CHAPTER

2

Vector components describe motion in two dimensions.

Key Concepts

In this chapter, you will learn about:

- two-dimensional motion
- vector methods

Learning Outcomes

When you have completed this chapter, you will be able to:

Knowledge

- explain two-dimensional motion in a horizontal or vertical plane
- interpret the motion of one object relative to another

Science, Technology, and Society

- explain that scientific knowledge is subject to change as new evidence comes to light and as laws and theories are tested, restricted, revised, or reinforced

▲ **Figure 2.1** The motion of Canada's Snowbird precision flight squad can be described using vectors.

Imagine being a pilot for the Canadian Snowbirds (Figure 2.1). This precision flight team, composed of highly trained military personnel, performs at air shows across the country. Unlike the flight crew in the cockpit of a commercial airliner, these pilots execute aerobatic manoeuvres that require motion in both horizontal and vertical directions, while being acutely aware of their positions relative to the ground and to each other.

In this chapter, you will study motion in one and two dimensions by building on the concepts you learned in Chapter 1. You will use vectors to define position, velocity, and acceleration, and their interrelationships. The vector methods you will learn will allow you to study more complex motions.

2-1 QuickLab

Taking a One-dimensional Vector Walk

Problem

How can you add vectors to determine displacement?

Materials

30-m tape measure
field marker (tape or flag)

Procedure

1. Starting at the centre of a football field (or gymnasium), work out a path sequence using six forward and backward displacements to move from your starting position to the goal line. At least two of your six displacements must be oriented in the direction opposite to the direction of the goal line.
2. On the field, mark your starting point with a flag or tape. Define direction axes.
3. Ask your partner to walk the displacements of the path sequence chosen in step 1 while holding the end of the measuring tape. Mark your partner's endpoint after each displacement (Figure 2.2).
4. Continue the journey, using the measuring tape, until you have walked all the displacements.
5. Mark the final endpoint.
6. Using the measuring tape, determine the displacement from your starting point.
7. Repeat steps 2–6 using two different sequences of the six displacements you originally chose.

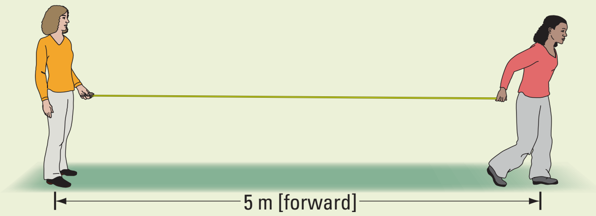


Figure 2.2

Questions

1. What was the total distance you travelled?
2. What was the total displacement?
3. What conclusion can you draw about the order of adding vectors?

Think About It

1. How does the order of a series of displacements affect the final position of an object?
2. In order to cross a river in the shortest possible time, is it better to aim yourself upstream so that you end up swimming straight across or to aim straight across and swim at an angle downstream?
3. Why does it take longer to fly across Canada from east to west rather than west to east in the same airplane?
4. How does the angle of a throw affect the time a ball spends in the air?
5. Two objects start from the same height at the same time. One is dropped while the other is given an initial horizontal velocity. Which one hits the ground first?

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes to your ideas.

2.1 Vector Methods in One Dimension

One of the fastest-growing sports in the world today is snowshoeing (Figure 2.3). The equipment required is minimal and the sport is easy to learn — you need to move forward in a straight line. Despite its simplicity, snowshoeing has great cardiovascular benefits: You can burn up to 1000 calories per hour, which makes it the ultimate cross-training program for athletes. It also allows athletes to explore different terrains and gain a greater appreciation of the outdoors, as well as to test their limits, especially by participating in endurance races!

The motions in a showshoe race can be broken up into one-dimensional vector segments. In this section, you will study motion in one dimension using vectors.

▲ **Figure 2.3** Snowshoeing is an excellent way of enjoying the great outdoors in winter while improving your health.

Vector Diagrams

▲ **Figure 2.4** A vector has a tail and a tip.

In Chapter 1, you used variables and graphs to represent vector quantities. You can also represent vector quantities using vector diagrams. In a diagram, a line segment with an arrowhead represents a vector quantity. Its point of origin is called the *tail*, and its terminal point (arrowhead) is the *tip* (Figure 2.4). If the magnitude of a vector is given, you can draw the vector to scale. The length of the line segment depends on the vector's magnitude. The arrowhead indicates direction. Drawing vector diagrams to represent motion helps you to visualize the motion of an object. Properly drawn, vector diagrams enable you to accurately add vectors and to determine an object's position.

Choosing Reference Coordinates

When describing the motion of an object, there are many ways to describe its direction. You could use adjectives such as forward or backward, up or down, into or out of, and left or right. You can also use compass directions, such as north [N], south [S], east [E], and west [W]. When drawing a vector diagram, it is important to choose which directions are positive and to include these directions on every vector diagram. As you learned in section 1.1 (Figure 1.6), in this unit, forward, up, right, north, and east are usually designated as positive, whereas their opposites are usually considered negative. You may choose your own designation of positive and negative when solving problems, but make sure your reference direction is consistent within each problem and clearly communicated at the beginning of your solution.

Practise drawing vectors in the next Skills Practice exercise.

SKILLS PRACTICE Representing a Vector

Using an appropriate scale and direction convention, draw each of the following vectors.

(a) 5 m [forward]
(b) 20 m [down]
(c) 30 km [north]
(d) 150 km [left]

Adding Vectors in One Dimension

Motion in one dimension involves vectors that are collinear. **Collinear** vectors lie along the same straight line. They may point in the same or in opposite directions (Figure 2.5).

collinear: along the same straight line, either in the same or in opposite directions

▲ **Figure 2.5** (a) Collinear vectors in the same direction (b) Collinear vectors in opposite directions

When more than one vector describes motion, you need to add the vectors. You can add and subtract vectors graphically as well as algebraically, provided they represent the same quantity or measurement. As in mathematics, in which only like terms can be added, you can only add vectors representing the same types of quantities. For example, you can add two or more position vectors, but not a position vector, 5 m [E], to a velocity vector, 5 m/s [E]. In addition, the unit of measurement must be the same. For example, before adding the position vectors 5 m [E] and 10 km [E], you must convert the units of one of the vectors so that both vectors have the same units.

In the next example, determine the sum of all the vector displacements graphically by adding them tip to tail. The sum of a series of vectors is called the **resultant vector**.

resultant vector: a vector drawn from the tail of the first vector to the tip of the last vector

Example 2.1

Contestants in a snowshoe race must move forward 10.0 m, untie a series of knots, move forward 5.0 m, solve a puzzle, and finally move forward 25.0 m to the finish line (Figure 2.6). Determine the resultant vector by adding the vectors graphically.

▲ **Figure 2.6**

Analysis and Solution

1. Choose an appropriate scale and reference direction.
   1.0 cm : 5.0 m, forward is positive.
2. Draw the first vector and label its magnitude and direction (Figure 2.7).

▲ **Figure 2.7**

3. Place the tail of the second vector at the tip of the first vector. Continue to place all the remaining vectors in order, tip to tail (Figure 2.8).

▲ **Figure 2.8**

4. Connect the tail of the first vector to the tip of the last vector. This new vector, which points toward the tip of the last vector, is the resultant vector, $\vec{R}$ (the purple arrow in Figure 2.9).

▲ **Figure 2.9**

Find the magnitude of the resultant vector by measuring with a ruler, then convert the measured value using the scale. Remember to include the direction.

$$\Delta\vec{d} = 8.0\ \cancel{\text{cm}}\ [\text{forward}] \times \frac{5.0\ \text{m}}{1.0\ \cancel{\text{cm}}}$$

$$= 40\ \text{m}\ [\text{forward}]$$

Practice Problems

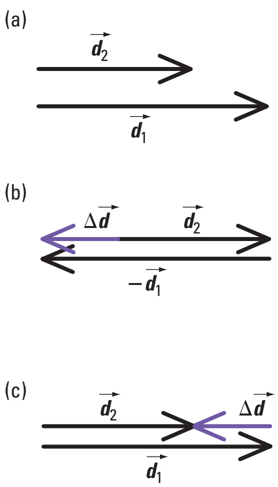
1. The coach of the high-school rugby team made the team members run a series of sprints: 5.0 m [forward], 10 m [backward], 10 m [forward], 10 m [backward], 20 m [forward], 10 m [backward], 40 m [forward], and 10 m [backward].
   (a) What is their total distance?
   (b) What is their displacement?

Answers

1. (a) 115 m
   (b) 35 m [forward]

In summary, you can see that adding vectors involves connecting them tip to tail. The plus sign in a vector equation tells you to connect the vectors tip to tail in the vector diagram.

To subtract collinear vectors graphically (Figure 2.10(a)), you may use one of two methods. For the first method, find $\Delta\vec{d}$ using the equation $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$: Add the negative of $\vec{d}_1$ to $\vec{d}_2$, tip to tail, as you did in Example 2.1. The negative of a vector creates a new vector that points in the opposite direction of the original vector (Figure 2.10(b)). For the second method, connect the vectors tail to tail. This time, $\Delta\vec{d}$ starts at the tip of $\vec{d}_1$ and ends at the tip of $\vec{d}_2$ (Figure 2.10(c)).

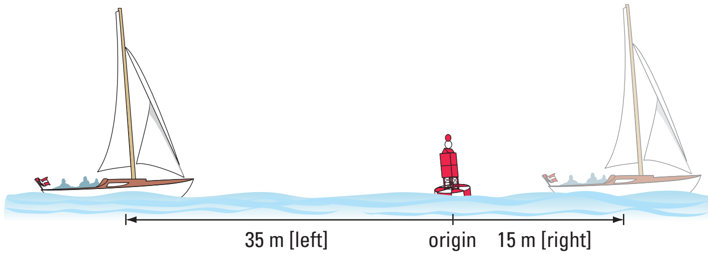


▲ **Figure 2.10**
(a) To subtract two collinear vectors, $\vec{d}_2 - \vec{d}_1$, graphically, (b) add the negative of $\vec{d}_1$ to $\vec{d}_2$ or (c) connect the vectors tail to tail and draw the resultant connecting the tip of $\vec{d}_1$ to the tip of $\vec{d}_2$.

Recall that the definition of displacement is final position minus initial position, or $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$. The next example reviews the algebraic subtraction of vectors, which you learned in Chapter 1, Example 1.1, and also shows you how to subtract vectors graphically.

Example 2.2

A sailboat that is initially 15 m to the right of a buoy sails 35 m to the left of the buoy (Figure 2.11). Determine the sailboat's displacement (a) algebraically and (b) graphically.



▲ **Figure 2.11**

Given

Consider right to be positive.

$\vec{d}_i = 15\text{ m [right]} = +15\text{ m}$

$\vec{d}_f = 35\text{ m [left]} = -35\text{ m}$

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

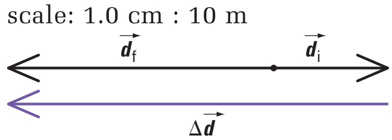
(a) To find displacement algebraically, use the equation

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i\\ &= -35\text{ m} - (+15\text{ m})\\ &= -35\text{ m} - 15\text{ m}\\ &= -50\text{ m}\end{aligned}$$

The sign is negative, so the direction is to the left.

