{net}\\ &= (F_{net})(\cos\theta)(\Delta d)\\ &= (ma)(\cos\theta)(\Delta d)\end{aligned}$$

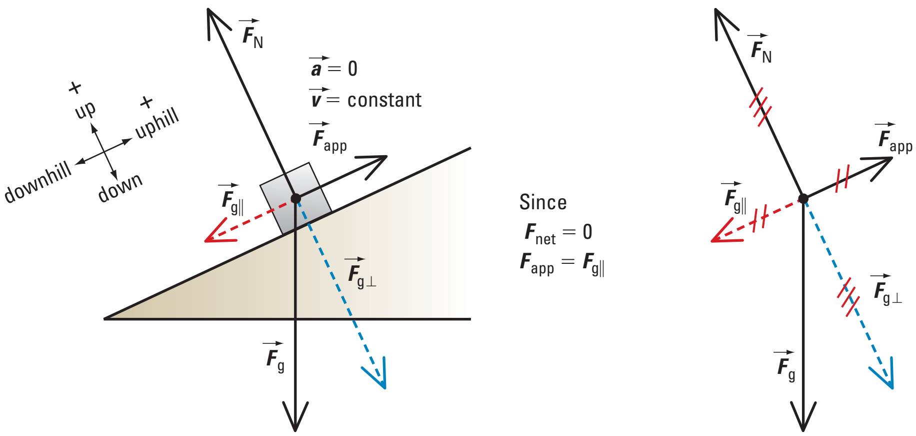
PHYSICS INSIGHT

In the example of the airplane takeoff, θ is the angle between the direction of F_{net} or a, and d.

In all cases, work done by a non-zero net force results in a change in kinetic energy but the applied forces on an object may cause changes in its potential energy, its kinetic energy, or both. For example, once the jet plane is in the air, the thrust produced by its engines must increase its speed (ΔE_k) as well as cause it to gain altitude (ΔE_p).

Zero and Non-zero Net Forces and Effects on Energy

A motor that is pulling a block up a frictionless inclined plane at a constant speed (Figure 6.24) is exerting a force that causes a change in gravitational potential energy but not kinetic energy. The constant speed indicates that the applied force (F_{app}) is exactly balanced by the $F_{g\parallel}$ component of F_g. There is zero net force; $F_{app} = F_{g\parallel}$.

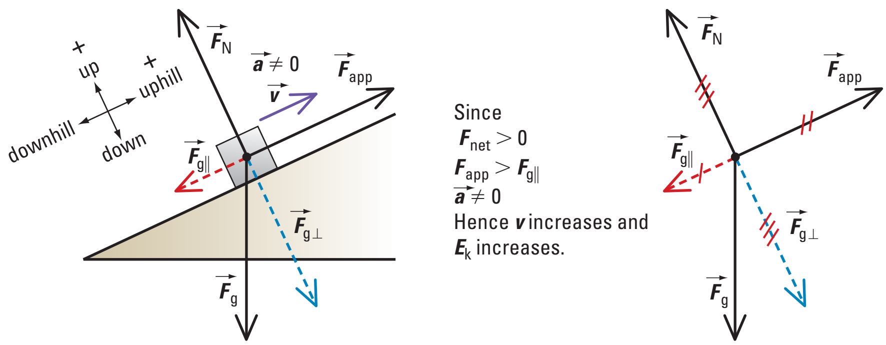


▲ **Figure 6.24** If all the forces acting on a block combine to produce a net force of zero, the block moves up the incline at a constant speed. It increases its gravitational potential energy but not its kinetic energy.

If, however, the force applied is now increased so that there is a non-zero net force and $F_{app} > F_{g\parallel}$ (Figure 6.25), the forces are no longer balanced, the block accelerates up the incline, and both kinetic energy and potential energy change. Now the work done on the block is transferred to both its kinetic energy and its gravitational potential energy. This is expressed mathematically as

$W = \Delta E$ or, in more detail, as

$W = \Delta E_k + \Delta E_p$ This is known as the **work-energy theorem**.



▲ **Figure 6.25** If the forces acting on a block are such that there is a non-zero net force up the plane, then $F_{app} > F_{g\parallel}$. Both the kinetic energy and the gravitational potential energy will increase as the block moves up the incline.

PHYSICS INSIGHT

The symbol ΔE_p could refer to gravitational potential energy, elastic potential energy, or the sum of the gravitational and elastic potential energies.

The work-energy theorem states that the work done on a system is equal to the sum of the changes in the potential and kinetic energies of the system.

Concept Check

A block is sliding down an inclined plane. If there is no friction, describe a situation where the net work might still be negative.

Example 6.7

A block is moved up a frictionless inclined plane by a force parallel to the plane. At the foot of the incline, the block is moving at 1.00 m/s. At the top of the incline, 0.850 m above the lower end, the block is moving at 4.00 m/s. The block has a mass of 1.20 kg. What is the work done on the block as it moves up the incline?

Given

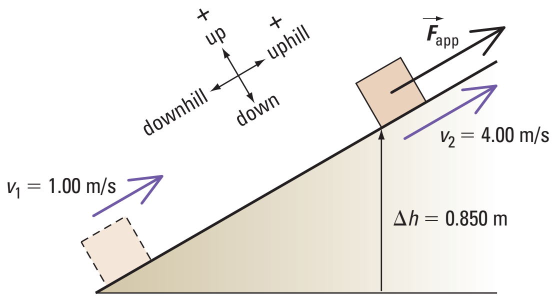
$m = 1.20$ kg

$v_1 = 1.00$ m/s

$v_2 = 4.00$ m/s

$\Delta h = 0.850$ m

$g = 9.81$ m/s^2



▲ **Figure 6.26**

Required

work done on the block as it moves up the incline (W)

Analysis and Solution

The work-energy theorem states that the work will be equal to the sum of the changes in the kinetic and potential energies. For the change in kinetic energy find the difference in the final and initial kinetic energy using the final and initial speeds. Change in gravitational potential energy can be found from the change in height (Figure 6.26).

$$
\begin{aligned}
W &= \Delta E_k + \Delta E_p \\
&= (E_{k_2} - E_{k_1}) + (mg\Delta h) \\
&= \left(\frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2\right) + (mg\Delta h) \\
&= \left[\frac{1}{2}(1.20 \text{ kg})(4.00 \text{ m/s})^2 - \frac{1}{2}(1.20 \text{ kg})(1.00 \text{ m/s})^2\right] \\
&\quad + (1.20 \text{ kg})(9.81 \text{ m/s}^2)(0.850 \text{ m}) \\
&= (9.60 \text{ J} - 0.60 \text{ J}) + (10.01 \text{ J}) \\
&= 19.0 \text{ J}
\end{aligned}
$$

Paraphrase and Verify

The work caused the block to gain a total of 19.0 J, the sum of 9.00 J of kinetic and 10.0 J of potential energy as it moved up the ramp.

Practice Problems

1. A mountain climber rappels down the face of a cliff that is 25.0 m high. When the climber, whose mass is 72.0 kg, reaches the bottom of the cliff he has a speed of 5.00 m/s. What is the work done on the climber by the rope?
2. A force of 150 N [up] acts on a 9.00-kg mass lifting it to a height of 5.00 m.
   (a) What is the work done on the mass by this force?
   (b) What is the change in gravitational potential energy?
   (c) What change in kinetic energy did the mass experience?
3. Draw a free-body diagram for the forces on the mass in question 2.
   (a) Calculate the net force acting on the mass.
   (b) Calculate the work done on the mass by the net force.
   (c) How does this relate to the answer to question 2(c)?

Answers

1. -1.68×10^4 J
2. (a) 750 J
   (b) 441 J
   (c) 309 J
3. (a) 61.7 N [up]
   (b) 309 J
   (c) $\Delta E_k = 309$ J

Calculations of Mechanical Energy

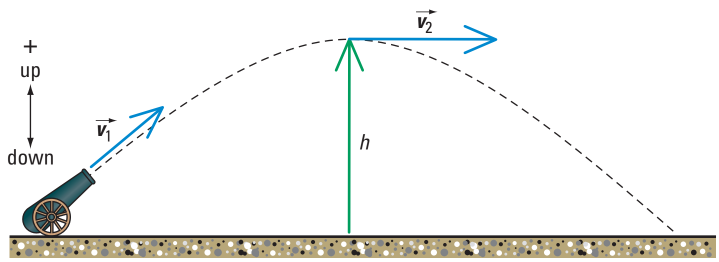
To calculate the mechanical energy of an object is simply to find the total of the kinetic energy and all forms of potential energy.

$$E_m = E_k + E_p$$

Because gravitational potential energy is defined relative to a reference point, mechanical energy will also depend on that reference point.

Example 6.8

A cannon ball is fired from Earth's surface. At the peak of its trajectory, it has a horizontal speed of 160 m/s and is 1.20×10^3 m above the ground. With reference to the ground, what is the mechanical energy of the cannon ball at the highest point on its trajectory, if the mass of the cannon ball is 5.20 kg?



▲ **Figure 6.27**

Given

$m = 5.20$ kg

$v = 160$ m/s

$h = 1.20 \times 10^3$ m

$g = 9.81$ m/s^2

Required

mechanical (total) energy of the cannon ball (E_m)

Analysis and Solution

At the top of its trajectory, the cannon ball has kinetic energy due to its horizontal motion, and gravitational potential energy because of its height above the ground.

$$\begin{aligned} E_m &= E_k + E_p \\ &= \frac{1}{2}mv^2 + mgh \\ &= \frac{1}{2}(5.20\text{ kg})\left(160\,\frac{\text{m}}{\text{s}}\right)^2 + (5.20\text{ kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(1.20 \times 10^3\text{ m}) \\ &= 6.656 \times 10^4\text{ J} + 6.121 \times 10^4\text{ J} \\ &= 1.28 \times 10^5\text{ J} \end{aligned}$$

Paraphrase and Verify

The total energy of the cannon ball at the top of the trajectory is 1.28×10^5 J. The gravitational potential energy is positive because the cannonball is higher than the reference point.

Practice Problems

1. A rocket is accelerating upward. When the rocket has reached an altitude of 5.00×10^3 m, it has reached a speed of 5.40×10^3 km/h. Relative to its launch site, what is its mechanical energy, if the mass of the rocket is 6.50×10^4 kg?
2. What is the speed of a 4.50-kg cannon ball if, at a height of 275 m above the ground, its mechanical energy relative to the ground is 6.27×10^4 J?
3. As a roller coaster trolley with a mass of 600 kg coasts down the first hill, it drops a vertical distance of 45.0 m from an initial height of 51.0 m above the ground. If, at the bottom of the hill, its speed is 30.0 m/s:
   (a) what is the trolley's mechanical energy relative to the top of the hill, and
   (b) what is the trolley's mechanical energy relative to the ground?

Answers

1. 7.63×10^{10} J
2. 150 m/s
3. (a) 5.13×10^3 J
   (b) 3.05×10^5 J

6.2 Check and Reflect

Knowledge

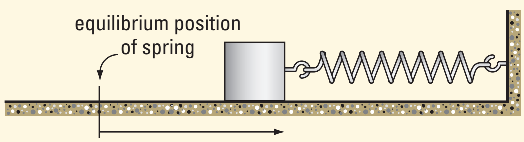
1. What are the forms of energy that make up mechanical energy?
2. Why does your choice of a frame of reference affect the calculated value of the mechanical energy?
3. What is the relationship between the net force and kinetic energy?
4. State the work-energy theorem.

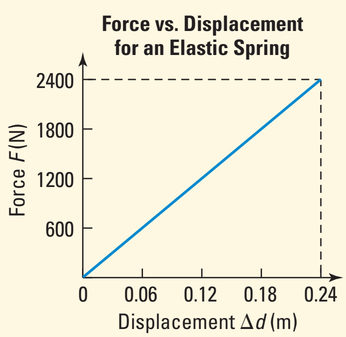
Applications

5. A net force of 5.75×10^3 N [180°] acts on a mass of 23.0 kg. If, while the force acts, the mass travels through a displacement of 360 m [210°], what work did the net force do on the object? Into what form of energy was this work transferred?
6. At a height of 75.0 m above the ground, a cannon ball is moving with a velocity of 240 m/s [up]. If the cannon ball has a mass of 12.0 kg, what is its total mechanical energy relative to the ground? What effect would there be on the answer, if the velocity of the cannon ball were downward instead of upward? Explain.
7. A mass of 8.50 kg is travelling 7.50 m/s [up]. It is acted on by a force of 340 N [up] over a displacement of 15.0 m [up].
   (a) What work does the applied force do on the object?
   (b) What is its gain in potential energy?
   (c) What is its change in kinetic energy?
   (d) What is its speed at the end of the 15.0-m displacement?