(b) To find displacement graphically, subtract the two position vectors. Draw the vectors tail to tail and draw the resultant from the tip of the initial position vector to the tip of the final position vector (Figure 2.12).

scale: 1.0 cm : 10 m



▲ **Figure 2.12**

$$\begin{aligned}\Delta\vec{d} &= 5.0\ \cancel{\text{cm}}\text{ [left]} \times \frac{10\text{ m}}{1.0\ \cancel{\text{cm}}}\\ &= 50\text{ m [left]}\end{aligned}$$

Paraphrase

The sailboat's displacement is 50 m [left].

Practice Problems

1. Sprinting drills include running 40.0 m [N], walking 20.0 m [N], and then sprinting 100.0 m [N]. Using vector diagrams, determine the sprinter's displacement from his initial position.
2. To perform a give and go, a basketball player fakes out the defence by moving 0.75 m [right] and then 3.50 m [left]. Using vector diagrams, determine the player's displacement from the starting position.
3. While building a wall, a bricklayer sweeps the cement back and forth. If she swings her hand back and forth, a distance of 1.70 m, four times, use vector diagrams to calculate the distance and displacement her hand travels during that time.

Check your answers against those in Example 1.1 Practice Problems 1-3.

Answers

1. 160.0 m [N]
2. 2.75 m [left]
3. 6.80 m, 0 m

For collinear vectors, find displacement by subtracting initial position from final position. Subtract vectors graphically by connecting them tail to tail or by reversing the direction of the initial position vector. Recall from Chapter 1 that direction for displacement is given with respect to initial position.

2.1 Check and Reflect

Knowledge

1. Describe the similarities and differences between the two vectors drawn below.

2. Using the same scale and reference coordinates, compare the vectors 5 m [N] and 10 m [S].

3. If the scale vector diagram of 5.0 m [S] is 6.0 cm long, what is the length of the scale vector diagram of 20 m [S]?

4. What scale is being used if 5.0 cm represents 100 km?

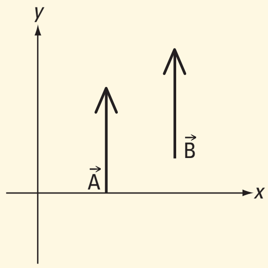
Applications

5. The scale on a *National Geographic* world map is 1.0 cm : 520 km. On the map, 4.0 cm separates Alberta's north and south provincial boundaries. What is the separation in kilometres?

6. During a tough drill on a field of length 100 yards, players run to each 10-yard line and back to the starting position until they reach the other end of the field.
   (a) Write a vector equation that includes all the legs of the run.
   (b) What is the players' final displacement?
   (c) How far did they run?

7. A car drives north 500 km. It then drives three sequential displacements south, each of which is 50 km longer than the previous displacement. If the final position of the car is 50 km [N], find the three displacements algebraically.

8. Are vectors A and B equal? Why or why not?



9. A bouncy ball dropped from a height of 10.0 m bounces back 8.0 m, then drops and rebounds 4.0 m and finally 2.0 m. Find the distance the ball travels and its displacement from the drop point.

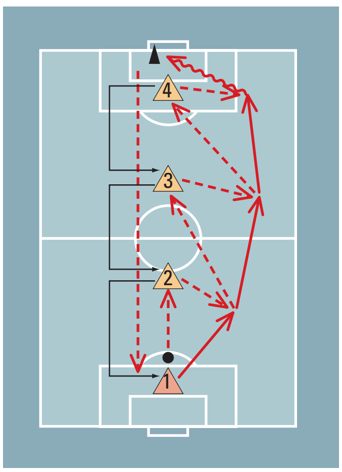
e TEST

To check your understanding of vectors in one dimension, follow the eTest links at www.pearsoned.ca/school/physicssource.

2.2 Motion in Two Dimensions

From the boot, the ball flies across the grass into the net and the crowd roars. The enormously successful FIFA Under-19 Women's World Championship, held in 2002, raised the profile of women's soccer in Canada and drew crowds totalling almost 200 000 to venues in Edmonton, Vancouver, and Victoria (Figure 2.13). Stars like Charmaine Hooper, Brittany Timko, and Christine Sinclair continue to amaze. From World Championship team members to the Under-6s on the local soccer pitch, performance depends on understanding and coordinating the movement of players and ball across the surface of the field.

▲ **Figure 2.13** Motion in sports such as soccer can be described by vectors in two dimensions.



▲ **Figure 2.14** This page is taken from a soccer playbook. How many players are involved in this wall pass-in-succession manoeuvre?

Playbooks are available for fast-paced games such as hockey and soccer to allow coaches and players to plan the strategies that they hope will lead to success. Sometimes a team can charge straight up the rink or field, but, more often, a series of angled movements is needed to advance the puck or ball (Figure 2.14). For everyone to understand the play, a system is needed to understand motion in two dimensions.

Components of Vectors

Imagine that you are at one corner of a soccer field and you have to get to the far opposite corner. The shortest path from one corner to the other is a straight diagonal line. Figure 2.15 shows this path to be 150 m.

Another way to describe this motion is to imagine an *x*-axis and a *y*-axis placed onto the soccer field, with you standing at the point (0, 0). You could move along the length of the field 120 m and then across the field 90 m and end up at the same spot (Figure 2.15).

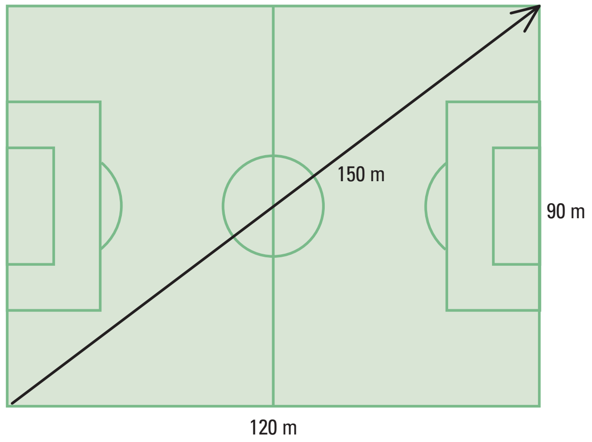


Figure 2.15 The diagonal distance from one corner to the opposite corner of a soccer field is 150 m.

In this example, the sideline of the soccer field could be considered the *x*-axis, and the goal line could be the *y*-axis. The diagonal motion vector can then be separated, or resolved, into two perpendicular parts, or **components**: the *x* component and the *y* component. The diagonal vector is the resultant vector.

components: perpendicular parts into which a vector can be separated

If you walked along the sideline or *x*-axis, you would move through a distance of 120 m. This distance is the *x* component of the diagonal vector. The second part of the walk along the goal line, parallel to the *y*-axis, is the *y* component of the diagonal vector. This motion has a distance of 90 m. Figure 2.16 shows the *x* and *y* components of the diagonal motion across the soccer field.

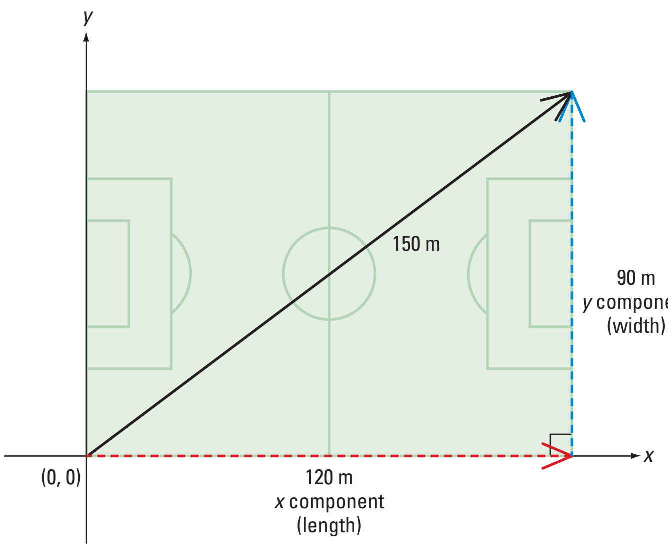


Figure 2.16 The resultant vector representing the diagonal walk across the soccer field can be resolved into *x* and *y* components.

Vector Directions

Recall that a vector must have a magnitude and a direction. You have just studied how to resolve a vector into its components. Before going further, you need to know how to indicate the direction of vectors in two dimensions. There are two methods commonly used to show direction for vector quantities in two dimensions: the **polar coordinates method** and the **navigator method**. Both methods are considered valid ways to describe the direction of a vector.

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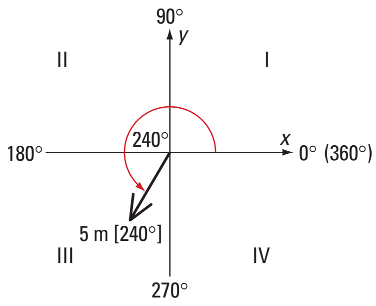
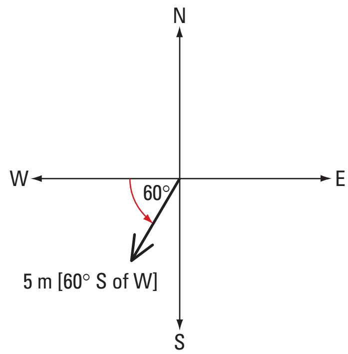
A third method for measuring direction is the bearing method, in which angles are measured clockwise from north, 0° to 360°, so east is 90°, south is 180°, and west is 270°.



▲ **Figure 2.17** The polar coordinates method for stating vector direction



▲ **Figure 2.18** The navigator method for stating vector direction

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Sailors can now create their sailing plans with a click of a mouse. Digitized maps and global positioning satellites have been combined to allow sailors to create a plan by using the mouse to place vectors on the desired path on screen. The computer calculates the total distance, identifies directions, and estimates the time required for the trip.

Polar Coordinates Method

With the polar coordinates method, the positive *x*-axis is at 0° and angles are measured by moving counterclockwise about the origin, or pole. One complete rotation is 360° — a complete circle. In Figure 2.17, the displacement vector, 5 m [240°], is located in quadrant III in the Cartesian plane. This vector is rotated 240° counterclockwise starting from the positive *x*-axis.

Navigator Method

Another method for indicating vector direction is the navigator method. This method uses the compass bearings north [N], south [S], east [E], and west [W] to identify vector directions. In Figure 2.18, the displacement vector 5 m [60° S of W] is between the west and south compass bearings. To draw this vector, start with the second compass bearing you are given in square brackets, west, then move 60° in the direction of the first compass bearing you are given, south.

The type of problem will determine the method you use for stating vector directions. Often, it will be clear from the context of the problem which method is preferred. For example, if the question is about a boat sailing [30° N of W], then use the navigator method. If a plane has a heading of 135°, then use the polar coordinates method.

In the problems below, you can practise identifying and drawing vectors using the two methods.

SKILLS PRACTICE Directions

1. For each of the following vectors, identify the method used for indicating direction. Then draw each vector in your notebook, using an appropriate scale and reference coordinates.
   (a) 3 m [0°]
   (b) 17 m/s [245°]
   (c) 7 m [65°]
   (d) 8 m/s [35° W of N]
   (e) 2 m [98°]
   (f) 12 m/s [30° S of E]

2. For each vector in question 1, state the direction using the alternative method. Then draw each vector using an appropriate scale and reference coordinates.

Concept Check

Write the direction [60° S of W] another way using a different starting axis but keeping the angle less than 90°.

2-2 QuickLab

Vector Walk

Problem

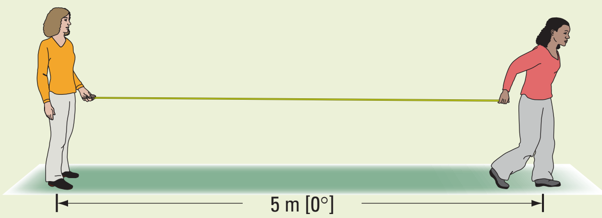
How can you add vectors to determine displacement?

Materials

30-m measuring tape
large chalkboard protractor
field marker (tape or flag)

Procedure

1. Using a tree diagram, determine the number of pathways you could take to walk the series of displacement vectors below. Assume that you will start on the centre line of a football field and mark it 0°.
   (a) 5 m [0°]
   (b) 12 m [270°]
   (c) 15 m [90°]
   (d) 3 m [180°]
2. On a football or large school field, mark your starting point with a flag or tape. Define direction axes.
3. Ask your partner to walk the displacement given in (a) while you hold the end of the measuring tape. Mark your partner's endpoint (Figure 2.19).
4. Continue the journey, using the protractor and measuring tape, until you have walked all the vectors.
5. Mark the final endpoint.
6. Using the measuring tape, determine the displacement from your starting point.
7. Use the protractor to estimate the angle of displacement.
8. Repeat steps 3–7 for all the pathways you determined in step 1.



▲ **Figure 2.19**

NOTE: Use the same method for determining direction throughout the lab.

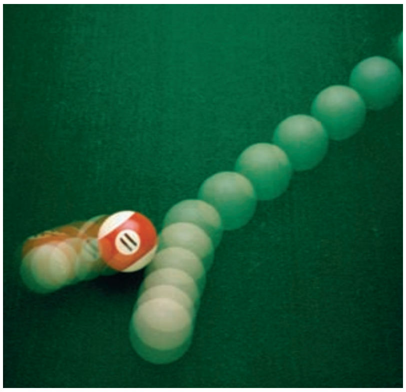
Questions

1. What was the total distance you travelled?
2. What was the total displacement?
3. What conclusion can you draw about the order of adding vectors?

Adding Two-dimensional Vectors Graphically

To sink the eight ball in the front side pocket of a billiard table, you must cause the ball to travel down and across the table (Figure 2.20). The ball's motion occurs in a plane, or two dimensions, even though its path is linear (Figure 2.21).

▲ **Figure 2.20** Playing billiards involves two-dimensional motion.



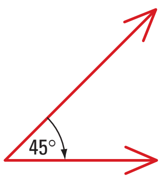
▲ **Figure 2.21** The path of the billiard ball is linear, but it occurs in two dimensions.

Recall from section 2.1 that the plus sign (+) in a vector equation indicates that you need to connect the vectors tip to tail. Up to this point, you have added collinear vectors only. In this section, you will learn how to add non-collinear vectors. The plus sign still indicates you need to connect the vectors tip to tail while keeping track of their directions.

Adding Non-collinear Vectors

non-collinear: not along a straight line

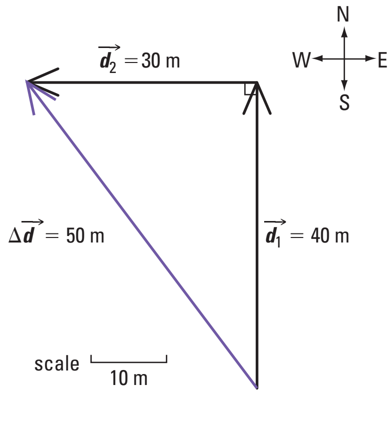
In section 2.1, you learned that vectors that lie along the same straight line are collinear. Vectors that are *not* along the same straight line are **non-collinear** (Figure 2.22). To determine the magnitude and direction of the sum of two or more non-collinear vectors *graphically*, use an accurately drawn scale vector diagram.



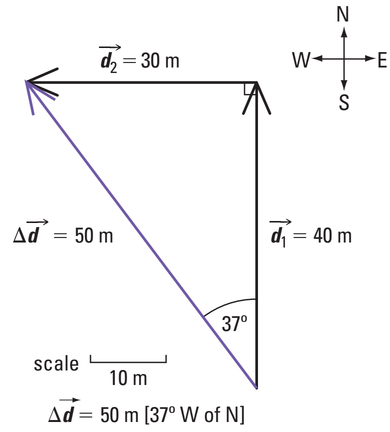
▲ **Figure 2.22** Non-collinear vectors lie along different lines.

Imagine you are walking north a distance of 40 m. Your initial position from your starting point is $\vec{d}_1$. You stop, head west a distance of 30 m, and stop again. Your final position is $\vec{d}_2$. To find your displacement, you cannot simply subtract your initial position from your final position because the vectors are not collinear. To find your displacement in two dimensions, you need to *add* the two position vectors: $\Delta\vec{d} = \vec{d}_1 + \vec{d}_2$.

From Figure 2.23, you can see that the answer is *not* 70 m. You would obtain the answer 70 m if you walked in the same direction for both parts of your walk. Because you did not, you cannot directly substitute values into the displacement equation $\Delta\vec{d} = \vec{d}_1 + \vec{d}_2$. Instead, you must draw the vectors to scale, connect them **tip to tail** (because of the plus sign), and measure the magnitude of the resultant. Since $\Delta\vec{d}$ is a vector quantity, you must also indicate its direction. You can find the direction of the resultant using a protractor (Figure 2.24).



▲ **Figure 2.23** What is the sum of $\vec{d}_1$ and $\vec{d}_2$?



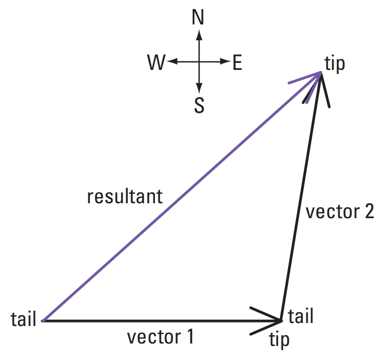
▲ **Figure 2.24** When adding non-collinear vectors graphically, use a protractor to find the direction of the resultant.

Eight Steps for Adding Non-collinear Vectors Graphically

To find the resultant vector in a non-collinear vector addition statement using the graphical method, follow these eight steps (see Figure 2.25):

1. Create an appropriate scale.
2. Choose a set of reference coordinates.
3. Draw vector 1 to scale. Measure its direction from the tail.
4. Draw vector 2 to scale. Draw its tail at the tip (arrowhead) of vector 1.
5. Draw the resultant vector by connecting the tail of vector 1 to the tip of vector 2.
6. Measure the magnitude (length) of the resultant. Measure the direction (angle) of the resultant from its tail.
7. Use your scale to convert the magnitude of the resultant to its original units.
8. State the resultant vector. Remember to include both magnitude and direction.

This method also works for more than two vectors. You can add the vectors in any order. The next example shows you how to add more than two non-collinear vectors graphically.



▲ **Figure 2.25** Adding vectors

Example 2.3

A camper left her tent to go to the lake. She walked 0.80 km [S], then 1.20 km [E] and 0.30 km [N]. Find her resultant displacement.

Given

$\Delta\vec{d}_1 = 0.80$ km [S]

$\Delta\vec{d}_2 = 1.20$ km [E]

$\Delta\vec{d}_3 = 0.30$ km [N]

Required

resultant displacement ($\Delta\vec{d}_R$)

Practice Problems

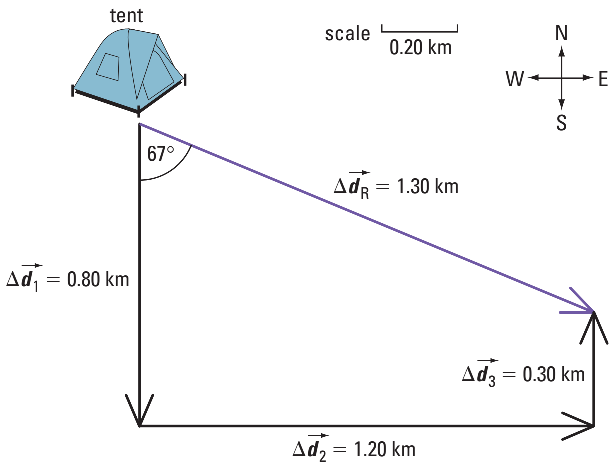
1. For Example 2.3, add the vectors in two different orders and obtain the resultant for each case.
2. A student runs through a field 100 m [E], then 200 m [S], and finally 50 m [45° S of E]. Find her final position relative to her starting point.

Answers

1. 1.30 km [67° E of S]
2. 272 m [60° S of E]

Analysis and Solution

The three vectors are non-collinear, so add them tip to tail to find the resultant (Figure 2.26).



$$\Delta\vec{d}_R = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$$

▲ **Figure 2.26**

Paraphrase

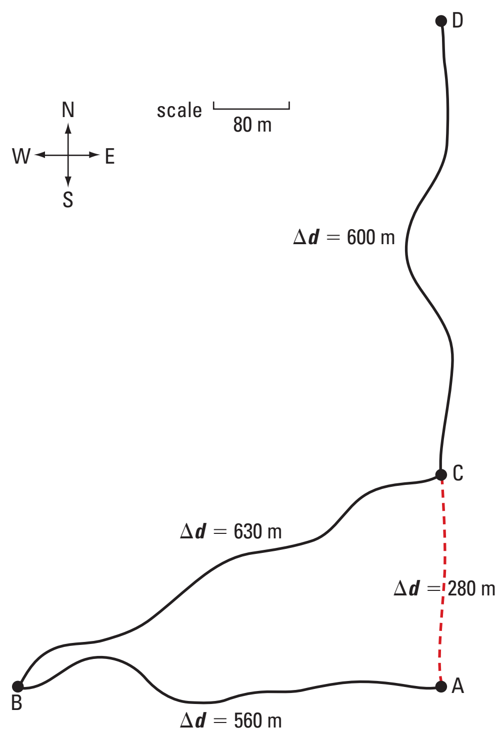
The camper's resultant displacement is 1.30 km [67° E of S].

SKILLS PRACTICE Distance, Displacement, and Position

Figure 2.27 shows the distances a bicycle courier travelled in going along the path from A to D, passing through B and C on the way. Use the information in the diagram, a ruler calibrated in mm, and a protractor to complete the distance, displacement, and position information required in Table 2.1. Assume the bicycle courier's reference point is A. Complete Table 2.1, then draw and label the displacement vectors AB, BC, and CD, and the position vectors AB, AC, and AD.

▼ **Table 2.1** Distance, Displacement, and Position

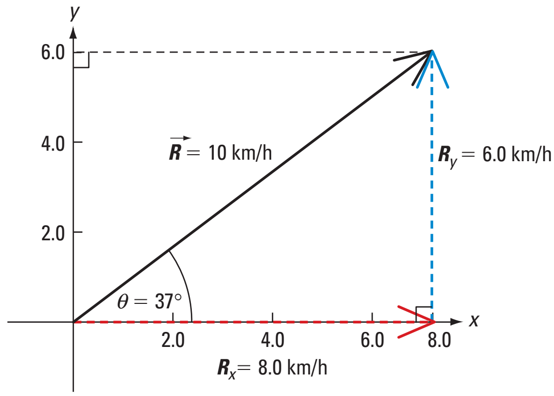
	Distance Δd (m)	Final position $\vec{d}$ (m) [direction] reference point	Displacement $\Delta\vec{d}$ (m) [direction]
AB			
BC			
CD			
AC			
AD			



▲ **Figure 2.27**

Determining Components

Figure 2.28 shows a vector $\vec{R}$ drawn in a Cartesian plane and its two components, R_x and R_y, in the x and y directions. The Greek letter theta, θ, denotes the angle between $\vec{R}$ and the x-axis. Vector $\vec{R}$ and its x and y components form a right triangle.