Extensions

8. The figure below shows the force versus displacement graph for an elastic spring as it is compressed a distance of 0.240 m from its equilibrium position by a force of magnitude 2.40×10^3 N. A 7.00-kg mass is placed at the end of the spring and released. As the spring expands, it accelerates the mass so that when the spring's compression is still 0.180 m from its equilibrium position, the mass has a speed of 6.00 m/s.
   (a) What is the mechanical energy in this system when the spring is compressed to 0.240 m?
   (b) What is the mechanical energy in the system when the spring is compressed to 0.180 m?
   (c) How much work has been done on the mass by this system as the spring expanded from a compression of 0.240 m to 0.180 m?
   (d) How does the work done on the mass by the spring compare to the kinetic energy of the mass?





e TEST

To check your understanding of the work-energy theorem and mechanical energy, follow the eTest links at www.pearsoned.ca/school/physicssource.

6.3 Mechanical Energy in Isolated and Non-isolated Systems

PHYSICS INSIGHT

An Open System can exchange both energy and matter with its surroundings. A Closed System can exchange energy but not matter with its surroundings. An Isolated System cannot exchange energy or matter with its surroundings.

Isolated Systems

Imagine two people are in an isolated (sealed) room. They may complete as many money transfers as they like but the total amount of money in the room before and after each transfer will be the same. We can say that the total amount of money in this system is conserved, in that it does not change during transactions.

▲ **Figure 6.28** In an isolated room, the total amount of money in the room, before and after a transaction, is constant.

▲ **Figure 6.29** In a non-isolated room, the amount of money in the room may change.

Now imagine that the room is not isolated. In this case, money may be taken out of (or put into) the room so that the total amount of money in the room is not necessarily constant. In this system, it cannot be said that money is conserved. It would be much more complex to keep track of the money transfers that occur in this non-isolated room compared with those occurring in the isolated room.

In physics, when the energy interactions of a group of objects need to be analyzed, we often assume that these objects are isolated from all other objects in the universe. Such a group is called an **isolated system**.

info **BIT**

In everyday terms, energy conservation means to use as little energy as possible to accomplish a task. In physics, energy conservation refers to systems, such as an ideal pendulum, in which the total amount of energy is constant.

isolated system: a group of objects assumed to be isolated from all other objects in the universe

Isolated Systems and Conservation of Energy

While objects within an isolated system are free to interact with each other, they cannot be subjected to unbalanced forces from outside that system. In terms of mechanical energy, that means that no force from outside the system may work to transfer energy to or from any object inside the system. The quantity of energy in the system must be constant. Even though friction may seem like an internal force, its effect is to allow energy to escape from a system as heat. Thus, an isolated system must also be frictionless. These ideas will be further explored later in the chapter.

Conservation of Mechanical Energy

Because the mechanical energy (the sum of potential and kinetic energies) for an isolated system must be a constant, it follows that if you calculate the mechanical energy (E_m) at any two randomly chosen times, the answers must be equal. Hence,

$$E_{m_2} = E_{m_1} \quad (1)$$

Within an isolated system, energy may be transferred from one object to another or transformed from one form to another, but it cannot be increased or decreased. This is the **law of conservation of energy**.

Relationship between kinetic and potential energy in an isolated system

The law of conservation of energy is one of the fundamental principles of science and is a powerful mathematical model for analysis and prediction of the behaviour of objects within systems. Viewed from a slightly different perspective, conservation of energy states that, in terms of mechanical energy, any gain in kinetic energy must be accompanied by an equal loss in potential energy.

$$\Delta E_k = -\Delta E_p \quad (2)$$

Statements (1) and (2) are equivalent. This can be verified as follows. If total energy remains constant regardless of time or position, then

$$E_{m_2} = E_{m_1}.$$

But mechanical energy is the sum of the kinetic and potential energies. Therefore,

$$E_{k_2} + E_{p_2} = E_{k_1} + E_{p_1}$$

Hence,

$$E_{k_2} - E_{k_1} = -E_{p_2} + E_{p_1}$$

$$\Delta E_k = -(E_{p_2} - E_{p_1})$$

Thus,

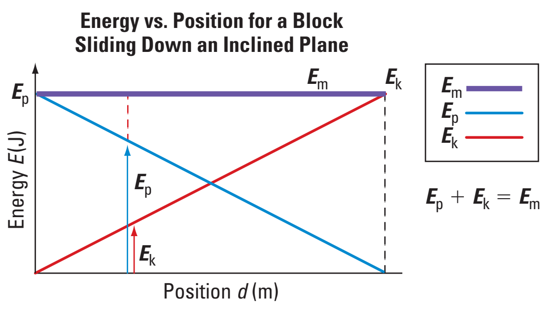
$$\Delta E_k = -\Delta E_p \text{ is true.}$$

eTECH

Consider the transformation of energy from potential energy to kinetic energy in a falling object, and in a ball bouncing on a trampoline. Follow the eTech links at www.pearsoned.ca/school/physicssource.

eSIM

Find out more about the mechanical energy, gravitational potential energy, and kinetic energy of a satellite-Earth system or a projectile-Earth system. Go to www.pearsoned.ca/school/physicssource.



▲ **Figure 6.30** In an isolated system the loss in gravitational potential energy is equal to the gain in kinetic energy.

As a block slides down a frictionless inclined plane, its gravitational potential energy will decrease and its kinetic energy will increase. Figure 6.30 shows the energy-position graph for this isolated system. As the block moves down the plane, the sum of the heights of the potential and kinetic energy curves (value of blue line plus value of red line) at any point is equal to the block's mechanical energy (E_m). The mechanical energy (shown by the purple line) is constant; therefore, energy is conserved. This graph is typical of an isolated system.

Example 6.9

A frictionless roller coaster car has a mass (m) of 8.00×10^2 kg. At one point on its journey, the car has a speed of 4.00 m/s and is 35.0 m above the ground. Later, its speed is measured to be 20.0 m/s.

(a) Calculate its total initial mechanical energy relative to the ground.

(b) What is its gravitational potential energy in the second instance?

Given

$m = 8.00 \times 10^2$ kg

$v_1 = 4.00$ m/s

$h_1 = 35.0$ m

$v_2 = 20.0$ m/s

$g = 9.81$ m/s^2

Required

(a) mechanical energy (E_{m_1})

(b) gravitational potential energy when the speed is 20.0 m/s (E_{p_2})

Analysis and Solution

A frictionless roller coaster can be treated as an isolated system.

(a) The mechanical energy at any point is the sum of its kinetic and potential energies.

$$E_{m_1} = E_{k_1} + E_{p_1}$$

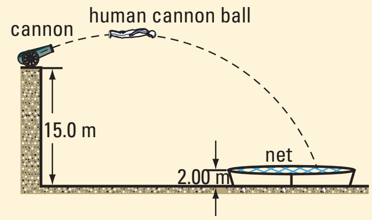
$$\begin{aligned} E_{m_1} &= \frac{1}{2}mv_1^2 + mgh_1 \\ &= \frac{1}{2}(8.00 \times 10^2\ \text{kg})\left(4.00\ \frac{\text{m}}{\text{s}}\right)^2 + (8.00 \times 10^2\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(35.0\ \text{m}) \\ &= 2.810 \times 10^5\ \text{J} \\ &= 2.81 \times 10^5\ \text{J} \end{aligned}$$

(b) The system is defined as isolated, meaning that energy is conserved. By the law of conservation of energy, the mechanical energy at any two points must be equal. The gravitational potential energy at the second point must be equal to the mechanical energy less the kinetic energy at the second point.

$$\begin{aligned} E_{m_2} &= E_{m_1} \\ E_{k_2} + E_{p_2} &= E_{m_1} \\ E_{p_2} &= E_{m_1} - E_{k_2} \\ &= E_{m_1} - \frac{1}{2}mv_2^2 \\ &= 2.810 \times 10^5\ \text{J} - \frac{1}{2}(8.00 \times 10^2\ \text{kg})\left(20.0\ \frac{\text{m}}{\text{s}}\right)^2 \\ &= 2.810 \times 10^5\ \text{J} - 1.600 \times 10^5\ \text{J} \\ &= 1.21 \times 10^5\ \text{J} \end{aligned}$$

Practice Problems

1. In an isolated system, a crate with an initial kinetic energy of 250 J and gravitational potential energy of 960 J is sliding down a frictionless ramp. If the crate loses 650 J of gravitational potential energy, what will be its final kinetic energy?
2. A mass of 55.0 kg is 225 m above the ground with a velocity of 36.0 m/s [down]. Use conservation of energy to calculate its velocity when it reaches a height of 115 m above the ground. Ignore the effects of air resistance.
3. A human “cannon ball” in the circus is shot at a speed of 21.0 m/s at an angle of 20° above the horizontal from a platform that is 15.0 m above the ground. See Figure 6.31.
   (a) If the acrobat has a mass of 56.0 kg, what is his gravitational potential energy relative to the ground when he is at the highest point of his flight? Ignore the effects of air resistance.
   (b) If the net in which he lands is 2.00 m above the ground, how fast is he travelling when he hits it?



▲ **Figure 6.31**

Answers

1. 900 J
2. 58.8 m/s [down]
3. (a) 9.69×10^3 J
   (b) 26.4 m/s

eSIM

Learn about the relationships among the mechanical, kinetic, and gravitational potential energies of a pendulum. Go to www.pearsoned.ca/school/physicssource.

Paraphrase and Verify

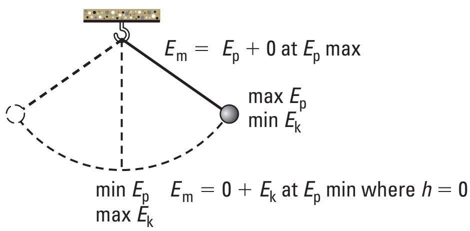
(a) The total initial mechanical energy relative to the ground is 2.81×10^5 J.

(b) The gravitational potential energy at a speed of 20.0 m/s is 1.21×10^5 J. The kinetic energy increased from 6.40×10^3 J to 1.60×10^5 J, while the gravitational potential energy decreased from 2.74×10^5 J to 1.21×10^5 J. As kinetic energy increases, potential energy decreases. When the speed is 20.0 m/s, the car must be below its starting point.

MINDS ON Energy and Earth's Orbit

At its closest point to the Sun (perihelion), around January 4th, Earth is about 147 million kilometres from the Sun. At its farthest point from the Sun (aphelion), around July 5th, Earth is about 152 million kilometres from the Sun.

- In terms of the conservation of energy, what conclusions can be made about Earth's speed as it moves around the Sun?
- What assumptions must you make to support your conclusions?



▲ **Figure 6.32** As a pendulum swings, gravity acts to convert energy back and forth between gravitational potential energy and kinetic energy.

A Simple Pendulum

A simple pendulum is an excellent approximation of an isolated system. During its downswing, Earth's gravity does work on the pendulum to transfer gravitational potential energy into kinetic energy. On the upswing, gravity transfers kinetic energy back into gravitational potential energy. The mechanical energy of the pendulum is constant (Figure 6.32).

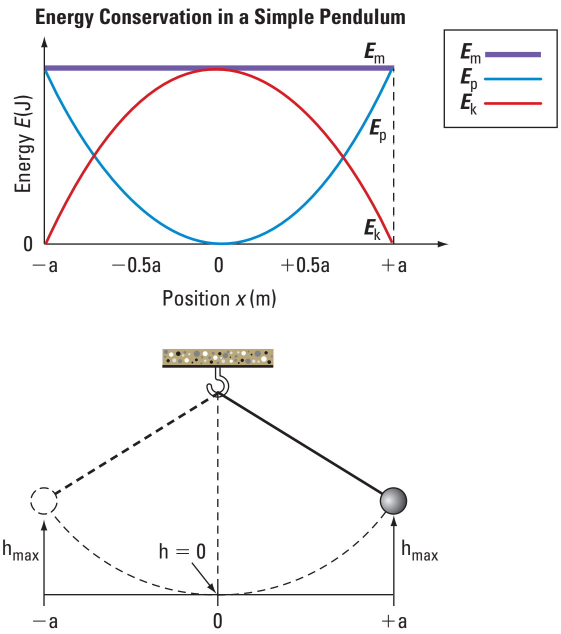
Newton's third law of motion states that for every action force there is an equal but opposite reaction force. This means that as Earth's gravity acts on the pendulum converting gravitational potential energy into kinetic energy, the pendulum must also act to convert gravitational potential energy to kinetic energy for Earth; Earth must be part of the isolated system that contains the pendulum. Earth's mass compared to that of the pendulum is enormous, so its reaction to the pendulum is immeasurably small. This explains why we can ignore the effects of the pendulum on Earth and analyze the pendulum as if it were an isolated system.

Treating the pendulum as an isolated system greatly simplifies the calculations of the system's mechanical energy. It means that the force of gravity works on the pendulum without changing the energy in the system. In fact, while work done by the force of gravity may transfer energy from one form to another it never causes a change in mechanical energy (Figure 6.33). Forces that act within systems but do not change their mechanical energy are defined as **conservative forces**. This type of force will be discussed in more detail later in this chapter.