◀ **Figure 2.28**
Drawing components

Because the triangle is a right triangle, you can determine components algebraically by using the trigonometric functions sine, cosine, and tangent. You can define each of the trigonometric functions in terms of the sides of a right triangle, like the one in Figure 2.29. Knowing these definitions, you can use the trigonometric functions to help you solve for the components of a vector.

To calculate R_x, use the cosine function:

$$\cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{R_x}{R} \text{ or } R_x = R \cos\ \theta$$

In Figure 2.28, the x component is:

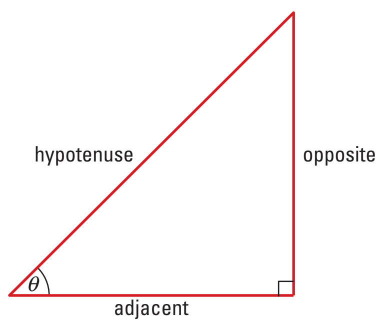
$$\begin{aligned} R_x &= (10\ \text{km/h})(\cos 37°) \\ &= 8.0\ \text{km/h} \end{aligned}$$

To calculate R_y, use the sine function:

$$\sin\ \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{R_y}{R} \text{ or } R_y = R \sin\ \theta$$

In Figure 2.28, the y component is:

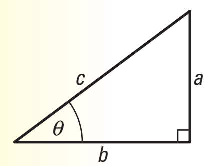
$$\begin{aligned} R_y &= (10\ \text{km/h})(\sin 37°) \\ &= 6.0\ \text{km/h} \end{aligned}$$



▲ **Figure 2.29** Labelled sides of a right triangle

Example 2.4 shows the steps for finding the velocity components of a car travelling in a northeasterly direction using trigonometry. This example uses the navigator method to indicate the direction of the velocity vector. Note that the east direction [E] is the same as the positive x direction in the Cartesian plane, and north [N] is the same as the positive y direction. So, for any vector $\vec{R}$, the x component is the same as the east component, and the y component is the same as the north component.

PHYSICS INSIGHT



For a right triangle with sides a and b forming the right angle, c is the hypotenuse. The Pythagorean theorem states that $a^2 + b^2 = c^2$. You can find the angle, θ, in one of three ways:

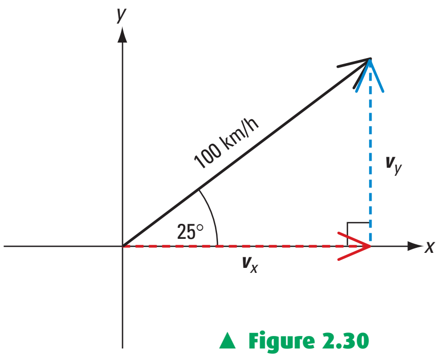
$$\sin\ \theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\ \theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\ \theta = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

Example 2.4

Determine the north and east velocity components of a car travelling at 100 km/h [25° N of E].



▲ **Figure 2.30**

Given

$\vec{v}$ = 100 km/h [25° N of E]

Required

velocity component north (y component, v_y)
velocity component east (x component, v_x)

Analysis and Solution

The vector lies between the north and east directions, so the x and y components are both positive. Since the north direction is parallel to the y-axis, use the sine function, $R_y = R \sin \theta$, to find the north component. Since the east direction lies along the x-axis, use the cosine function, $R_x = R \cos \theta$, to find the east component.

$$\begin{aligned} R_y &= R \sin \theta \\ v_y &= (100 \text{ km/h})(\sin 25°) \\ &= 42.3 \text{ km/h} \end{aligned}$$

$$\begin{aligned} R_x &= R \cos \theta \\ v_x &= (100 \text{ km/h})(\cos 25°) \\ &= 90.6 \text{ km/h} \end{aligned}$$

Paraphrase

The north component of the car's velocity is 42.3 km/h and the east component is 90.6 km/h.

Practice Problems

1. A hiker's displacement is 15 km [40° E of N]. What is the north component of his displacement?
2. A cyclist's velocity is 10 m/s [245°]. Determine the x and y components of her velocity.
3. A snowmobile travels 65 km [37° E of S]. How far east does it travel?

Answers

1. 11 km [N]
2. $v_x = -4.2$ m/s, $v_y = -9.1$ m/s
3. 39 km [E]

Concept Check

For a vector $\vec{R}$ in quadrant I (Cartesian method), are R_x and R_y always positive? Determine whether R_x and R_y are positive or negative for vectors in quadrants II, III, and IV. Display your answers in a chart.

Adding Vectors Using Components

You can write the magnitude of any two-dimensional vector as the sum of its x and y components. Note that x and y components are perpendicular. Because motion along the x direction is perpendicular to motion along the y direction, a change in one component does not affect the other component. Whether it is movement across a soccer field or any other type of two-dimensional motion, you can describe the motion in terms of x and y components.

In Example 2.4, you learned how to determine the x and y components, R_x and R_y, for a general vector $\vec{R}$. In some situations, you already know R_x and R_y, and you must find the magnitude and direction of the resultant vector $\vec{R}$. For example, a toy moves 9.0 m right and then 12.0 m across a classroom floor (Figure 2.31). What is the toy's displacement? Solving this problem algebraically requires two steps:

Step 1: Find the magnitude of $\vec{R}$.

To find the magnitude of the resultant vector, use the Pythagorean theorem. You can use this theorem because the two components, R_x and R_y, form a right triangle with the resultant vector. You are given that $R_x = 9.0$ m and $R_y = 12.0$ m.

$$R^2 = R_x^2 + R_y^2$$

$$R = \sqrt{(9.0\ \text{m})^2 + (12.0\ \text{m})^2}$$

$$= 15\ \text{m}$$

Step 2: Find the angle of $\vec{R}$.

To find the angle of $\vec{R}$, use the tangent function:

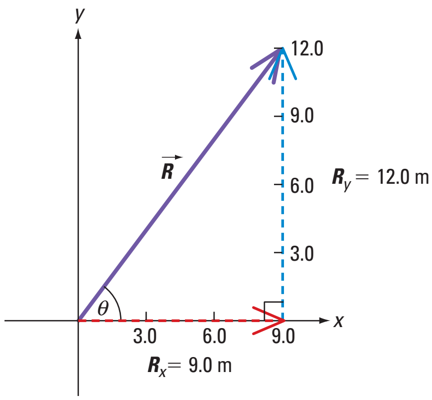
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{12.0\ \cancel{\text{m}}}{9.0\ \cancel{\text{m}}}$$

$$= 1.33$$

$$\theta = \tan^{-1}(1.33)$$

$$= 53.1°$$



▲ **Figure 2.31** Vector components of the movement of a toy across a classroom floor

Using the polar coordinates method, the resultant vector direction is [53.1°]. Using the navigator method, the direction is [53.1° N of E].

SKILLS PRACTICE Using Components

1. Find R_x and R_y for the following vectors:
   (a) A boat travelling at 15 km/h [45° N of W]
   (b) A plane flying at 200 km/h [25° E of S]
   (c) A mountain bike travelling at 10 km/h [N]

2. Find $\vec{R}$ and θ for the following R_x and R_y values:
   (a) $R_x = 12$ m, $R_y = 7$ m
   (b) $R_x = 40$ km/h, $R_y = 55$ km/h
   (c) $R_x = 30$ cm, $R_y = 10$ cm

▲ **Figure 2.32** The movement of the players and the ball in a lacrosse game could be tracked using vectors.

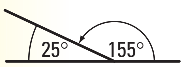
In general, most vector motion involves adding non-collinear vectors. Consider the following scenario. During a lacrosse game, players pass the ball from one person to another (Figure 2.32). The ball can then be redirected for a shot on goal. Each of the displacements could involve different angles. In order to find the net displacement, you would use the following sequence of calculations.

Four Steps for Adding Non-collinear Vectors Algebraically

1. Determine the x and y components of each vector.
2. Add all components in the x direction. Add all components in the y direction. The sums of the x and y components are the two (perpendicular) components of the resultant vector.
3. To find the magnitude of the resultant vector, use the Pythagorean theorem.
4. To find the angle of the resultant vector, use trigonometric ratios. (See Physics Insight on page 83.)

PHYSICS INSIGHT

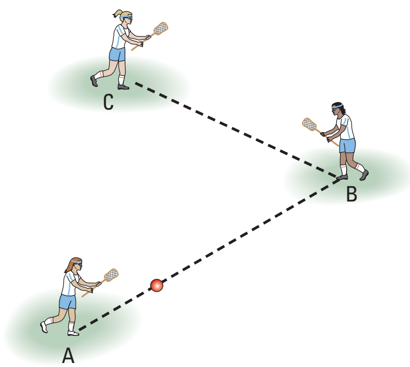
To simplify calculations for finding components, use acute ($< 90°$) angles. To determine the acute angle when given an obtuse ($> 90°$) angle, subtract the obtuse angle from 180°.



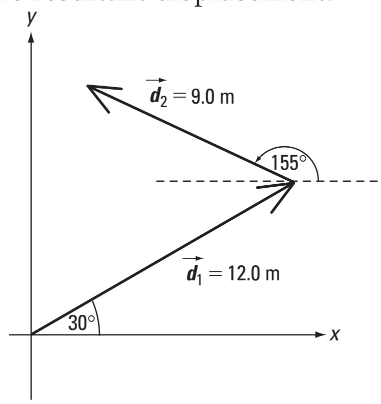
$180° - 155° = 25°$
For an angle greater than 180°, subtract 180° from the angle. For example, $240° - 180° = 60°$

The following example illustrates how to apply these steps.

In a lacrosse game (Figure 2.33(a)), player A passes the ball 12.0 m to player B at an angle of 30°. Player B relays the ball to player C, 9.0 m away, at an angle of 155°. Find the ball's resultant displacement.



▲ **Figure 2.33(a)** The path of the ball on the lacrosse field



▲ **Figure 2.33(b)** The path of the ball as vectors

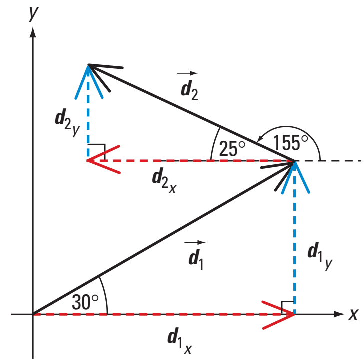
Figure 2.33(b) shows the path of the lacrosse ball as vectors. This problem is different from previous examples because the two vectors are not at right angles to each other. Even with this difference, you can follow the same general steps to solve the problem.

Step 1: Determine the x and y components of each vector.
Since you are solving for displacement, resolve each displacement vector into its components (Figure 2.34). Table 2.2 shows how to calculate the x and y components. In this case, designate up and right as positive directions.

▼ **Table 2.2** Resolution of Components in Figure 2.34

x direction	*y* direction
$d_{1_x} = (12.0\ \text{m})(\cos 30°)$ $= 10.39\ \text{m}$	$d_{1_y} = (12.0\ \text{m})(\sin 30°)$ $= 6.00\ \text{m}$
$d_{2_x} = -(9.0\ \text{m})(\cos 25°)$ $= -8.16\ \text{m}$	$d_{2_y} = (9.0\ \text{m})(\sin 25°)$ $= 3.80\ \text{m}$

(Note that d_{2_x} is negative because it points to the left, and up and right were designated as positive.)