PHYSICS INSIGHT

If $h = 0$ had been defined to occur at the lowest point of the pendulum's swing, the gravitational potential energy at the lowest point would be zero and the mechanical energy would be equal to the kinetic energy. Then, at the highest point on the swing, where movement stops and kinetic energy is zero, the mechanical energy would be equal to the potential energy.

Because the pendulum acts as an isolated system, energy is conserved. To calculate the mechanical energy of a pendulum it is necessary to know its mass, its height above the reference point, and its speed. If all of those values are known at any one point on its swing, then the mechanical energy of the pendulum is known at all points on its swing. Once the mechanical energy is known, it can be used to predict the pendulum's motion at any instant along its path, and to correlate kinetic and potential energy with the amplitude of the swing.



▶ **Figure 6.33** The force of gravity acts to change the gravitational potential energy and kinetic energy of the pendulum so that the mechanical energy remains constant.

Example 6.10

An ideal pendulum, as shown in Figure 6.32, is suspended by a string that is 2.00 m long. It is pulled sideways and released. At the highest point of its swing the pendulum bob is 25.0 cm above the floor. At the lowest point of its swing the pendulum bob is 5.00 cm above the floor. The mass of the pendulum bob is 250 g.

(a) What is the mechanical energy of the pendulum, relative to the floor, when the bob is at its highest point?
(b) What is the mechanical energy of the pendulum, relative to the floor, when the bob is at its lowest point?
(c) What is the kinetic energy of the bob when it is at its lowest point?
(d) What is the speed of the pendulum bob when the bob is at its lowest point?

Given

$m = 250 \text{ g} = 0.250 \text{ kg}$

$g = 9.81 \text{ m/s}^2$

$h_1 = 25.0 \text{ cm} = 0.250 \text{ m}$

$h_2 = 5.00 \text{ cm} = 0.0500 \text{ m}$

$v_1 = 0$

Required

(a) sum of gravitational potential and kinetic energies of the pendulum at the highest point (E_{m_1})
(b) mechanical energy of the pendulum at the lowest point (E_{m_2})
(c) kinetic energy of the bob at the lowest point (E_{k_2})
(d) speed of the bob at the lowest point (v_2)

Practice Problems

1. When the pendulum bob in Example 6.10 is 15.0 cm above the floor, calculate:
   (a) its mechanical energy relative to the floor
   (b) its kinetic energy
   (c) its speed
2. A model rocket has a mass of 3.00 kg. It is fired so that when it is 220 m above the ground it is travelling vertically upward at 165 m/s. At that point its fuel runs out so that the rest of its flight is without power. Assume that the effect of air friction is negligible and that all potential energies are measured from the ground.
   (a) What is the mechanical energy of the rocket, relative to the ground, when it is 220 m above the ground?
   (b) When it reaches the highest point on its trajectory, what will its gravitational potential energy be?
   (c) How far above the ground is the rocket at its highest point?
   (d) When it hits the ground, what is its speed?

3. A roller coaster trolley and its passengers have a mass of 840 kg. The trolley comes over the top of the first hill with a speed of 0.200 m/s. The hill is 85.0 m above the ground. The trolley goes down the first hill and up to the crest of the second hill 64.0 m above the ground. Ignore the effect of frictional forces. What is the kinetic energy of the trolley at the top of the second hill?
4. A pole-vaulter with a mass of 56.0 kg tries to convert the kinetic energy of her approach into height.
   (a) What is the maximum height she can expect to attain if her approach speed is 8.00 m/s? Assume that the centre of mass of the vaulter is initially 0.850 m above the ground.
   (b) Describe the energy changes that occur from the time the vaulter starts to run until she reaches the highest point of her jump.

Answers

1. (a) 0.613 J
   (b) 0.245 J
   (c) 1.40 m/s
2. (a) 4.73×10^4 J
   (b) 4.73×10^4 J
   (c) 1.61×10^3 m
   (d) 178 m/s
3. 1.73×10^5 J
4. (a) 4.11 m

Analysis and Solution

(a) At its highest point, the speed of the pendulum is zero. Thus, the mechanical energy at that point is equal to its gravitational potential energy.

$$E_{m_1} = E_{p_1} + E_{k_1}$$

$$\begin{aligned}
E_{m_1} &= mgh_1 + \frac{1}{2}mv_1^2 \\
&= (0.250\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.250\text{ m}) + \frac{1}{2}(0.250\text{ kg})(0)^2 \\
&= 0.6131\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\
&= 0.613\text{ J}
\end{aligned}$$

(b) In an isolated system, the mechanical energy is constant. Thus, by the law of conservation of energy the mechanical energy at its lowest point is equal to the mechanical energy at its highest point.

$$E_{m_2} = E_{m_1}$$

$$\therefore E_{m_2} = 0.613\text{ J}$$

(c) The kinetic energy at the lowest point is the difference between the mechanical and gravitational potential energies at that point.

$$\begin{aligned}
E_{m_2} &= E_{p_2} + E_{k_2} \\
E_{k_2} &= E_{m_2} - E_{p_2} \\
&= E_{m_2} - (mgh_2) \\
&= 0.6131\text{ J} - (0.250\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.0500\text{ m}) \\
&= 0.6131\text{ J} - (0.1226\text{ J}) \\
&= 0.491\text{ J}
\end{aligned}$$

(d) The speed at the lowest point can be found from kinetic energy.

$$\begin{aligned}
E_{k_2} &= \frac{1}{2}mv_2^2 \\
v_2 &= \sqrt{\frac{2E_{k_2}}{m}} = \sqrt{\frac{2(0.491\text{ J})}{0.250\text{ kg}}} \\
&= 1.98\ \frac{\text{m}}{\text{s}}
\end{aligned}$$

Paraphrase and Verify

(a) At the highest point, the total energy of the pendulum is 0.613 J.
(b) As the bob swings lower, gravitational potential energy is lost and kinetic energy is gained. The total energy remains 0.613 J.
(c) At the lowest point, the kinetic energy is 0.491 J, the difference between its total energy and its gravitational potential energy.
(d) At the lowest point of its swing, the bob has a speed of 1.98 m/s.

6-2 Inquiry Lab

Conservation of Mechanical Energy

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Question

Is energy conserved during the motion of a pendulum?

Hypothesis

State a hypothesis concerning the energy status of a pendulum. Remember to write this in the form of an if/then statement.

Variables

The variables in this lab are the values used to calculate the gravitational potential energy (mass, gravitational acceleration, and height) and the kinetic energy (mass and speed) at various points on the swing of the pendulum. Consider and identify which are controlled variable(s), which manipulated variable(s), and which responding variables.

Materials and Equipment

string (at least 2.0 m long)
pendulum bob (a 1-kg mass or greater)
metre-stick
ticker tape timer
masking tape
stopwatch (for timer calibration)

Procedure

For some interval timers, the period of the timer varies with the operating voltage. If your timer is of that type, begin by calibrating the timer. This is done by pulling a strip of ticker tape through the timer for a measured time, as set out below.

Calibrate the timer:

1. Start the tape moving steadily through the timer, then connect the timer to the power supply for an exact measure of time (3 to 5 s works well). Be sure the tape does not stop moving while the timer is running.
2. Count the number of intervals between the dots *(N)* and divide that number into the measured time (t) to determine the time lapse per interval (Δt). Do at least one more calibration trial (without changing the voltage) to check if the time per interval (Δt) remains constant.

(NOTE: The voltage to the timer must not vary from trial to trial. If you are using a variable-voltage power supply, adjust the voltage so that the timer runs smoothly, and leave the power supply untouched for the remainder of the experiment. Stop and start the timer by disconnecting and reconnecting the lead attached to the black post of the power supply rather than turning the power supply off and on.)

3. Record your results in a table of data (Table 6.3).

▼ Table 6.3 Calibration Data

Test Number	Total Time t (s)	Number of intervals N	Time/Interval t (s)
1			
2			

Set up the apparatus:

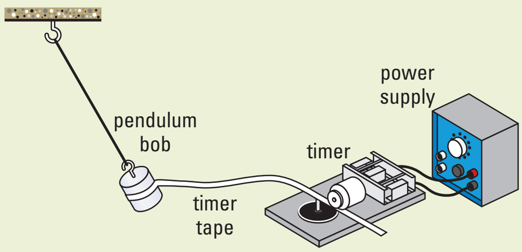
4. Suspend the pendulum from a suitable solid point and allow the pendulum bob to come to rest at its lowest point. Place a marker (e.g., a piece of masking tape) on the floor below the centre of mass of the bob to indicate the pendulum's rest position. Measure and record the length (l) of the pendulum, and mass (m) of the pendulum bob.

(NOTE: The length of a pendulum is measured from the point at which it pivots to the centre of mass of the pendulum bob. If the shape of the bob is a symmetrical solid, such as a sphere or a cylinder, the centre of mass is at its geometric centre.)

5. Pull the pendulum sideways so that its horizontal displacement (x) is about one-half its length. Place a marker, such as an iron stand, at this point, which will be x_{max} for the experiment. Ensure the path of the pendulum is clear, then release it to check the path of its swing. One team member should be positioned to catch the pendulum so that it does not swing back.

CAUTION: Make sure that the path of the pendulum is clear before you allow it to begin its swing.

6 Position the ticker tape timer at a distance approximately equal to the length of the pendulum from the pendulum's rest position. Locate the timer so that its height is just above the lowest point of the pendulum bob's path. Align the timer so that the tape does not bind as the bob pulls it through the timer. See Figure 6.34. Anchor the timer firmly so that it does not shift during trials.



▲ **Figure 6.34**

7 With ticker tape attached to the pendulum but without the timer running, do a trial run of the system. Attach the tape to the pendulum bob at its centre of mass, so that the pull of the tape does not cause the bob to twist. Use a length of ticker tape that will reach from the timer to a point slightly beyond the bob's rest position. Move the pendulum bob sideways to its starting point. Hold the bob in place while you pull the tape tight, then gently allow the tape to take up the weight of the bob. Be sure that the tape is not twisted so that it does not rip as it passes through the timer. Release the tape and allow the pendulum to pull it through the timer.

Collect data:

8 Once the timer is positioned so that the tape moves smoothly through it, you are ready to do a trial with the timer running. First, with the bob at its rest position, have one team member hold the bob stationary and pull the tape through the timer until it is just taut. Place a mark on the tape at the location where the timer records its dots. This mark on the tape records the position of the bob when its horizontal displacement is equal to zero ($x = 0$).

9 Move the pendulum bob sideways to the starting point of its swing (x_{max}). Again hold the bob steady while a team member pulls the tape tight. Gently allow the tape to take up the weight of the pendulum bob. Start the timer, then release the pendulum.

10 Lay the tape out on a table and place a line across the tape through the first dot the timer put on the tape. At that position, the speed is zero ($v = 0$) and the position is the maximum displacement (x_{max}). Measure the length of the tape from $x = 0$ to x_{max}.

11 Locate the two dots that define the interval containing the mark that indicates the position of the bob at its rest position ($x = 0$). Label this interval as interval 1. Measure the length (Δx_1) of this interval (the space between the two dots on either side of the mark) and record it in a data table (Table 6.4). Calculate the speed v of the pendulum for interval 1 by dividing Δx_1 by the interval time Δt.

12 Along the length of the tape, between $x = 0$ and x_{max}, choose at least four more time intervals and draw a line across the tape at the midpoint of each chosen interval. Starting from interval 1, number the selected intervals as 2, 3, etc. For each of the chosen intervals, measure (a) the length of the interval (Δx), and (b) the distance (x) to the midpoint of the interval from the line indicating $x = 0$. Record your measurements in a data table (Table 6.4).

Analysis

1. Use a table similar to Table 6.4 to organize your data.

2. Calculate the height (h) of the pendulum above its rest position by using the relationship $h = l - \sqrt{l^2 - x^2}$. (See the diagram in Figure 6.35.)

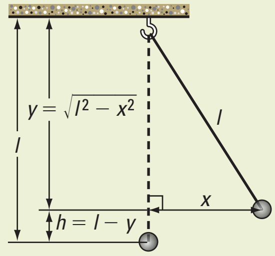
3. Calculate the values for the gravitational potential (E_p), the kinetic (E_k) and the mechanical (E_m) energies for each of the intervals you marked on your tape.

4. On the same set of axes, plot graphs for E_p and E_k against the horizontal displacement (x) of the pendulum. Describe the relationship between E_k and the position of the pendulum that is indicated by the graph. Does E_k change uniformly as the pendulum swings? Does E_p change uniformly as the pendulum swings? What relationship does the graph suggest exists between E_p and E_k for the pendulum?