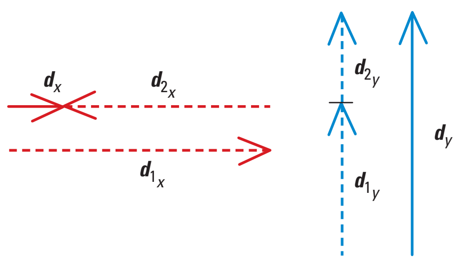


▲ **Figure 2.34** The path of the lacrosse ball

Step 2: Add the *x* components and the *y* components separately.
Add all the *x* components together, then add all the *y* components (see Table 2.3 and Figure 2.35).

▼ **Table 2.3** Adding *x* and *y* Components in Figure 2.35

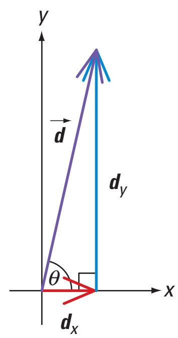
x direction	*y* direction
$d_x = d_{1_x} + d_{2_x}$ $= 10.39\ \text{m} + (-8.16\ \text{m})$ $= 10.39\ \text{m} - 8.16\ \text{m}$ $= 2.23\ \text{m}$	$d_y = d_{1_y} + d_{2_y}$ $= 6.00\ \text{m} + 3.80\ \text{m}$ $= 9.80\ \text{m}$



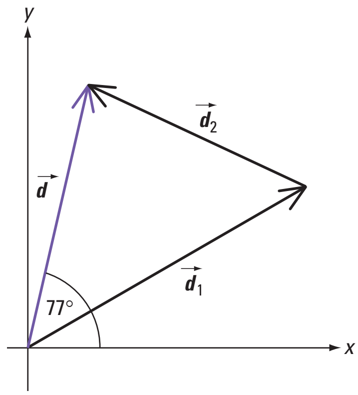
▲ **Figure 2.35** Add the *x* and *y* components separately first to obtain two perpendicular vectors.

Step 3: Find the magnitude of the resultant, $\vec{d}$.
To find the magnitude of the resultant, use the Pythagorean theorem (Figure 2.36).

$$
\begin{aligned}
d^2 &= (d_x)^2 + (d_y)^2 \\
d &= \sqrt{(d_x)^2 + (d_y)^2} \\
&= \sqrt{(2.23\ \text{m})^2 + (9.80\ \text{m})^2} \\
&= 10\ \text{m}
\end{aligned}
$$



▲ **Figure 2.36** The component method allows you to convert non-perpendicular vectors into perpendicular vectors that you can then combine using the Pythagorean theorem.



▲ **Figure 2.37** $\vec{d}$ is the resultant displacement of the ball.

Step 4: Find the angle of $\vec{d}$.
Use the tangent function to find the angle (Figure 2.36).

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{9.80\ \cancel{\text{m}}}{2.23\ \cancel{\text{m}}}\\ &= 4.39\end{aligned}$$

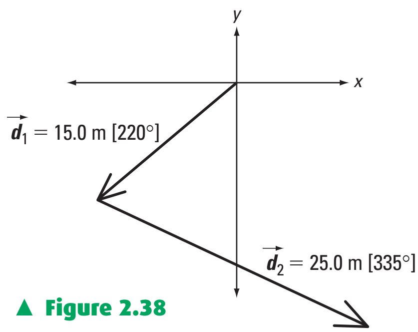
$$\begin{aligned}\theta &= \tan^{-1}(4.39)\\ &= 77°\end{aligned}$$

The ball's displacement is, therefore, 10 m [77°], as shown in Figure 2.37.

The following example illustrates another situation where the displacement vectors are not at right angles.

Example 2.5

Use components to determine the displacement of a cross-country skier who travelled 15.0 m [220°] and then 25.0 m [335°] (Figure 2.38).

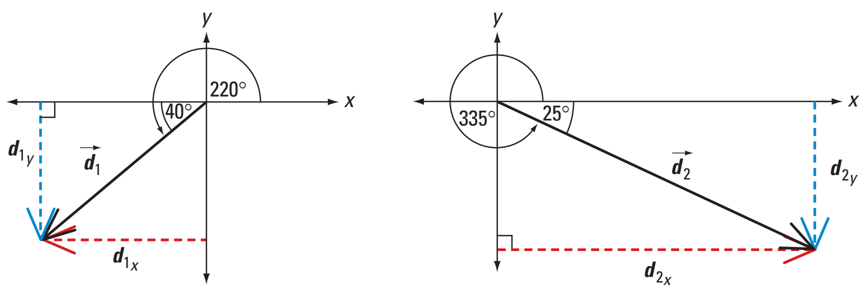


▲ **Figure 2.38**

Given
$\vec{d}_1 = 15.0$ m [220°]
$\vec{d}_2 = 25.0$ m [335°]

Required
displacement ($\vec{d}$)

Analysis and Solution
Step 1: Use $R_x = R\cos\theta$ and $R_y = R\sin\theta$ to resolve each vector into its x and y components. Designate up and to the right as positive. Work with acute angles (Figure 2.39).



▲ **Figure 2.39**

Practice Problems

1. Find the displacement of a farmer who walked 80.0 m [0°] and then 60.0 m [335°].
2. Find the displacement of a soccer player who runs 15 m [15° N of E] and then 13 m [5° W of N].
3. While tracking a polar bear, a wildlife biologist travels 300 m [S] and then 550 m [75° N of E]. What is her displacement?

Answers

1. 137 m [349°]
2. 21 m [52° N of E]
3. 272 m [58° N of E]

x direction:

$d_{1_x} = -(15.0 \text{ m})(\cos 40°)$
$= -11.49 \text{ m}$

$d_{2_x} = (25.0 \text{ m})(\cos 25°)$
$= 22.66 \text{ m}$

y direction:

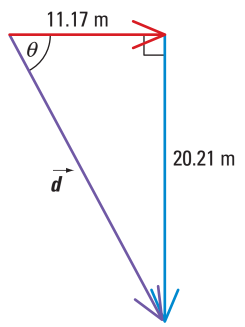
$d_{1_y} = -(15.0 \text{ m})(\sin 40°)$
$= -9.642 \text{ m}$

$d_{2_y} = -(25.0 \text{ m})(\sin 25°)$
$= -10.57 \text{ m}$

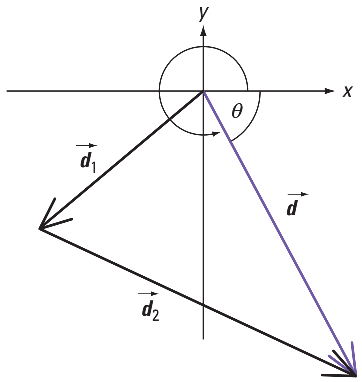
Step 2: Add the x and y components.

$d_x = d_{1_x} + d_{2_x}$
$= -11.49 \text{ m} + 22.66 \text{ m}$
$= 11.17 \text{ m}$

$d_y = d_{1_y} + d_{2_y}$
$= -9.642 \text{ m} + (-10.57 \text{ m})$
$= -20.21 \text{ m}$



▲ **Figure 2.40**



▲ **Figure 2.41**

Step 3: To find the magnitude of the resultant, calculate d using the Pythagorean theorem (Figure 2.40).

$$d^2 = (d_x)^2 + (d_y)^2$$
$$= (11.17 \text{ m})^2 + (20.21 \text{ m})^2$$
$$d = \sqrt{(11.17 \text{ m})^2 + (20.21 \text{ m})^2}$$
$$= 23.09 \text{ m}$$

Figure 2.41 shows that the resultant lies in quadrant IV.

Step 4: To find the angle, use the tangent function (see Figure 2.40).

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{20.21 \cancel{\text{m}}}{11.17 \cancel{\text{m}}}$$
$$= 1.810$$
$$\theta = \tan^{-1}(1.810)$$
$$= 61°$$

From Figure 2.41, note that the angle, θ, lies below the positive x-axis. Using the polar coordinates method, the angle is 299°.

Paraphrase

The cross-country skier's displacement is 23.1 m [299°].

eSIM

Practise the numerical addition of two or more vectors. Follow the eSim links at www.pearsoned.ca/school/physicssource.

In summary, in order to solve a two-dimensional motion problem, you need to split the motion into two one-dimensional problems by using the vectors' x and y components. Then add the x and y components separately. To find the magnitude of the resultant, use the Pythagorean theorem. To find the angle of the resultant, use the tangent function.

2.2 Check and Reflect

Knowledge

1. What trigonometric functions can be used to determine the x or horizontal component of a vector? Draw diagrams to illustrate your answers.

2. Are the following statements true or false? Justify your answer.

(a) The order in which vectors are added is important.

(b) Displacement and distance are always equal.

3. Describe when you would use the navigator method to indicate the direction of a vector.

Applications

4. A student has created a short computer program that calculates components of vectors drawn with a computer mouse. To demonstrate his program, he drags the mouse to create a vector at 55 cm [30° W of S]. What are the components of the vector?

5. Determine the distance travelled and the displacement for each of the following.

(a) Blading through Fish Creek Park in Calgary takes you 5.0 km [W], 3.0 km [N], 2.0 km [E], and 1.5 km [S].

(b) A swimmer travels in a northerly direction across a 500-m-wide lake. Once across, the swimmer notices that she is 150 m east of her original starting position.

(c) After leaving her cabin, a camper snowshoes 750 m [90°] and then 2.20 km [270°].

6. A boat sails 5.0 km [45° W of N]. It then changes direction and sails 7.0 km [45° S of E]. Where does the boat end up with reference to its starting point?

7. A pellet gun fires a pellet with a velocity of 355 m/s [30°]. What is the magnitude of the vertical component of the velocity at the moment the pellet is fired?

8. Tourists on a jet ski move 1.20 km [55° N of E] and then 3.15 km [70° S of E]. Determine the jet ski's displacement.

9. A jogger runs with a velocity of 6.0 km/h [25° N of W] for 35 min and then changes direction, jogging for 20 min at 4.5 km/h [65° E of N]. Using a vector diagram, determine the jogger's total displacement and his average velocity for the workout.

10. Given that a baseball diamond is a square, assume that the first-base line is the horizontal axis. On second base, a baseball player's displacement from home plate is 38 m [45°].

(a) What are the components of the player's displacement from home plate?

(b) Has the runner standing on second base travelled a distance of 38 m? Why or why not?

11. Determine the resultant displacement of a skateboarder who rolls 45.0 m [310°] and 35.0 m [135°].

eTEST

To check your understanding of two-dimensional motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

2.3 Relative Motion

Conveyor belts in the oil sands mines of Northern Alberta are 50 km long (Figure 2.42). Every hour, they move 25 200 t of oil sand from the mine to the extraction plant. What is the speed of the oil sand with respect to the conveyor belt? How fast is the oil sand moving with respect to the ground? How fast is it moving relative to a 21 240-t mechanical drive truck going in the opposite direction?

▲ **Figure 2.42** A conveyor belt represents an example of uniform and relative motion.

Sometimes objects move within a medium that is itself moving. Wind (moving air) affects the motion of objects such as kites, sailboats, and airplanes. Current (moving water) affects the motion of watercraft, wildlife, and swimmers. An Olympic kayak competitor who can paddle with a speed of 5.0 m/s in still water may appear to be going faster to an observer on shore if she is paddling in the same direction as the current. In this case, the velocity of the moving object depends on the location of the observer: whether the observer is on the moving object or observing the moving object from a stationary position. **Relative motion** is motion measured with respect to an observer.

relative motion: motion measured with respect to an observer

Concept Check

An observer is on a train moving at a velocity of 25 m/s [forward]. A ball rolls at 25 m/s [forward] with respect to the floor of the moving train. What is the velocity of the ball relative to the observer on the train? What is the velocity of the ball relative to an observer standing on the ground? What happens if the ball moves 25 m/s [backward]?

How does a moving medium affect the motion of a table tennis ball?

2-3 QuickLab

Table Tennis in the Wind

Problem

How does air movement affect the motion of a table tennis ball?

Materials

large upright fan
table tennis table
paddles
table tennis ball

Procedure

1. With a partner, practise hitting the table tennis ball to each other (Figure 2.43).
2. Set up the fan on one side of the table tennis table.
3. Hit the table tennis ball straight across the length of the table
   (a) against the wind
   (b) with the wind
   (c) perpendicular to the wind's direction
4. Record how the moving air influences the motion of the ball in each case.

▲ **Figure 2.43**

Questions

1. When did the ball move the fastest? the slowest?
2. When the air movement was perpendicular to the ball's path, did it change the ball's speed? Did it change the ball's velocity? Explain.
3. Given your results, speculate as to why golfers release a tuft of grass into the wind before driving the ball.
4. Describe how wind direction might influence a beach volleyball player's serve.

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Relative Motion in the Air

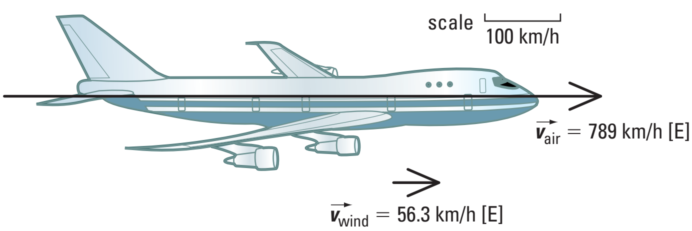
ground velocity: velocity relative to an observer on the ground

air velocity: an object's velocity relative to still air

wind velocity: velocity of the wind relative to the ground

A flight from Edmonton to Toronto takes about 3.5 h. The return flight on the same aircraft takes 4.0 h. If the plane's air speed is the same in both directions, why does the trip east take less time? The reason is that, when travelling eastward from Edmonton, a tailwind (a wind that blows from the rear of the plane, in the same direction as the plane's motion) increases the airplane's **ground velocity** (velocity relative to an observer on the ground), hence reducing the time of travel and, therefore, fuel consumption and cost.

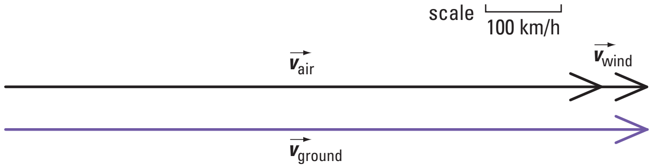
A Canadian regional jet travels with an **air velocity** (the plane's velocity in still air) of 789 km/h [E]. The jet encounters a **wind velocity** (the wind's velocity with respect to the ground) of 56.3 km/h [E] (Figure 2.44). (This wind is a west wind, blowing eastward from the west.) What is the velocity of the airplane relative to an observer on the ground? The resultant velocity of the airplane, or ground velocity, is the vector sum of the plane's air velocity and the wind velocity (Figure 2.45). Let the positive direction be east.



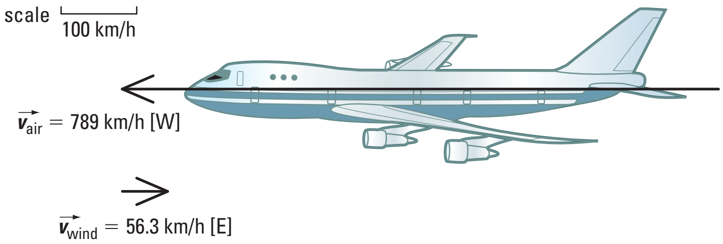
▲ **Figure 2.44** The air velocity and wind velocity are in the same direction.

$$\begin{aligned}\vec{v}_{\text{ground}} &= \vec{v}_{\text{air}} + \vec{v}_{\text{wind}} \\ &= +789\ \text{km/h} + 56.3\ \text{km/h} \\ &= +845\ \text{km/h}\end{aligned}$$

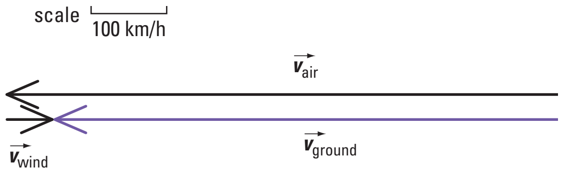
The sign is positive, so the ground velocity is 845 km/h [E].



▲ **Figure 2.45**



▲ **Figure 2.46** The air velocity and wind velocity are in opposite directions.



▲ **Figure 2.47**

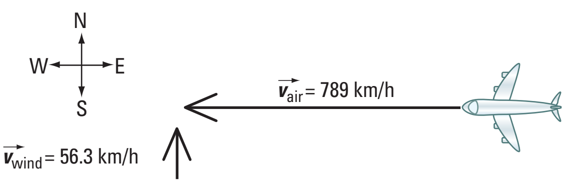
If the jet heads west, from Toronto to Edmonton (Figure 2.46), its resultant velocity becomes

$$\begin{aligned}\vec{v}_{\text{ground}} &= \vec{v}_{\text{air}} + \vec{v}_{\text{wind}} \\ &= -789\ \text{km/h} + 56.3\ \text{km/h} \\ &= -733\ \text{km/h}\end{aligned}$$

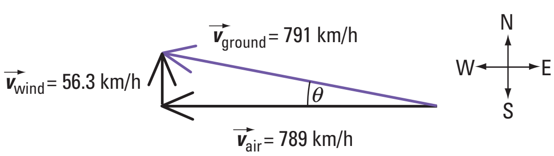
(See Figure 2.47.) The sign is negative, so the ground velocity is 733 km/h [W]. The plane's speed decreases due to the headwind (wind that approaches from the front).

Non-collinear Relative Motion

Suppose the jet travelling west from Toronto encounters a crosswind of 56.3 km [N] (Figure 2.48).



▲ **Figure 2.48** A plane flies in a crosswind.



▲ **Figure 2.49** A plane that flies in a crosswind needs to adjust its direction of motion.

In this case, the velocity of the plane is not aligned with the wind's velocity. The defining equation for this case is still the same as for the collinear case: $\vec{v}_{\text{ground}} = \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}$. From section 2.2, recall that the plus sign in a two-dimensional vector equation tells you to connect the vectors tip to tail. The resultant vector is the ground velocity, $\vec{v}_{\text{ground}}$. The ground velocity indicates the actual path of the plane (Figure 2.49).

To solve for the ground velocity, notice that the triangle formed is a right triangle, meaning that you can use the Pythagorean theorem to solve for the magnitude of the ground velocity.

$$(v_{\text{ground}})^2 = (v_{\text{air}})^2 + (v_{\text{wind}})^2$$

$$\begin{aligned} v_{\text{ground}} &= \sqrt{(v_{\text{air}})^2 + (v_{\text{wind}})^2} \\ &= \sqrt{(789 \text{ km/h})^2 + (56.3 \text{ km/h})^2} \\ &= 791 \text{ km/h} \end{aligned}$$

PHYSICS INSIGHT

To determine the angle, substitute the magnitudes of the relative velocities into the tangent function. To determine the direction, refer to the vector diagram for the problem.

Using the tangent function, the direction of the ground velocity is

$$\begin{aligned} \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{56.3 \cancel{\text{km/h}}}{789 \cancel{\text{km/h}}} \\ &= 0.07136 \\ \theta &= \tan^{-1}(0.07136) \\ &= 4.1° \end{aligned}$$

From Figure 2.49, the wind blows the airplane off its westerly course in the northerly direction. Hence, the airplane's ground velocity is 791 km/h [4.1° N of W]. The pilot must take into consideration the effect of the wind blowing the airplane off course to ensure that the plane reaches its destination.

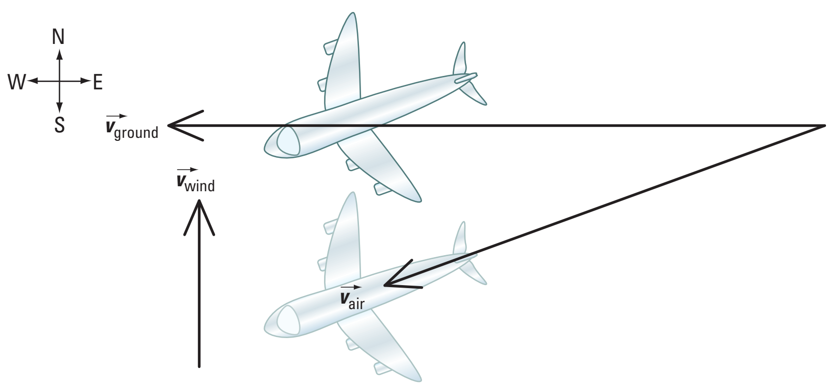
What path would the pilot have to take to arrive at a point due west of the point of departure? Remember that, if the vectors are not perpendicular, resolve them into components first before adding them algebraically.

Example 2.6

A plane flies west from Toronto to Edmonton with an air speed of 789 km/h.

(a) Find the direction the plane would have to fly to compensate for a wind velocity of 56.3 km/h [N].

(b) Find the plane's speed relative to the ground.



▲ **Figure 2.50**

Given

$\vec{v}_{\text{wind}} = 56.3$ km/h [N]

$\vec{v}_{\text{air}} = 789$ km/h

direction of ground velocity is west

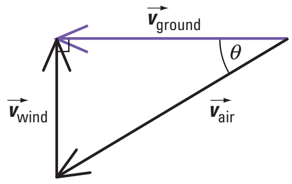
Required

(a) the plane's direction (direction of air velocity)

(b) ground speed (v_{ground})

Analysis and Solution

(a) First construct a diagram based on the defining equation,



▲ **Figure 2.51**

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}$$

The rules of vector addition tell you to connect the vectors $\vec{v}_{\text{air}}$ and $\vec{v}_{\text{wind}}$ tip to tail.

To find the direction required in order to compensate for the wind velocity, find the angle, θ. Because the connection of the vectors forms a right triangle, and you know the magnitude of the opposite side ($\vec{v}_{\text{wind}}$) and the hypotenuse ($\vec{v}_{\text{air}}$), you can use the sine function to find the angle (Figure 2.51).

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\theta = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$$

$$= \sin^{-1}\left(\frac{56.3\ \cancel{\text{km/h}}}{789\ \cancel{\text{km/h}}}\right)$$

$$= 4.1°$$

From Figure 2.51, the angle is [4.1° S of W].

(b) To find the magnitude of the ground velocity, use the Pythagorean theorem. From Figure 2.51, note that the hypotenuse in this case is the air velocity, $\vec{v}_{\text{air}}$.

$$(v_{\text{air}})^2 = (v_{\text{wind}})^2 + (v_{\text{ground}})^2$$

$$(v_{\text{ground}})^2 = (v_{\text{air}})^2 - (v_{\text{wind}})^2$$

$$= (789\ \text{km/h})^2 - (56.3\ \text{km/h})^2$$

$$= 6.1935 \times 10^5\ (\text{km/h})^2$$

$$v_{\text{ground}} = 787\ \text{km/h}$$

Notice that there is a small change in the magnitude of the ground velocity from the previous example of the plane heading west. As the magnitude of the wind velocity increases, the magnitude of the ground velocity and the compensating angle will significantly change.