5. On the same set of axes, plot a graph of the total mechanical energy (E_m) of the system against horizontal position (x). What does the graph suggest is the nature of the total mechanical energy for the pendulum? Suggest a reason for this relationship.

6. Within experimental error, can the mechanical energy of the system be considered constant? If the mechanical energy is assumed constant, what value would you choose to be the most representative of this energy? Explain why. For each of the intervals that you chose for analysis, what is the percent error in the mechanical energy at that interval? Does your analysis indicate a systemic error change for the pendulum as it swings? What would be the cause of this error?

7. How is your hypothesis affected by your data? Explain.



◀ **Figure 6.35**

▼ **Table 6.4** Pendulum Data

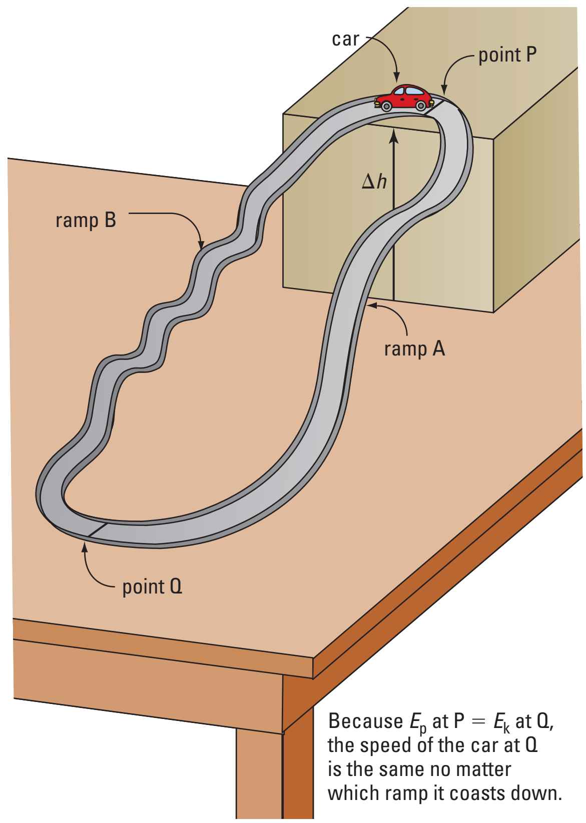
Interval Number	Horizontal Displacement x (m)	Height h (m)	E_p (J)	Interval Length Δx (cm)	Interval Speed v (m/s)	E_k (J)	E_m (J) ($E_p + E_k$)
1	0	0	0				
2							

Conservative and Non-conservative Forces

To understand the law of conservation of energy you must understand that some forces, such as gravity and elastic forces, act within systems without affecting the mechanical energy of the system. When such forces operate, energy is conserved. These are called **conservative forces.** Other forces, such as friction, and forces applied from outside a system, cause the energy of the system to change so that energy is not conserved. These are known as **non-conservative forces**.

Figure 6.36 shows a system of two ramps joining point P to point Q. The drop Δh is the same for the two ramps, but ramp A is shorter than ramp B, because of the hills in ramp B. If a frictionless car is released from P and moves down one ramp to Q, the amount of kinetic energy the car gains in moving from P to Q does not depend on which ramp (A or B) the car coasts down. If energy is conserved, the kinetic energy at point Q is equal to the potential energy at point P, so the speed of the car at point Q will be the same whether it comes down ramp A or ramp B.

Since a conservative force does not affect the mechanical energy of a system, the work done by a conservative force to move an object from one point to another within the system is independent of the path the object follows.



▲ **Figure 6.36** If a conservative force acts on an object, then the work it does is independent of the path the object follows between two points.

PHYSICS INSIGHT

No matter how closely conditions approximate an isolated system, the force of friction is never truly zero. Friction is continually converting kinetic energy to thermal energy. This thermal energy, which cannot be converted back into mechanical energy, radiates out of the system as heat.

non-isolated system: a system in which there is an energy exchange with the surroundings

Friction Is a Non-conservative Force

In the absence of friction, the car in Figure 6.36 would return to point P with no change in its mechanical energy. However, if there is friction, any motion of the car will be subject to it. When you analyze the work done by friction, you can see that path length does affect the work done on the car. The term Δd, the distance through which the force of friction acts, is not the displacement, but is always the actual distance the object travels.

W_f is the work done by the force of friction, F_f, on the system. Therefore:

$$W_f = F_f \Delta d$$

but $\Delta d_B > \Delta d_A$,

so $W_{f_B} > W_{f_A}$ and the car on Ramp B would lose more energy.

Since the potential energy at the bottom of the ramp is the same regardless of the route, the loss in mechanical energy must be a loss in kinetic energy. Therefore, friction is not a conservative force. Because thermal energy is being radiated out of the system, the system is, by definition, a **non-isolated system**. The amount of work done by friction will cause the mechanical energy of the system to change so that

$$\Delta E_m = W_f$$

Therefore,

$$E_{m_2} - E_{m_1} = W_f \text{ or}$$

$$E_{m_2} = E_{m_1} + W_f$$

The direction of the force of friction is always exactly opposite to the direction of the motion; therefore, the calculated value of W_f is always negative. Friction always reduces the mechanical energy of a system.

Concept Check

What assumptions must be made if you wish to use the law of conservation of energy to solve a problem in physics?

MINDS ON

That's the Way the Ball Bounces

With each successive bounce, the height attained by a bouncing ball becomes less. Assuming the elastic forces that cause the ball to bounce are conservative in nature, and no outside forces act on the ball, you might expect it to behave as an isolated system. If so, the energy of the ball should be conserved. Use the concept of systems and conservation of energy to explain why the height decreases with each bounce.

Energy Changes in Non-isolated Systems

Not all external forces remove mechanical energy from a system. Motors, in general, are used to add mechanical energy to a system. A ski-lift motor, for example, increases the gravitational potential energy of the skiers. More generally, if several external forces (A, B, C, . . .), as well as friction, act on a system, then the total work done by all of these forces produces the change in mechanical energy.

$$\begin{aligned} E_{m_2} &= E_{m_1} + W \\ &= E_{m_1} + (W_A + W_B + W_C + \ldots + W_f) \end{aligned}$$

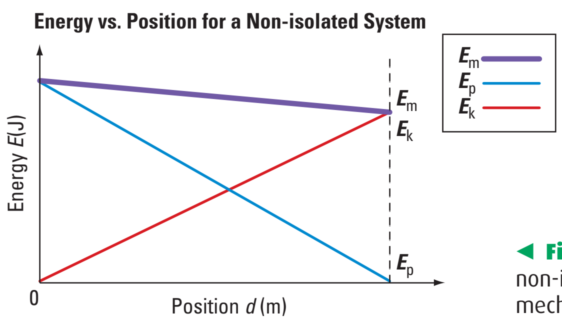
This is simply another version of the work–energy theorem.

PHYSICS INSIGHT

In a non-isolated system, W_f has a negative value and decreases the total mechanical energy. The effect of friction is opposite to forces that are adding mechanical energy.

Comparison of Energy-Position Graphs for Isolated and Non-isolated Systems

Figure 6.37 shows the energy-position graphs for a block sliding down an inclined plane in a non-isolated system. In an energy-position graph, the mechanical energy E_m is the sum of the potential energy E_p and kinetic energy E_k. So, the height of the mechanical energy line above the axis is the sum of the heights of the potential and kinetic energy lines. The purple line is the sum of the values of the red line and the blue line.



◀ **Figure 6.37** Friction acts on a non-isolated system to remove mechanical energy.

By rearranging the equation for the definition of work,

$$\Delta E = F\Delta d$$

$$\therefore F = \frac{\Delta E}{\Delta d}$$

it can be easily seen that force is equal to the slope of an energy-position graph. The units of the slope are $\frac{\text{N}\cdot\cancel{\text{m}}}{\cancel{\text{m}}}$, or units of force. In particular:

- The component of the force of gravity parallel to the motion can be determined by calculating the slope of the gravitational potential energy-position graph.
- The net force can be determined by calculating the slope of the kinetic energy-position graph.

e **MATH**

To determine the forces along an incline in an isolated system by using an energy-position graph, visit www.pearsoned.ca/physicssource.

For example, in the isolated system in Figure 6.30 on page 312, the slope of the potential energy curve gives the component of gravity parallel to the inclined plane. The slope of the kinetic energy curve gives the net force. The slope of the mechanical energy curve is zero indicating that no outside forces act on this system.

In the non-isolated system shown in Figure 6.37, E_m is not constant so friction is present. The slope of the total energy curve gives the force of friction. As an example of a non-isolated system, imagine a cart accelerating down an inclined plane. The force of friction removes energy from the system, but it is not sufficient to stop the cart from speeding up. The magnitude of the change in kinetic energy is less than the magnitude of the change in gravitational potential energy. In this case, the mechanical energy decreases by the amount of energy that friction removes from the system. The graph would be similar to Figure 6.37.

Project **LINK**

How will the design of your persuader apparatus allow for the energy changes in a system during a collision?

6-3 Design a Lab

The Energy Involved in a Collision

The Question

What happens to the energy of the system when two carts collide?

Design and Conduct Your Investigation

Design an experiment to investigate the energy of a system in which two carts collide. In one case, compare the energy before and after the collision for simulated **"elastic" collisions** in which the carts interact via a spring bumper. In a second case, compare the energy before and after a collision when the carts stick together in what is called an **"inelastic" collision**. You will need to develop a list of materials and a detailed procedure. Use the work-energy theorem to explain your results and form conclusions.

e **LAB**

If probeware is available, perform *6-3 Design a Lab* using a motion sensor. For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Concept Check

A block slides down an inclined plane, radiating energy out of the system as heat due to friction. Yet when you measure the mechanical energy at the bottom of the ramp, the total energy in the system is unchanged. Explain how this might occur.

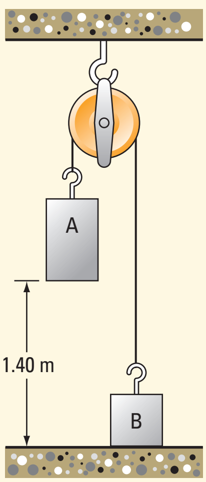
6.3 Check and Reflect

Knowledge

1. What is meant by an isolated system?
2. If energy is conserved in a system, how can work be done in the system?
3. Describe the changes in the forms of energy as an acrobat bounces on a trampoline so that she goes higher after each bounce.
4. Can a system be classified as isolated if a non-conservative force acts on it? Explain.
5. A golfer drives a golf ball from the top of a cliff. The ball's initial velocity is at an angle above the horizontal. If there were no air friction, describe the energy transformations from the time the golfer starts her swing until the golf ball lands on the ground at a distance from the bottom of the cliff. Include the energy transformation at the point of impact.
6. The pendulum of a clock is given a tiny push at the beginning of each swing. Why?

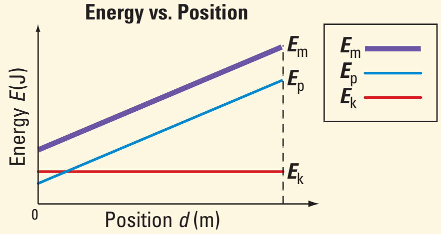
Applications

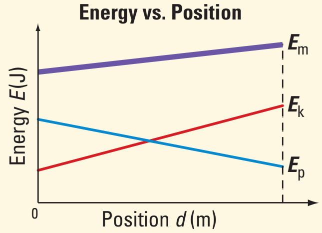
7. Two masses are suspended by a light string over a frictionless pulley. Mass A is 2.40 kg, mass B is 1.50 kg. Can this be considered an isolated system? Explain. On release, mass A falls to the tabletop, 1.40 m below. What is the kinetic energy of this system the instant before mass A hits the tabletop?
8. Draw graphs showing the gravitational potential, kinetic, and mechanical energy of the system in question 7 against the change in position of mass A if there is (a) no friction, (b) a force of friction, but mass A still accelerates.



▲ Diagram for questions 7 and 8.

9. The figure below shows the energy-position graphs for two different systems. For each graph, describe what is happening to the object(s) in the system in terms of their energies. Describe for each the nature of the forces acting on the object(s).



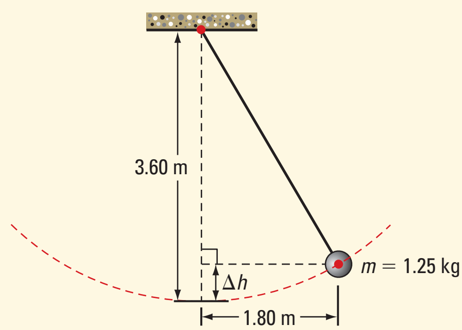


Extensions

10. A 3.60-m-long pendulum with a 1.25-kg bob is pulled sideways until it is displaced 1.80 m horizontally from its rest position.
    (a) Use the Pythagorean theorem to calculate the bob's gain in height.