Paraphrase

(a) The plane's heading must be [4.1° S of W].

(b) The plane's ground speed is 787 km/h.

Practice Problems

1. A swimmer can swim at a speed of 1.8 m/s. The river is 200 m wide and has a current of 1.2 m/s [W]. If the swimmer points herself north, directly across the river, find
   (a) her velocity relative to the ground.
   (b) the time it takes her to cross.
2. For a river flowing west with a current of 1.2 m/s, a swimmer decides she wants to swim directly across. If she can swim with a speed of 1.8 m/s, find
   (a) the angle at which she must direct herself.
   (b) the time it takes her to cross if the river is 200 m wide.

Answers

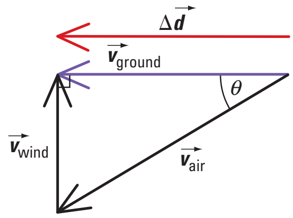
1. (a) 2.2 m/s [34° W of N]
   (b) 1.1×10^2 s
2. (a) [42° E of N]
   (b) 1.5×10^2 s

PHYSICS INSIGHT

For simplicity, Edmonton was assumed to be directly west of Toronto, which, of course, it is not! However, the calculation is still valid because this problem involves straight-line motion.

To calculate the time it takes to fly from Toronto to Edmonton, use the equation $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$. The distance between Edmonton and Toronto is about 2335 km, but you must decide which value for velocity to use: air velocity or ground velocity. The displacement and velocity vectors in this equation must be aligned.

Since you are assuming that displacement is in the west direction, the appropriate velocity to use is the one that is in the westerly direction. In this example, it is the ground velocity, $\vec{v}_{\text{ground}}$ (Figure 2.52).



◀ **Figure 2.52** For calculating time, choose the velocity vector that matches the direction of the displacement vector.

Consider west to be positive. Since both vectors are in the same direction (west), use the scalar form of the equation to solve for time.

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{2335\ \cancel{\text{km}}}{787\ \frac{\cancel{\text{km}}}{\text{h}}} \\ &= 2.97\ \text{h}\end{aligned}$$

It takes 2.97 h to fly from Toronto to Edmonton.

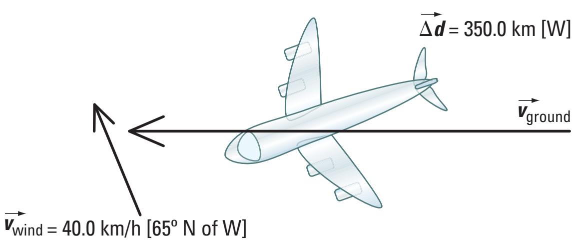
In the following example, the three velocity vectors do not form a right triangle. In order to solve the problem, you will need to use components.

PHYSICS INSIGHT

When substituting values into a vector equation, make sure that the values have the same direction (are collinear). If the vectors are not collinear, you need to use graphical or algebraic methods to find the answer.

Example 2.7

As a pilot of a small plane, you need to transport three people to an airstrip 350.0 km due west in 2.25 h. If the wind is blowing at 40.0 km/h [65° N of W], what should be the plane's air velocity in order to reach the airstrip on time?



▲ **Figure 2.53**

Given

$\vec{v}_{\text{wind}} = 40.0$ km/h [65° N of W]

$\Delta\vec{d} = 350.0$ km [W]

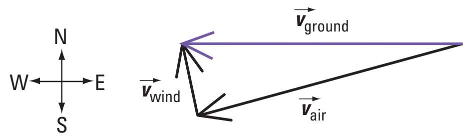
$\Delta t = 2.25$ h

Required

plane's air velocity ($\vec{v}_{\text{air}}$)

Analysis and Solution

First draw a vector diagram of the problem (Figure 2.54).



▲ **Figure 2.54**

Designate north and west as the positive directions. Then calculate the ground velocity from the given displacement and time.

If the plane must fly 350.0 km [W] in 2.25 h, its ground velocity is

$$\begin{aligned}\vec{v}_{\text{ground}} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{350.0 \text{ km [W]}}{2.25 \text{ h}}\\ &= 155.6 \text{ km/h [W]}\end{aligned}$$

Now find the components of the wind velocity (Figure 2.55).

x direction:

$$\begin{aligned}v_{\text{wind}_x} &= (40.0 \text{ km/h})(\cos 65°)\\ &= 16.9 \text{ km/h}\end{aligned}$$

y direction:

$$\begin{aligned}v_{\text{wind}_y} &= (40.0 \text{ km/h})(\sin 65°)\\ &= 36.25 \text{ km/h}\end{aligned}$$

The ground velocity is directed west, so its x component is 155.6 km/h and its y component is zero.

Since $\vec{v}_{\text{ground}} = \vec{v}_{\text{air}} + \vec{v}_{\text{wind}}$, rearrange this equation to solve for $\vec{v}_{\text{air}}$.

$$\vec{v}_{\text{air}} = \vec{v}_{\text{ground}} - \vec{v}_{\text{wind}}$$

Use this form of the equation to solve for the components of the air velocity.

Add the x (west) components:

$$\begin{aligned}v_{\text{air}_x} &= v_{\text{ground}_x} - v_{\text{wind}_x}\\ &= 155.6 \text{ km/h} - 16.9 \text{ km/h}\\ &= 138.7 \text{ km/h}\end{aligned}$$

Add the y (north) components:

$$\begin{aligned}v_{\text{air}_y} &= v_{\text{ground}_y} - v_{\text{wind}_y}\\ &= 0 - 36.25 \text{ km/h}\\ &= -36.25 \text{ km/h}\end{aligned}$$

Use the Pythagorean theorem to find the magnitude of the air velocity.

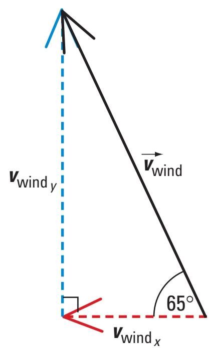
$$\begin{aligned}v_{\text{air}} &= \sqrt{\left(v_{\text{air}_x}\right)^2 + \left(v_{\text{air}_y}\right)^2}\\ &= \sqrt{(138.7 \text{ km/h})^2 + (36.25 \text{ km/h})^2}\\ &= 143 \text{ km/h}\end{aligned}$$

Practice Problems

1. An airplane can fly with a maximum air velocity of 750 km/h [N]. If the wind velocity is 60 km/h [15° E of N], what must be the plane's ground velocity if it is to remain on a course going straight north?
2. What is the air velocity of a jetliner if its ground velocity is 856 km/h [25.0° W of S] and the wind velocity is 65.0 km/h [S]?
3. How long will it take a plane to travel 100 km [N] if its ground velocity is 795 km/h [25° W of N]?

Answers

1. 8.1×10^2 km/h [1° W of N]
2. 798 km/h [27.0° W of S]
3. 0.139 h



▲ **Figure 2.55**

To find the direction of air velocity, use the tangent function (Figure 2.56).

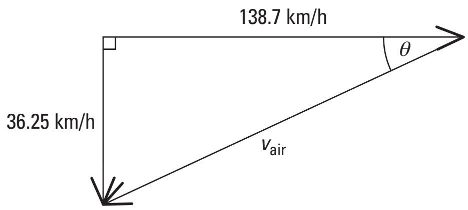
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{36.25 \cancel{\text{ km/h}}}{138.7 \cancel{\text{ km/h}}}$$

$$= 0.261$$

$$\theta = \tan^{-1}(0.261)$$

$$= 15°$$



▲ **Figure 2.56**

The x component is positive, so its direction is west. Since the y component is negative, its direction is to the south. Thus, the direction of the air velocity is [15° S of W].

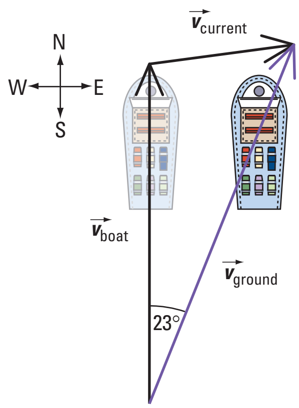
Paraphrase
The airplane's air velocity is 143 km/h [15° S of W].

Relative Motion in the Water

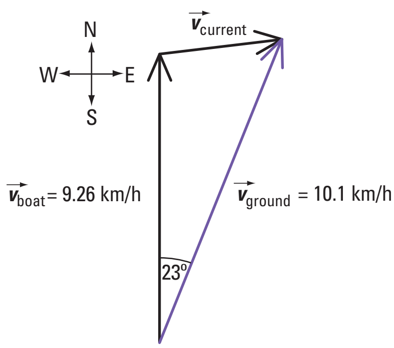
Whereas wind velocity affects the speed and direction of flying objects, watercraft and swimmers experience currents. As with flying objects, an object in the water can move with the current (ground velocity increases), against the current (ground velocity decreases), or at an angle (ground velocity increases or decreases). When the object moves at an angle to the current that is not 90°, both the object's speed and direction change. The following example illustrates how to use components to find velocity.

Example 2.8

The *Edmonton Queen* paddleboat travels north on the Saskatchewan River at a speed of 5.00 knots or 9.26 km/h. If the *Queen*'s ground velocity is 10.1 km/h [23° E of N], what is the velocity of the Saskatchewan River?



▲ **Figure 2.57(a)**



▲ **Figure 2.57(b)**

Given

$\vec{v}_{\text{boat}} = 9.26 \text{ km/h [N]}$

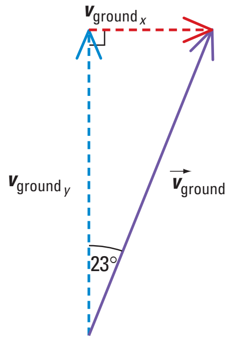
$\vec{v}_{\text{ground}} = 10.1 \text{ km/h [23° E of N]}$

(Note that the angle is given with respect to the vertical (y) axis (Figure 2.57(a).)

Required

velocity of current ($\vec{v}_{\text{current}}$)

Analysis and Solution



▲ **Figure 2.58(a)**

Let north and east be positive.

Calculate the current's velocity using components.

First find the components of the ground velocity (Figure 2.58(a)).

x direction:

$$\begin{aligned} v_{\text{ground}_x} &= v_{\text{ground}} \sin\theta \\ &= (10.1 \text{ km/h})(\sin 23°) \\ &= 3.946 \text{ km/h} \end{aligned}$$

y direction:

$$\begin{aligned} v_{\text{ground}_y} &= v_{\text{ground}} \cos\theta \\ &= (10.1 \text{ km/h})(\cos 23°) \\ &= 9.297 \text{ km/h} \end{aligned}$$

Since the boat's velocity is directed north, its y component is 9.26 km/h and its x component is zero.

$$\vec{v}_{\text{ground}} = \vec{v}_{\text{boat}} + \vec{v}_{\text{current}}$$

You are asked to find $\vec{v}_{\text{current}}$, so rearrange the vector equation accordingly:

$$\vec{v}_{\text{current}} = \vec{v}_{\text{ground}} - \vec{v}_{\text{boat}}$$

Use this form of the equation to solve for the components of the current's velocity.

$$\begin{aligned} v_{\text{current}_x} &= v_{\text{ground}_x} - v_{\text{boat}_x} \\ &= 3.946 \text{ km/h} - 0 \\ &= 3.946 \text{ km/h} \end{aligned}$$

$$\begin{aligned} v_{\text{current}_y} &= v_{\text{ground}_y} - v_{\text{boat}_y} \\ &= 9.297 \text{ km/h} - 9.26 \text{ km/h} \\ &= 0.037 \text{ km/h} \end{aligned}$$

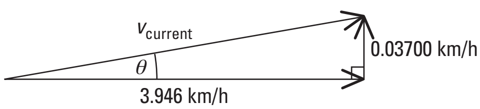
To find the magnitude of the current's velocity, use the Pythagorean theorem.

$$\begin{aligned} v_{\text{current}} &= \sqrt{(v_{\text{current}_x})^2 + (v_{\text{current}_y})^2} \\ &= \sqrt{(3.946 \text{ km/h})^2 + (0.037 \text{ km/h})^2} \\ &= 3.946 \text{ km/h} \end{aligned}$$

To find the direction of the current's velocity, use the tangent function (Figure 2.58(b)).