    If the bob is released, calculate the speed of the bob when it

    (b) passes through its rest position
    (c) is 0.250 m above its rest position



eTEST

To check your understanding of mechanical energy in isolated and non-isolated systems, follow the eTest links at www.pearsoned.ca/school/physicssource.

6.4 Work and Power

Power, Work, and Time

info **BIT**

In metric terms, 1.00 hp = 746 W or about 0.75 kW.

Toward the end of the 18th century, horses were the main source of energy used to drive the pumps that removed water from mines. Thus, when James Watt (1736–1819) wanted to know how his newly improved steam engine compared with existing methods of pumping water out of mines, he compared its effectiveness to that of horses. Today, even though it is a rather awkward unit, we still use his concept of **horsepower (hp)** to identify the power output of motors, especially in the automotive industry.

▲ **Figure 6.38** This drag racer's 7000 hp engine burns a special fuel mixture called nitromethane. Each of its eight cylinders generates approximately three times the power of a normal car engine. The distortion of the tires, as seen above, is evidence of the magnitude of the forces exerted during acceleration.

The high-performance race car in Figure 6.38 accelerates to speeds over 530 km/h in about 4.4 s. A family sedan with a 250-hp engine can accelerate to 100 km/h, from rest, in about 8.0 s. Aside from acceleration, what aspect of a car's performance is affected by the horsepower rating of its motor? On the highway, cars with 100-hp motors and cars with 300-hp motors both easily travel at the speed limit. What factors decide how much power is required to move a car along the highway?

power: the rate of doing work

In physics, **power** (P) is defined as the rate of doing work. Thus, the equation for power is

$$P = \frac{W}{\Delta t} \text{ or } P = \frac{\Delta E}{\Delta t}$$

The unit of power, the watt (W), is named in recognition of James Watt's contributions to physics. Using the equation for power we see that a power output of one watt results when one joule of work is done per second.

$$\begin{aligned} 1\ \text{W} &= \frac{1\ \text{J}}{1\ \text{s}} \\ &= 1\ \frac{\text{J}}{\text{s}} \end{aligned}$$

Efficiency

efficiency: ratio of the energy output to the energy input of any system

Efficiency may be defined in terms of energy or in terms of power. In both cases, the ratio of the output (useful work) to the input (energy expended) defines the efficiency of the system. Thus, efficiency can be calculated as either,

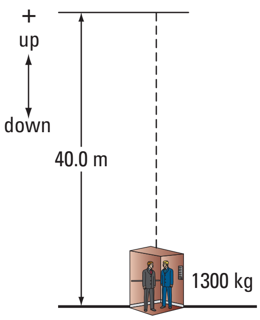
$$\begin{aligned} \text{Efficiency} &= \frac{\text{Energy output } (\Delta E_{\text{out}})}{\text{Energy input } (\Delta E_{\text{in}})}, \text{ or} \\ &= \frac{\text{Power output } (P_{\text{out}})}{\text{Power input } (P_{\text{in}})} \end{aligned}$$

Concept Check

In terms of kg, m, and s, what is the unit for power?

Example 6.11

An elevator and its occupants have a mass of 1300 kg. The elevator motor lifts the elevator to the 12th floor, a distance of 40.0 m, in 75.0 s. (a) What is the power output of the elevator? (b) What is the efficiency of the system if the motor must generate 9.40 kW of power to do the specified work?



▲ **Figure 6.39**

Given

$m = 1.300 \times 10^3$ kg

$g = 9.81$ m/s^2

$\Delta h = +40.0$ m

$\Delta t = 75.0$ s

$P_{\text{in}} = 9.40 \times 10^3$ W

Required

(a) power output of the elevator
(b) efficiency of the system

Analysis and Solution

(a) The work done by the elevator is equal to its gain in gravitational potential energy. The power output of the elevator is the change in potential energy divided by the time.

$$\begin{aligned} P &= \frac{\Delta E_{\text{p}}}{\Delta t} \\ &= \frac{mg\Delta h}{\Delta t} \\ &= \frac{(1.300 \times 10^3 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(40.0 \text{ m})}{75.0 \text{ s}} \\ &= 6.802 \times 10^3 \frac{\text{J}}{\text{s}} \\ &= 6.80 \times 10^3 \text{ W} \end{aligned}$$

(b) Efficiency is the ratio of the power output to the power input.

$$\begin{aligned} \textit{Efficiency} &= \frac{P_{\text{out}}}{P_{\text{in}}} \\ &= \frac{6.802 \times 10^3 \cancel{\text{W}}}{9.40 \times 10^3 \cancel{\text{W}}} \\ &= 0.724 \end{aligned}$$

Paraphrase and Verify

(a) The power output of the elevator is 6.80×10^3 W. The answer has the right order of magnitude for the given data. The power output is equivalent to about sixty-eight 100-W light bulbs.
(b) The efficiency of the system is 0.724 (72.4%).

Practice Problems

1. The engine of a crane lifts a mass of 1.50 t to a height of 65.0 m in 3.50 min. What is the power output of the crane? Convert the SI unit answer to hp.
2. If a motor is rated at 5.60 kW, how much work can it do in 20.0 min?
3. A tractor, pulling a plough, exerts a pulling force of 7.50×10^3 N over a distance of 3.20 km. If the tractor's power output is 25.0 kW, how long does it take to do the work?

Answers

1. 4.55 kW (6.11 hp)
2. 6.72×10^6 J
3. 960 s (16.0 min)

6-4 Inquiry Lab

Measuring the Power Output of a Motor

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

The Question

How much power can a small electric motor generate?

The Problem

The problem in the lab is to measure the power output of a motor by timing how long it takes for the motor to do a fixed amount of work.

Variables

The variables for measuring power are the work done against gravity (ΔE_p) and the time (Δt) it takes to do the work. Calculating ΔE_p requires mass (m), gravitational acceleration (g), and change in height (Δh).

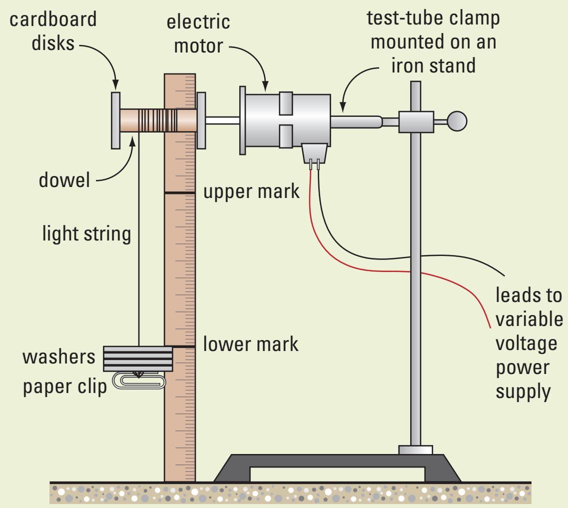
Materials

- small dc electric motor
- alligator clip leads
- iron stand
- 1-kg mass
- test-tube clamps
- low-voltage power supply
- dowel (about 3 cm long and 1 cm in diameter)
- thread (about 2.5 m long)
- tape
- paper clip
- washers
- scale (sensitive to at least 0.1 g)
- stopwatch
- metre-stick

Procedure

1. Use the balance to determine the mass of the paper clip. Record your measurement.
2. Place 10 washers on the balance scale and determine their mass. Calculate the average mass of the washers. Record your measurement.
3. Assemble the apparatus as shown in Figure 6.40. Set up a measuring scale behind the string holding the washers. The distance between the upper and lower timing marks on the scale may be adjusted if your apparatus permits, but should be 1.5 m or greater.

CAUTION: Close the test-tube clamp just tight enough to hold the motor in place. If you tighten it too much, it could warp the body of the motor.



▲ **Figure 6.40**

CAUTION: Check with your instructor to be sure the connections are correct before you plug in the power supply. If the motor is incorrectly connected, it could be damaged when the current is turned on.

4. Connect the power supply to the electric motor. Once your instructor has approved your connection, disconnect the lead connected to the red post of the power supply and turn on the power supply.
5. Place five washers on the string, as shown in Figure 6.40. Complete the circuit by holding the insulated alligator clip lead on the red post of the power supply and observe the speed with which the motor lifts the washers. Adjust the number of washers until the motor moves the load upward at a uniform speed. (If the speed is too great, it will be difficult to time the motion of the washers. If the speed is too slow, then the motor may not run smoothly.)

6. Pull the thread to unwind thread from the dowel until the washers rest on the floor. Start the motor. Measure the time the washers take to travel between the lower and upper timing marks.

7. Record your measurements in a table such as Table 6.5.

8. Vary the number of washers on the paper clip and repeat the trial. Do trials with at least three different masses.

9. Calculate the work that the motor did in lifting the washers the measured distance.

10. Calculate the power output for each trial.

Analysis

1. Does the power output of the motor vary with the force it is exerting?
2. Make a graph of the power output versus the mass being lifted.
3. For what mass does the motor produce the most power?
4. What is the advantage of lifting the weights over a long distance?
5. Suggest reasons why the motor might generate more power when different masses are used.
6. Does the motor feel warm after it has done some work? What does that tell you about this system?
7. Would it make sense to use a very large motor to lift very tiny masses? Explain.
8. In terms of car engines, what are the implications for engine size?

▼ Table 6.5 Power Output of a Motor

Trial Number	Number of Washers	Mass m (kg)	Time Δt (s)	Change in Potential Energy $\Delta E_p = mg\Delta h$ (J)	Power $P = \Delta E_p/\Delta t$ (W)

Power and Speed

When a motor, such as the electric motor in 6-4 Inquiry Lab, applies a constant force to move an object at a constant speed, the power output of the motor can be shown to be the product of the force and the speed. When the force is constant, the work done can be found by the equation

$$W = F\Delta d$$

Inserting the equation for work into the equation for power gives:

$$P = \frac{F\Delta d}{\Delta t}$$

$$= F\frac{\Delta d}{\Delta t}$$

But the expression $\Delta d/\Delta t$ is just average speed v_{ave}; therefore,

$$P = (F)(v_{ave})$$

e WEB

Why is it that if you double the speed of a car, the rate at which it consumes fuel more than doubles? Is lowering the speed limit the most effective way to conserve energy or would design changes (e.g., hybrid fuel systems or fuel cells) be more effective? With a group of classmates, investigate how best to improve the energy efficiency of automobiles. Use your library and the Internet. Present the results of your investigation in a report using presentation software such as PowerPoint ™. Begin your search at www.pearsoned.ca/school/physicssource.

Example 6.12

A car, of mass 2000 kg, is travelling up a hill at a constant speed of 90.0 km/h (25.0 m/s). The force of air resistance, which opposes this motion, is 450 N. The slope of the hill is 6.0° (Figure 6.41).

(a) Draw a free-body diagram to show the external forces acting on the car as it moves up the hill.

(b) Determine the forward force needed to maintain the car's speed.

(c) Assuming that all the power output of the car engine goes into maintaining the car's forward motion, calculate the power output of the engine.

Given

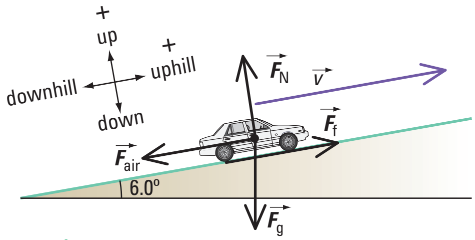
$m = 2.000 \times 10^3$ kg

$\vec{F}_{air} = 4.50 \times 10^2$ N [downhill]

$g = 9.81$ m/s^2

$v = 25.0$ m/s

$\theta = 6.0°$



▲ **Figure 6.41**

Required

(a) a free-body diagram for the car

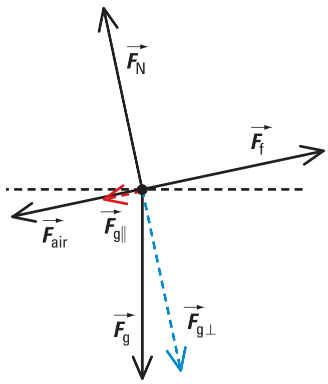
(b) forward force (F_f)

(c) power (p)

Analysis and Solution

(a) Figure 6.42(a) shows the free-body diagram.

(b) Since the car is not accelerating, the net force on the car must be zero, $F_{net} = 0$, both parallel and perpendicular to the incline of the hill. The forward force (F_f) must be equal to the sum of the magnitudes of the force of air resistance (F_{air}) and the component of the gravitational force that acts parallel to the incline ($F_{g\parallel}$).



▲ **Figure 6.42(a)**

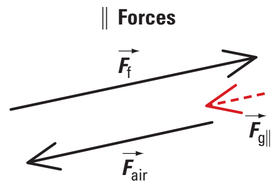
In the parallel direction

$$\vec{F}_{net\parallel} = \vec{F}_f + \vec{F}_{g\parallel} + \vec{F}_{air}$$

$$F_{net\parallel} = F_f + F_{g\parallel} + F_{air}$$

Now,

$$\begin{aligned} F_{g\parallel} &= -mg\sin\theta \\ &= -(2.000 \times 10^3 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(\sin 6.0°) \\ &= -2.051 \times 10^3 \text{ N} \end{aligned}$$



▲ **Figure 6.42(b)**

Therefore,

$$\begin{aligned} 0 &= F_f + (-2.051 \times 10^3 \text{ N}) + (-4.50 \times 10^2 \text{ N}) \\ F_f &= 2.051 \times 10^3 \text{ N} + 4.50 \times 10^2 \text{ N} \\ &= 2.50 \times 10^3 \text{ N} \end{aligned}$$

Practice Problems

1. What is the power output of an electric motor that lifts an elevator with a mass of 1500 kg at a speed of 0.750 m/s?
2. An engine's power is rated at 150 kW. Assume there is no loss of force due to air resistance. What is the greatest average speed at which this engine could lift a mass of 2.00 t?
3. A 1250-kg race car accelerates uniformly from rest to 30.0 m/s in 4.00 s on a horizontal surface with no friction. What must be the average power output of its motor?
4. Each car in a freight train experiences a drag force of 6.00×10^2 N due to air resistance.
   (a) If the engine of the train is to pull a train of 75 cars at a constant speed of 72.0 km/h, what power is required to move the cars?
   (b) If the engine operates at 15.0% efficiency, what must be the power generated by the engine to move these cars?

Answers:

1. 11.0 kW
2. 7.65 m/s
3. 141 kW
4. (a) 900 kW
   (b) 6.00×10^3 kW

(c) Calculate the power output of the engine using the forward force and the speed with which the car moves along the ramp.

$$P = F_f v_{ave}$$
$$= (2.50 \times 10^3 \text{ N})(25.0 \text{ m/s})$$
$$= 6.25 \times 10^4 \text{ W}$$
$$= 62.5 \text{ kW}$$

Paraphrase

(a) The free-body diagram shows the external forces acting on the car.
(b) If the car moves at a constant speed, then the forward force must be a force of 2.50×10^3 N.
(c) The power output of the car is 62.5 kW.

MINDS ON Power and Dance

At the moment of takeoff, Cossack dancers must generate considerable power to perform their spectacular leaps (Figure 6.43). Discuss techniques you could use to measure the power the dancers must generate to make such a jump. University Faculties of Kinesiology study this and other aspects of how humans move.

1. What factors involved in the jump will you need to determine?
2. What equipment might you require to measure those factors?
3. How would you measure the dancer's maximum power output compared with the power he can generate over a sustained period of time?

▲ **Figure 6.43**

e WEB

To learn more about power generated in human activities, follow the links at www.pearsoned.ca/school/physicssource.

THEN, NOW, AND FUTURE Fuel for the Future?

While the automobile in Figure 6.44 may look like a normal car, nothing could be further from the truth. When this vehicle travels along one of Vancouver's streets, its motor is barely audible. Perhaps even more surprising is the fact that the exhaust this car produces is pure water. While the motor that drives the car is actually an electric motor, it is the source of the electricity that is getting all the attention. The "battery" in this car is called a fuel cell.

At present, fuel cells are not an economically viable replacement for the internal combustion engine although successful trials of fuel-cell buses have been made in several cities around the world.

▲ **Figure 6.44** This car's exhaust is pure water.

The impact of fossil fuel (oil and coal)-burning systems on the environment has made the search for alternative energy sources much more attractive. This search is further enhanced by the realization that the supply of fossil fuels is finite. Even though Canada has, at present, an abundant supply of fossil fuels, it is still a world leader in fuel-cell research.

For further information, go to the Internet. Start your research at www.pearsoned.ca/school/physicssource.

Questions

1. What are the advantages and disadvantages of a fuel-cell-driven motor over an internal combustion engine?
2. Which of the advantages and disadvantages identified above can be further improved on or overcome by scientific and technological research? Explain.
3. What are the limitations of science and technology to finding answers to the problems associated with energy production and use?

6-5 Problem-Solving Lab

Power and Gears

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Recognize a Need

Modern bicycles have many gears to enable riders to make best use of their efforts. In the automotive industry, manufacturers use a device called a dynamometer (Prony brake) to measure the power output of the motors they install in their vehicles. A Prony brake for bicycles would be a useful thing.

The Problem

How does the gear used by a cyclist affect the power output at the drive wheel of the bicycle? In which gear do cyclists generate the greatest power?

Criteria for Success

A successful experiment will determine if there is a relationship between the power at the drive wheel of the bicycle and the gear in which the bicycle is being ridden.

Brainstorm Ideas

Investigate the design of a Prony brake, then brainstorm how that design might be adapted to measure the power output of a bicycle. Remember, for your results to be useful the design must allow a rider to "ride" the bicycle in a normal manner. If a computer and probeware are available, consider using probeware in your experimental design, to measure the speed of the Prony brake.

e WEB

Use the Internet to investigate Prony brake design. Begin your search at www.pearsoned.ca/school/physicssource.

Build a Prototype

Build a Prony brake that can measure the power output of a student riding a bicycle.

Test and Evaluate

Make measurements of the power output of a student riding a bicycle using various gear settings.

Communicate

Prepare a report of your research using a computer spreadsheet program to organize your data and to generate a graph of the power output versus the gear level. Print your graph, in colour if possible, as part of your report.

6.4 Check and Reflect

Knowledge

1. What is the relationship between the amount of work that is done and the power output of the machine that does the work?
2. A farmer says, "My tractor with its 60-hp engine easily pulls a plough while my car with a 280-hp engine cannot even budge it." How can you explain this fact?
3. What is the relationship between the speed of an object and the power required to move it?

Applications

4. You lift a 25.0-kg mass to your waist (0.800 m) in 1.20 s. What is your power output?
5. An airplane's engine exerts a thrust of 1.20×10^4 N to maintain a speed of 450 km/h. What power is the engine generating?
6. An electric motor has a power rating of 1.50 kW. If it operates at 75% efficiency, what work can it do in an hour?
7. A motor of a car must generate 9.50 kW to move the car at a constant speed of 25.0 m/s. What is the force of friction on the car?

Extension

8. A cannon fires a ball with a muzzle velocity of 240 m/s. The cannon ball has a mass of 3.60 kg. The barrel of the cannon is 1.20 m long and exerts a force of friction on the cannon ball of 650 N. What is the average power provided to fire the cannon ball?

e TEST

To check your understanding of power, work, and efficiency, follow the eTest links at www.pearsoned.ca/school/physicssource.

Key Terms

energy
work
gravitational potential energy
reference point
elastic potential energy
kinetic energy
mechanics
mechanical energy
work-energy theorem
isolated system
non-isolated system
conservation of energy
power
efficiency

Key Equations

$W = (F\cos\theta)\Delta d$

$\Delta E_p = mg\Delta h$

$E_p = mgh$

$E_p = \frac{1}{2}kx^2$

$E_k = \frac{1}{2}mv^2$

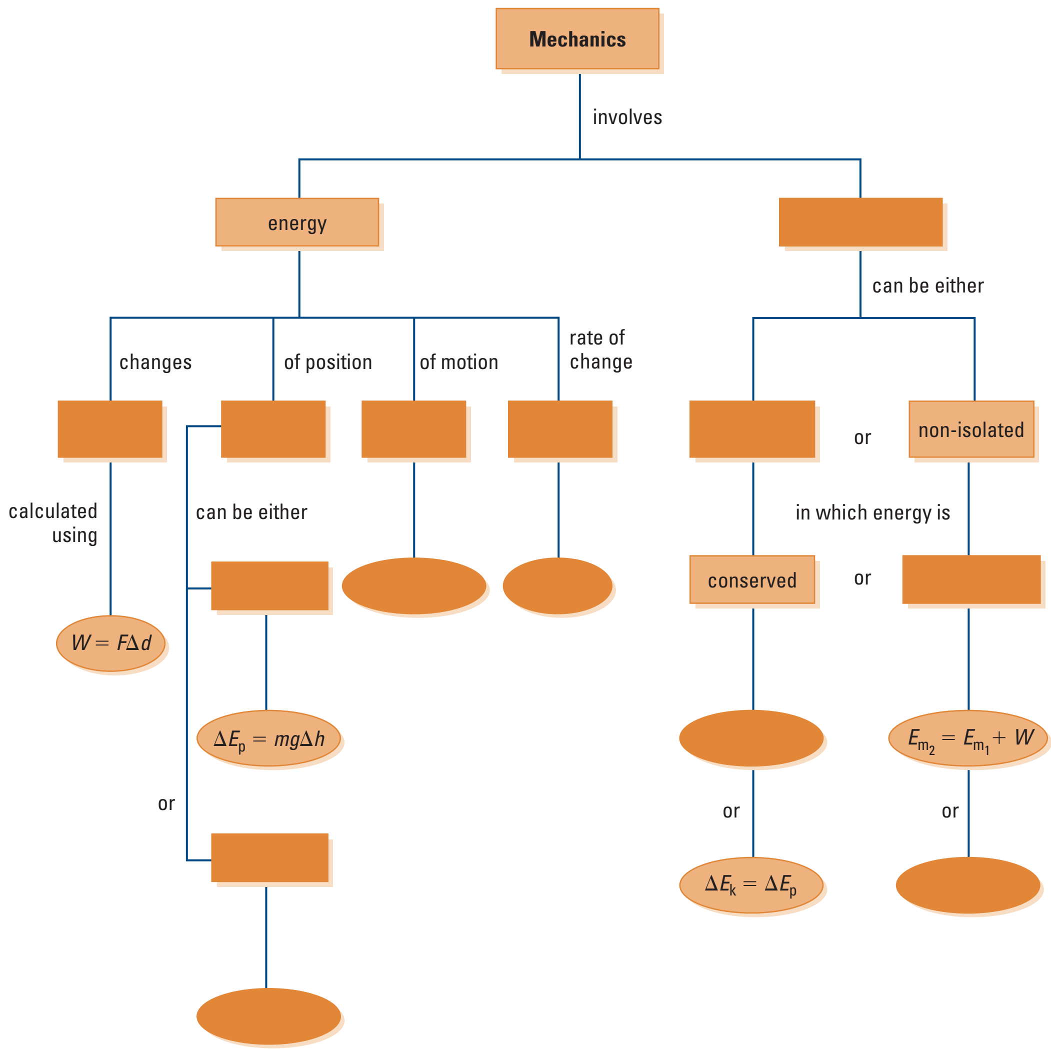
$W = \Delta E_k + \Delta E_p$

$E_m = E_k + E_p$

$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$

Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to produce a full summary of the chapter.



Knowledge

1. (6.1) (a) When a force acts on an object to do work, why do you need to know the angle between the direction of the force and the direction of the displacement?
   (b) If you know how the magnitude of a force changes while it acts over a displacement, how can you find the amount of work it does?
   (c) Describe the nature of the energy transfers for the work done on a bungee jumper from the time he leaps off the platform until his velocity is zero at the lowest point of his jump.
   (d) Two students calculate the gravitational potential energy of a mass resting on a shelf. One student calculates that it has 12.0 J of energy while the other calculates the gravitational potential energy to be 35.0 J. Is it possible that they are both right? Explain.
   (e) Two masses, A and B, are at rest on a horizontal frictionless surface. Mass A is twice as great as mass B. The same force acts on these masses over the same displacement. Which mass will have the greater (i) speed, and (ii) kinetic energy at the end of the displacement? Explain.
   (f) Many fitness facilities have treadmills as exercise machines. Is running on a treadmill work since the runner is not really moving?
   (g) Explain why it takes less force to push a cart up an inclined plane onto a platform than it does to lift the cart straight up from the floor. Assume you are able to move the cart by either method. Does it also take less work to lift it or to roll it up the plane?
   (h) An object sits on a tabletop. In terms of describing the object's gravitational potential energy, which reference point is better: the ground outside the room, the floor of the room, or the tabletop? Explain.

2. (6.2) (a) What is the effect of the work done by a net force?
   (b) If a force acts upward on an object, does all the work done by this force become potential energy?
   (c) What forms of energy are considered to be part of mechanical energy?
   (d) Can two people calculate the mechanical energy of an object and get two different correct answers? Explain.

3. (6.3) (a) Explain why the force of gravity but not the force of friction is called a conservative force.
   (b) An ideal spring is one where no energy is lost to internal friction. Is the force exerted by the spring considered a conservative force? Explain. Is a mass that is oscillating up and down on the end of an ideal spring a good approximation of an isolated system?
   (c) Since no system on Earth is truly an isolated system, why is it advantageous to assume that a system is isolated?
   (d) A truly isolated system does not exist on Earth. How does that affect the fundamental principle of conservation of energy?
   (e) If a system is not isolated, how can one calculate the change in mechanical energy in the system?
   (f) The mechanical energy of a system is measured at two different times and is the same each time. Is this an isolated system? Explain.

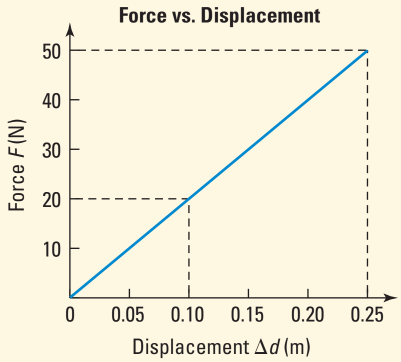
4. (6.4) (a) An elevator takes 2.50 min to travel from the ground floor to the 10^{th} floor of an apartment block. The tenants want the landlord to increase the speed of the elevator but the landlord argues that speeding up the elevator means that it will need to work harder and that would take more energy. Is he correct? Explain.
   (b) The transmission of an automobile allows the work done in the engine to be transmitted to the wheels. For a given power output by the engine, the wheels are not rotated as fast by a low gear as they are by a high gear. What advantage does having gears give the driver of a car?

Applications

5. What is the change in kinetic energy if a net force of 3.80×10^3 N [0°] acts on a mass while it undergoes a displacement of 95.0 m [335°]?

6. For gravitational potential energy, when the height doubles so does the potential energy. However, for elastic potential energy, if the stretch of the spring doubles, the energy does not.
   (a) How does the stored elastic potential energy change if the stretch doubles?
   (b) Explain in terms of force-position graphs for gravitational and elastic potential energies why this happens.

7. The figure below shows the graph of the force as a function of displacement for an elastic spring stretched horizontally 25.0 cm from its equilibrium position. A mass of 0.400 kg is attached to the spring and released. If the mass is sliding on a horizontal frictionless surface, what is the speed of the mass when the spring has contracted to (a) 10.0 cm from its equilibrium position, and (b) its equilibrium position?



▲ Graph for question 7

8. A bungee jumper with a mass of 65.0 kg leaps from a bridge. At the lowest point of the jump he is 30.0 m below the point from which the jump began. If, at equilibrium, the bungee cord is 15.0 m long, what is the elastic constant for the cord? **HINT:** Assume an isolated system. At the lowest point the bungee cord must convert all of the jumper's lost gravitational potential energy into elastic potential energy in the cord.

9. A motorcycle stuntman wants to jump over a line of city buses. He builds a takeoff ramp that has a slope of 20°, with its end 3.20 m above the ground. The combined mass of the motorcycle and rider is 185 kg. To clear the buses, the cyclist needs to be travelling 144 km/h when he leaves the end of the ramp. How high above the ground is the motorcycle at its highest point?

10. A skydiver reaches a maximum speed (or terminal velocity) of 36.0 m/s due to the force of air resistance. If the diver has a mass of 65.0 kg, what is the power output of the air resistance acting on him?

Extensions

11. Even when making short flights, jets climb to altitudes of about 10 000 m. Gaining altitude requires a great expenditure of fuel. Prepare a short research report to explain the advantages and disadvantages of travelling at these altitudes.

Consolidate Your Understanding

You have been hired to tutor a student on the topics in this chapter. Describe how you would answer the questions below. In each instance, include an example.

1. What is the difference between work and energy?
2. When an object moves up a hill, how does the length of the hill affect the increase in the gravitational potential energy?
3. If a cart, at rest, is allowed to coast from the top of a hill to the bottom, is the kinetic energy at the bottom of the hill always equal to the loss in gravitational potential energy?
4. A given force is to act on a block to accelerate it from rest as it slides up the length of an inclined plane. Using accurate spring balances and rulers, how could you gather data to enable you to calculate, with reasonable accuracy, the kinetic energy of the block when it reaches the top of the incline? You may not use timing devices.
5. What is the difference between an isolated and a non-isolated system in terms of work and energy?
6. What factor of a car's motion is most directly affected by the power of its engine?

Think About It

Review the answers you gave to the questions in Think About It on page 291. How would you change these answers?

e TEST

To check your understanding of energy, work, and power, follow the eTest links at www.pearsoned.ca/school/physicssource.

Building a Persuader Apparatus

Scenario

It is 1965. Seatbelts are oddities used only by airline passengers at takeoff and landings. The term "airbag" hasn't even been invented. Speed limits and traffic deaths are on the rise. Imagine that you are part of a team of engineers who design and build automobiles. Your company challenges you to design and build a model that can be used to convince its shareholders and the public that it is possible to build much safer automobiles. The automobile company has challenged your design team to produce safety features for its vehicles that will allow its passengers to survive crashes under severe conditions. Your team must determine how best to protect the passenger while, at the same time, keeping the size and mass of the car itself to reasonable proportions. Your presentation to the company will be used to persuade them of the benefits of your design. Finally, your report should persuade the public of the advantages of using the safety equipment you recommend for automobiles.

Planning

Your team should consist of three to five members. Your first task is to formulate your research question. This done, you will need to identify the assumptions about the nature of the collisions in which your vehicle may be involved. Since not all collisions may be head-on, your passenger (a fresh raw egg) should survive unscathed from a wide variety of crash scenarios. While all team members should be active participants in all aspects of the project, you should identify and draw on any special talents of your team. Create a team structure that assigns responsibilities such as team manager, data analyst, and record keeper. Begin by brainstorming possible design features and research strategies. Where might you find information on existing safety features? Which features are the most effective? How will you compare results for the various types of crashes? You may wish to draw on information from all topics in this course to improve the safety features of your vehicle. Create timelines for all phases of the project. Create a report that incorporates written, graphic, and photographic analysis of your project.

Materials

- material for construction of the vehicle
- mass-measuring equipment
- equipment to provide known energy crash conditions
- egg passengers
- digital camera
- computer

Assessing Results

Assess the success of your project based on a rubric* designed in class that considers:

- research strategies
- thoroughness of the experimental design
- effectiveness of the experimental technique
- effectiveness of the team's public presentation

Procedure

1. Research existing safety features used in automotive production. Identify which features are the most effective in reducing crash injuries. Keep a record of the sources of your information. If information comes from the Internet, be sure to identify the site, and who sponsors the information. Be alert to Internet sites that may contain biased information. Identify the most common types of injuries resulting from automobile accidents.
2. Design the persuader vehicle and gather the materials required for its construction.
3. Design the experiment that your team will use to test the effectiveness of your vehicle's design. Make sure that your experimental design makes it possible to accurately compare the energy of the vehicle when it is involved in different crash scenarios.

CAUTION: Your vehicle will need to gain considerable energy, which may or may not result in unexpected behaviour when it crashes. Take proper precautions to ensure that the vehicle path is clear during trials.

4. Test your vehicle's safety features under a variety of crash conditions. Assess how effective your system is when it is involved in crashes happening from different directions.
5. Prepare a report using an audiovisual format that will dramatically emphasize for your audience the value of the safety features that you recommend.

Thinking Further

Write a short appendix to your report (two or three paragraphs) to identify possible directions that future research might take to make automobile travel even safer. Suggest steps that government, technology, and industry should take in making automobile travel safer.

*Note: Your instructor will assess the project using a similar assessment rubric.

UNIT III SUMMARY

Unit Concepts and Skills: Quick Reference

Concepts	Summary	Resources and Skill Building
Chapter 5	**Newton's laws can explain circular motion.**	
	5.1 Defining Circular Motion	
Speed and velocity	The velocity of an object moving with circular motion is tangent to the circle and 90° to the radial line.	Figures 5.3–5.6, 5.8–5.11, 5.13; QuickLab 5-1; Inquiry Lab 5-2
Centripetal acceleration and force	Centripetal acceleration and centripetal force are both directed toward the centre of the circle.	Figures 5.8–5.11; Table 5.2; eSIM
	5.2 Circular Motion and Newton's Laws	
Velocity and circular motion	The velocity of circular motion can be determined by dividing the circumference by the period.	Example 5.2; Minds On
Centripetal acceleration	The centripetal acceleration of an object is determined by the velocity squared divided by the radius.	Figures 5.18–5.20; Example 5.3
Centripetal force — a horizontal system in circular motion	Newton's second law states $F = ma$ and can be applied to centripetal acceleration. A car making a turn experiences a centripetal acceleration and force that is created by the force of friction between the tires and the road.	Inquiry Lab 5-3; eTECH; Example 5.4; Figures 5.24, 5.25
Centripetal force — a vertical system in circular motion	The minimum speed necessary to move an object through a vertical loop equates centripetal force with gravitational force. Centripetal force can be equated to the gravitational force for planetary objects.	Figures 5.27–5.30; eTECH; Example 5.5; eSIM
Centripetal force — acceleration and frequency	Centripetal acceleration and force can be determined using period and frequency instead of speed.	Figures 5.32–5.34; Example 5.7
	5.3 Satellites and Celestial Bodies in Circular Motion	
Kepler's laws	Kepler formulated three laws that explained the motion of planets in the solar system.	Figures 5.36–5.38; Tables 5.4–5.6; Examples 5.8, 5.9; eSIM; Design a Lab 5-4
Newton's version of Kepler's third law	Newton recognized the reason that Kepler's laws were correct: $F_g = F_c$ for Earth–Moon system. He also found a way to determine the mass of an object from the period of a celestial body orbiting it.	Figures 5.41, 5.42; Examples 5.10, 5.11; eTECH
Orbital perturbations	The discovery of Uranus and Pluto occurred because of the apparent disturbances in the orbit of the planets. Extrasolar planets have been discovered by examining perturbations in stars' movements.	Then, Now, and Future; Figures 5.46, 5.47
Artificial satellites	Humans have placed a variety of artificial satellites into orbit to meet society's needs.	5-5 Decision-Making Analysis; Figures 5.48–5.51
Chapter 6	**In an isolated system, energy is transferred from one object to another whenever work is done.**	
	6.1 Work and Energy	
Work	Work is the transfer of energy that occurs when a force acts over a displacement. It is a scalar quantity measured in joules. (1 J = 1 N·m)	Example 6.1
Potential energy	Potential energy is the energy a body has because of its position or configuration. It is a scalar quantity measured in joules.	QuickLab 6-1; Example 6.2; Example 6.3; Example 6.4
Kinetic energy	Kinetic energy is the energy a body has because of its motion. It is a scalar quantity measured in joules.	QuickLab 6-1; Example 6.5; Example 6.6
	6.2 Mechanical Energy	
Work-energy theorem	Work done by a net force causes a change in kinetic energy.	Example 6.7
	The work–energy theorem states that the work done on a system is equal to the sum of the changes in the potential and kinetic energies.	Example 6.7
Mechanical energy	Mechanical energy is the sum of the potential and kinetic energies.	Example 6.8
	6.3 Mechanical Energy in Isolated and Non-isolated Systems	
Isolated systems	The law of conservation of energy states that in an isolated system, the mechanical energy is constant.	Example 6.9; Example 6.10
Conservation of energy	A simple pendulum is a good approximation of an isolated system in which energy is conserved.	Inquiry Lab 6-2
Conservative forces	A conservative force does not affect the mechanical energy of a system.	Example 6.10; Inquiry Lab 6-2
Non-isolated systems	In non-isolated systems, the mechanical energy may change due to the action of non-conservative forces.	Design a Lab 6-3
	6.4 Work and Power	
Power	Power is defined as the rate of doing work. Power is calculated by finding the ratio of the work done to the time required to do the work. It is measured in watts. (1 W = 1 J/s)	Example 6.11; Inquiry Lab 6-4; Example 6.12
	Power may be calculated by taking the product of the force doing the work and the average speed.	Problem-Solving Lab 6-5

UNIT III REVIEW

Vocabulary

1. Using your own words, define these terms:
   artificial satellite
   axis of rotation
   axle
   centripetal acceleration
   centripetal force
   conservation of energy
   conservative force
   cycle
   eccentricity
   efficiency
   elastic potential energy
   ellipse
   energy
   frequency
   gravitational potential energy
   isolated system
   Kepler's constant
   Kepler's laws
   kinetic energy
   mean orbital radius
   mechanical energy
   non-isolated system
   orbital period
   orbital perturbations
   period
   potential energy
   power
   reference point
   revolution
   rpm
   satellite
   uniform circular motion
   work
   work-energy theorem

Knowledge

CHAPTER 5

2. An object is moving in a circular path with a uniform centripetal acceleration that doesn't change. What will happen to the velocity if the radius is reduced?

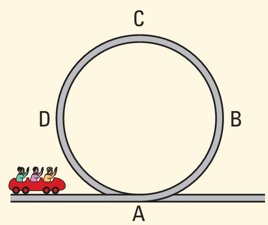
3. The centripetal acceleration of a car as it goes around a turn is inward, but the car will not skid in that direction if it is moving too quickly. Explain.

4. A bucket of water is spun in a vertical circle on the end of a rope.
   (a) Explain what force or forces act as the centripetal force when the bucket is in the highest position.
   (b) In which position is the rope most likely to break? Why?

5. Explain why centripetal force is inward when the force acting on your hand as you spin an object in a circular path is outward.

6. Using what you have learned about the force of gravity and circular motion, provide a thorough explanation why the magnitude of centripetal force changes for planets orbiting the Sun.

7. A roller coaster goes around a vertical loop with just enough velocity to keep it on the track.
   (a) In which position or positions is the force of gravity the centripetal force? Explain.
   (b) In which position or positions is there a force exerted on the track by the roller coaster? Explain.
   (c) Using the equation $F_g = mg$ and equation 6 from Chapter 5 on page 256, show why mass does not affect the speed required for the roller coaster to successfully enter and exit the loop as shown in the diagram below.

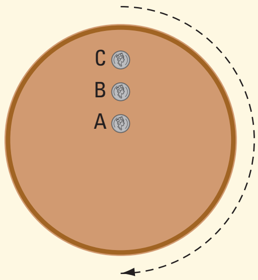


8. What is the relationship between frequency and radius for a rotating disc?

9. The motor of a table saw is rated for its horsepower and rotational frequency. Explain why rotational frequency is used instead of rotational speed.

10. What physical quantities must be known for the mass of Earth to be determined?

11. Kepler showed that planets follow elliptical orbits.
    (a) Which planet has the least elliptical orbit?
    (b) Which planet's semi-major axis is the closest in length to its semi-minor axis?



12. Three identical coins are placed on a rotating platter as shown. As the frequency of rotation increases, identify which coin will begin to slide off first. Explain your answer.

13. Briefly explain how Neptune was discovered. Use the terms orbital perturbation, force of gravity, and orbital velocity in your explanation.

14. Your friend argues that Neptune would not have been discovered as soon as it was if Neptune were a much smaller planet and Uranus much bigger. Is she right? Defend your answer.

15. What difficulties do astronomers face when searching for extrasolar planets that might have life as we know it?

CHAPTER 6

16. Express a joule in terms of kilograms, metres, and seconds.

17. If work is a scalar quantity, why is it affected by the directions of the force and displacement?

18. What happens to an object's gravitational potential energy when it is in free fall?

19. Explain why doubling the speed of an object does not result in a doubling of its kinetic energy.

20. A large mass and a small mass with the same kinetic energy are sliding on a horizontal frictionless surface. If forces of equal magnitude act on each of these bodies to bring them to rest, which one will stop in the shorter distance? Explain.

21. Describe the energy changes of a roller coaster car from the time when it is just coming over the crest of one hill until it arrives at the crest of the next hill.

22. A cart is pulled up an inclined plane by a force that is just large enough to keep the cart moving without a change in its speed. Is this an isolated system? Explain why or why not. Is the force used to move the cart up the incline a conservative force?

23. A cart at the top of an inclined plane is allowed to roll down the plane. Under what conditions can this system be considered isolated? If the conditions that make this an isolated system do not exist, is the force that moves the cart down the plane still considered a conservative force? Explain.

24. A slingshot is used to propel a stone vertically upward. Describe the energy changes that are involved from the time the stone is placed in the slingshot until the stone reaches its maximum height.

25. If a force that acts on an object results in a change in the object's kinetic energy, what can be said about the nature of this force?

26. How do you calculate work from a force-displacement graph?

27. According to the work-energy theorem, how much work is done on an isolated system?

28. Does power affect the amount of work you are able to do? Explain why or why not.

Applications

29. Electrons in an electric (AC) circuit vibrate at 60 Hz. What is their period?

30. A cell phone is set to vibrate when it rings. It vibrates with a period of 0.0160 s. What is the frequency of the ring?

31. A toy top spins at 300.0 rpm. What is the frequency (in Hz) and the period of the top?

32. The Moon orbits Earth once every 27.3 days at a mean orbital radius of 3.844×10^5 km. What is its speed?

33. A child sits in a pretend airplane on a ride at an amusement park. The airplane is at the end of a long arm that moves in a circular path with a radius of 4.0 m at a speed of 1.57 m/s. What is the period of the ride?

34. A person sliding down a water slide at a speed of 5.56 m/s encounters a turn with a radius of 10.0 m. Determine the acceleration that he experiences in the turn.

35. A pilot of a jet airplane makes a sharp turn to create an acceleration of 4.00 times the acceleration of gravity. If the turn has a radius of 500.0 m, what is the speed of the plane?

36. A cork ($m = 2.88$ g) is caught in a small whirlpool created in the basin of a sink. What is the centripetal force acting on the cork, if its speed is 0.314 m/s at a radius of 4.00 cm?

37. When braking to a stop, the maximum force that friction exerts on a 1250-kg auto is 3200 N.
    (a) If the original speed of the auto is 12.0 m/s, what is the minimum stopping distance?
    (b) If the speed of the car were twice as great, how would that affect the minimum stopping distance?

38. A force of 250 N [up] is applied to a mass of 15.0 kg over a displacement of 9.60 m [up].

(a) How much work does the force do on the mass?

(b) What is the change in gravitational potential energy?

(c) If it is an isolated system, explain the difference between the answers for (a) and (b).

39. A car with a mass of 2.00×10^3 kg is travelling at a velocity of 15.0 m/s [0°] on a horizontal stretch of highway. The driver presses on the accelerator so that the force propelling the car forward is increased to 3.30×10^3 N [0°]. The force acts over a displacement of 55.0 m [0°] during which force of friction on the car is 5.00×10^2 N in magnitude.

(a) Draw a free-body diagram to analyze the forces acting on the car. What is the net force on the car?

(b) What is the work done by net force over the displacement?

(c) What is the final kinetic energy of the car?

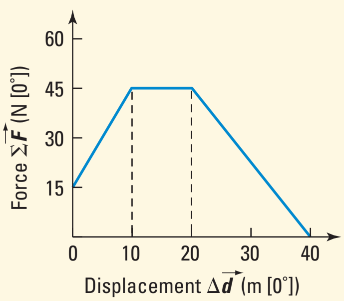
(d) What is the final speed of the car?

40. A block with a mass of 0.800 kg is initially at rest on a frictionless inclined plane. A force of 5.00 N, applied parallel to the inclined plane, moves the block a distance of 4.50 m up the plane. If, at the end of the effort, the block has a speed of 6.00 m/s up the incline, what is the change in height through which it moved?

41. A varying force acts on a 25.0-kg mass over a displacement as shown in the graph below. The mass has an initial velocity of 12.0 m/s [0°]. Recall that the area of a force-displacement graph is equivalent to the work done by the force.

(a) What is the work that the force did on the mass?

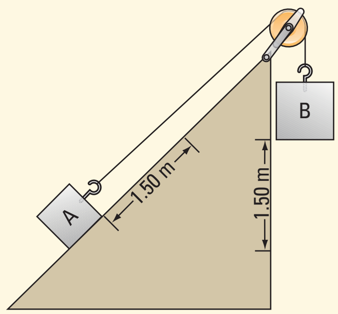
(b) What is the final speed of the mass?



42. In the following diagram, block A is at rest on a frictionless inclined plane. It is attached to block B by a light cord over a frictionless pulley. Block A has a mass of 4.50 kg and B has a mass of 5.50 kg. When they are released, block A moves up the incline so that after it has moved a distance of 1.50 m along the incline it has a speed of 3.00 m/s.

(a) What is the change in gravitational potential energy for block A?

(b) What was the change in height through which block A moved?



43. For question 42, draw the graph that shows the potential, kinetic, and mechanical energies for the system as a function of the displacement assuming (a) there is no friction, (b) there is friction, but block A still accelerates up the hill.

44. A pendulum bob with a mass of 0.750 kg is initially at rest at its equilibrium position. You give the bob a push so that when it is at a height of 0.150 m above its equilibrium position it is moving at a speed of 2.00 m/s.

(a) How much work did you do on the bob?

(b) If you pushed on the bob with a force of 40.0 N parallel to the displacement, how far did you push it?

45. A billiard ball with a speed of 2.00 m/s strikes a second ball initially at rest. After the collision, the first ball is moving at a speed of 1.50 m/s. If this is an elastic collision (i.e., energy is conserved), what is the speed of the second ball after the collision? Assume that the balls have identical masses of 0.200 kg.

46. A cannon ball (m = 3.00 kg) is fired at a velocity of 280 m/s at an angle of 20° above the horizontal. The cannon is on a cliff that is 450 m above the ocean.

(a) What is the mechanical energy of the cannon ball relative to the base of the cliff?

(b) What is the greatest height above the ocean that the cannon ball reaches?

(c) What is the speed of the cannon ball when it lands in the ocean?

47. A Styrofoam™ ball is dropped from a height of 5.00 m. The mass of the ball is 0.200 kg. When the ball hits the ground it has a speed of 3.00 m/s.

(a) What change in mechanical energy does the ball undergo while it falls?

(b) What is the average force that air friction exerted on the falling ball?

48. What is the average power output if an engine lifts a 250-kg mass a distance of 30.0 m in 20.0 s?

49. What is the effective power required to maintain a constant speed of 108 km/h if the force opposing the motion is 540 N in magnitude?

50. An airplane engine has an effective power output of 150 kW. What will be the speed of the plane if the drag (air friction opposing the motion of the plane) exerts a force of 2.50×10^3 N?

Extensions

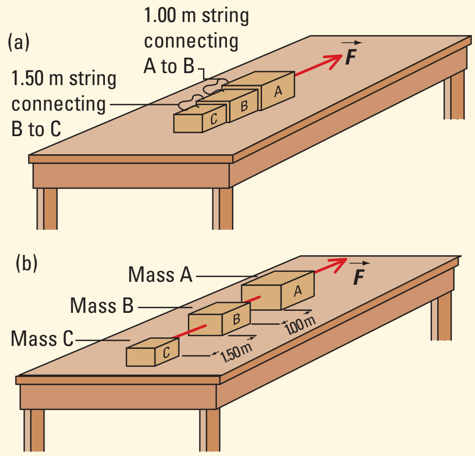
51. A 90.9-kg gymnast swings around a horizontal bar in a vertical circle. When he is directly over top of the bar, his arms experience a tug of 108.18 N. What is the speed of his body in this position? (Assume that the gymnast's mass is centred 1.20 m from the bar.)

52. Suppose a solar system, with three planets circling a star, exists in a galaxy far far away. One of the planets orbits the star with the period of 6.31×10^7 s and a radius of 3.00×10^{11} m. It has a moon that orbits it with a period of 1.73×10^6 s at a radius of 6.00×10^8 m.
    (a) What is the mass of the planet's star?
    (b) What is the mass of the planet?
    (c) What is the speed of the planet's moon?

53. A soil-moving machine called a bucket wheel loader has a large metallic wheel with a radius of 3.05 m that has many scoops attached to it. The scoops are designed to dig into the ground and lift soil out as the wheel turns around. If the wheel turns with a frequency of 0.270 Hz, will the soil fall out of a scoop when it gets to the top of the wheel?

54. Three blocks (A, B, and C), with masses 6.00 kg, 4.00 kg, and 2.00 kg, respectively, are initially at rest on a horizontal frictionless surface as shown in diagram (a) on the right. A force of 48.0 N [90°] acts on block A over a displacement of 7.50 m [90°]. Block A is connected to block B by a string that is 1.00 m long and block B is connected to block C by a string that is 1.50 m long. Initially, the three blocks are touching each other. As the blocks move and the strings become taut, they end up as shown in diagram (b). Is this an isolated or a non-isolated system? Explain.
    (a) What is the speed of the blocks after the force has acted for the full 7.50 m?
    (b) What is the speed of block A when the force has acted over a displacement of 2.00 m?

    Hint: Find the work done by the force.



Skills Practice

55. Your cousin doesn't understand how a satellite can stay in orbit without falling toward Earth. Using the knowledge you have gained from this unit, provide a short explanation.

56. Figure 6.21 on page 303 shows the impact crater for a meteor that landed in Arizona. How widespread is the evidence of meteors striking Earth? Where is the impact crater closest to where you live? Do Internet research to identify locations of impact craters in Alberta and Canada. On a map of Alberta, show the location of meteor impacts. What clues on maps show meteor landings? For each crater, identify how much kinetic energy the meteor would have had when it struck Earth.

57. Explain how you would experiment to determine the quantity of external work done on a system for a cart accelerating down an inclined plane. Could you confirm this quantity by measuring forces?

Self-assessment

58. Describe to a classmate one misconception you had about circular motion before studying this unit. Explain what you know about this concept now.

59. Describe to a classmate the relationship between the roles of science and technology in the development of new energy resources.

60. Can science provide solutions to all of the problems associated with the impact of energy consumption on the environment? Give reasons for your answer.

e TEST

To check your understanding of circular motion, work, and energy, follow the eTest links at www.pearsoned.ca/school/physicssource.