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{0.037 \cancel{\text{km/h}}}{3.946 \cancel{\text{km/h}}}\right) \\ &= 0.5° \end{aligned}$$



▲ **Figure 2.58(b)**

Since both the x and y components are positive, the directions are east and north, respectively. Therefore, the current's direction is [0.5° N of E].

Paraphrase

The current's velocity is 3.95 km/h [0.5° N of E].

Practice Problems

1. Determine a Sea Doo's ground velocity if it travels with a constant velocity of 4.50 m/s [W] and encounters a current of 2.0 m/s [20° W of N].
2. A jogger runs with a velocity of 3.75 m/s [20° N of E] on an Alaskan cruise ship heading north at 13 m/s. What is the jogger's ground velocity?
3. A ship travelling 55° [W of N] is 65.0 km farther north after 3.0 h. What is the ship's velocity?

Answers

1. 5.5 m/s [20° N of W]
2. 15 m/s [76° N of E]
3. 38 km/h [55° W of N]

In order to find the time required to cross the river, you need to use the velocity value that corresponds to the direction of the object's displacement, as you will see in the next example.

Example 2.9

From Example 2.8, if the river is 200 m wide and the banks run from east to west, how much time, in seconds, does it take for the *Edmonton Queen* to travel from the south bank to the north bank?

Given

$\vec{v}_{\text{ground}} = 10.1 \text{ km/h [23° E of N]}$

width of river = 200 m = 0.200 km

Required

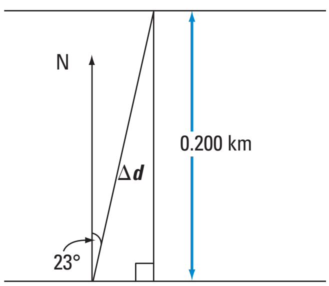
time of travel (Δt)

Analysis and Solution

Determine the distance, Δd, the boat travels in the direction of the boat's ground velocity, 23° E of N. From Figure 2.59,

$$\Delta d = \frac{0.200 \text{ km}}{\cos 23°}$$

$$= 0.2173 \text{ km}$$



▲ **Figure 2.59**

The boat's ground velocity is 10.1 km/h [23° E of N].

$$v_{\text{ground}} = 10.1 \text{ km/h}$$

$$\Delta t = \frac{\Delta d}{v_{\text{ground}}}$$

$$= \frac{0.2173 \text{ km}}{10.1 \text{ km/h}}$$

$$= 0.02151 \cancel{\text{h}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{h}}} \times \frac{60 \text{ s}}{1 \cancel{\text{min}}}$$

$$= 77.4 \text{ s}$$

Paraphrase

It takes the *Edmonton Queen* 77.4 s to cross the river.

Practice Problems

1. A river flows east to west at 3.0 m/s and is 80 m wide. A boat, capable of moving at 4.0 m/s, crosses in two different ways.
   (a) Find the time to cross if the boat is pointed directly north and moves at an angle downstream.
   (b) Find the time to cross if the boat is pointed at an angle upstream and moves directly north.

Answers

1. (a) 20 s
   (b) 30 s

Relative motion problems describe the motion of an object travelling in a medium that is also moving. Both wind and current can affect the magnitude and direction of velocity. To solve relative motion problems in two dimensions, resolve the vectors into components and then add them using trigonometry.

2.3 Check and Reflect

Knowledge

1. Describe a situation where a wind or current will increase an object's ground speed.

2. Describe a situation where a wind or current will change an object's direction relative to the ground, but not its speed in the original direction (i.e., the velocity component in the original, intended direction of motion).

3. Describe a situation when a wind or current will cause zero displacement as seen from the ground.

4. Provide an example other than in the text that illustrates that perpendicular components of motion are independent of one another.

Applications

5. A swimmer needs to cross a river as quickly as possible. The swimmer's speed in still water is 1.35 m/s.
   (a) If the river's current speed is 0.60 m/s and the river is 106.68 m wide, how long will it take the swimmer to cross the river if he swims so that his body is angled slightly upstream while crossing, and he ends up on the far bank directly across from where he started?
   (b) If he points his body directly across the river and is therefore carried downstream, how long will it take to get across the river and how far downstream from his starting point will he end up?

6. A small plane can travel with a speed of 265 km/h with respect to the air. If the plane heads north, determine its resultant velocity if it encounters
   (a) a 32.0-km/h headwind
   (b) a 32.0-km/h tailwind
   (c) a 32.0-km/h [W] crosswind

7. The current in a river has a speed of 1.0 m/s. A woman swims 300 m downstream and then back to her starting point without stopping. If she can swim 1.5 m/s in still water, find the time of her round trip.

8. What is the ground velocity of an airplane if its air velocity is 800 km/h [E] and the wind velocity is 60 km/h [42° E of N]?

9. A radio-controlled plane has a measured air velocity of 3.0 m/s [E]. If the plane drifts off course due to a light wind with velocity 1.75 m/s [25° W of S], find the velocity of the plane relative to the ground. If the distance travelled by the plane was 3.2 km, find the time it took the plane to travel that distance.

10. An airplane is observed to be flying at a speed of 600 km/h. The plane's nose points west. The wind's velocity is 40 km/h [45° W of S]. Find the plane's velocity relative to the ground.

11. A canoe can move at a speed of 4.0 m/s [N] in still water. If the velocity of the current is 2.5 m/s [W] and the river is 0.80 km wide, find
    (a) the velocity of the canoe relative to the ground
    (b) the time it takes to cross the river

eTEST

To check your understanding of relative motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

2.4 Projectile Motion

Sports are really science experiments in action. Consider golf balls, footballs, and tennis balls. All of these objects are projectiles (Figure 2.60). You know from personal experience that there is a relationship between the distance you can throw a ball and the angle of loft. In this section, you will learn the theory behind projectile motion and how to calculate the values you need to throw the fastball or hit the target dead on.

Try the next QuickLab and discover what factors affect the trajectory of a projectile.

Figure 2.60 Sports is projectile motion in action.

2-4 QuickLab

Projectiles

Problem

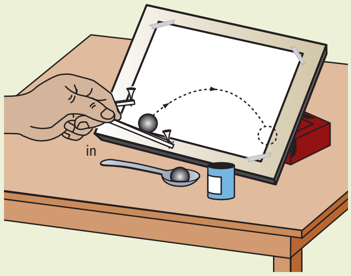
What factors affect the trajectory of a marble?

Materials

wooden board (1 m × 1 m)	two nails
hammer	elastic band
paint	spoon
marble	brick
newspaper	masking tape
white paper to cover the board	gloves

Procedure

1. Spread enough newspaper on the floor so that it covers a larger workspace than the wooden board.
2. Hammer two nails, 7.0 cm apart, at the bottom left corner of the board. Stretch the elastic between them.
3. Cover the board with white paper and affix the paper to the board using masking tape.
4. Prop the board up on the brick (Figure 2.61).
5. Wearing gloves, roll the marble in a spoonful of paint.
6. Pull the elastic band back at an angle and rest the marble in it.
7. Release the elastic band and marble. Label the marble's trajectory on the paper track 1.
8. Repeat steps 5–7 for different launch angles and extensions of the elastic band.



◀ **Figure 2.61**

Questions

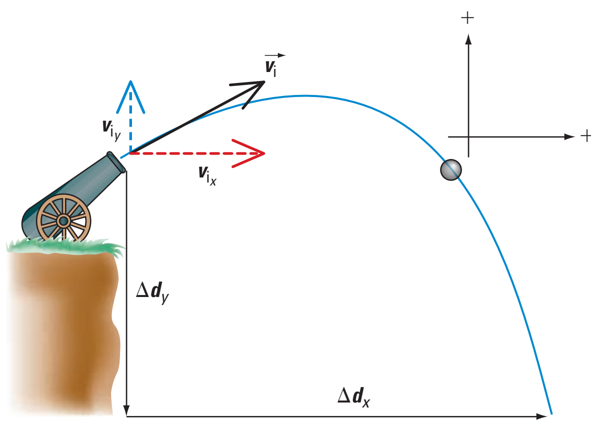
1. What is the shape of the marble's trajectory, regardless of speed and angle?
2. How did a change in the elastic band's extension affect the marble's path?
3. How did a change in launch angle affect the marble's path?

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Galileo studied projectiles and found that they moved in two directions at the same time. He determined that the motion of a projectile, neglecting air resistance, follows the curved path of a parabola. The parabolic path of a projectile is called its **trajectory** (Figure 2.62). The shape of a projectile's trajectory depends on its initial velocity — both its initial speed and direction — and on the acceleration due to gravity. To understand and analyze projectile motion, you need to consider the horizontal (x direction) and vertical (y direction) components of the object's motion separately.

trajectory: the parabolic motion of a projectile



▶ **Figure 2.62** A projectile has a parabolic trajectory.

2-5 QuickLab

Which Lands First?

Problem

What is the relationship between horizontal and vertical motion of objects on a ramp?

Materials

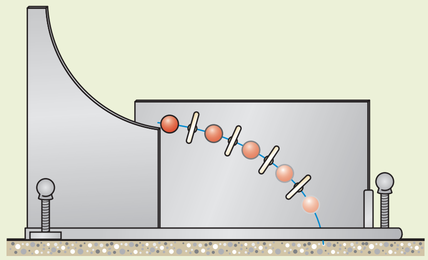
Galileo apparatus (Figure 2.63)
steel balls

Procedure

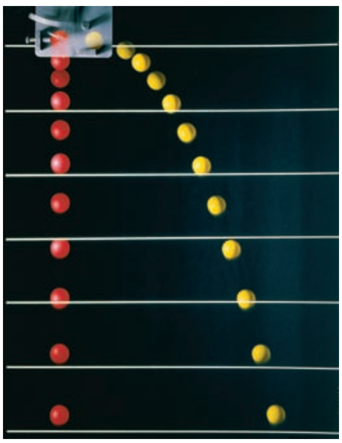
1. Set up the Galileo apparatus at the edge of a lab bench.
2. Place a steel ball at the top of each ramp.
3. Release the balls at the same time.
4. Listen for when each ball hits the ground.
5. Using a different ramp, repeat steps 1–4.

Questions

1. Which ball landed first?
2. Did the balls' initial velocity affect the result? If so, how?
3. What inference can you make about the relationship between horizontal and vertical motion?



▲ **Figure 2.63**



▲ **Figure 2.64** Gravity does not affect the horizontal motion of a projectile because perpendicular components of motion are independent.

PHYSICS INSIGHT

When a projectile is launched, for a fraction of a second, it accelerates from rest to a velocity that has x and y components.

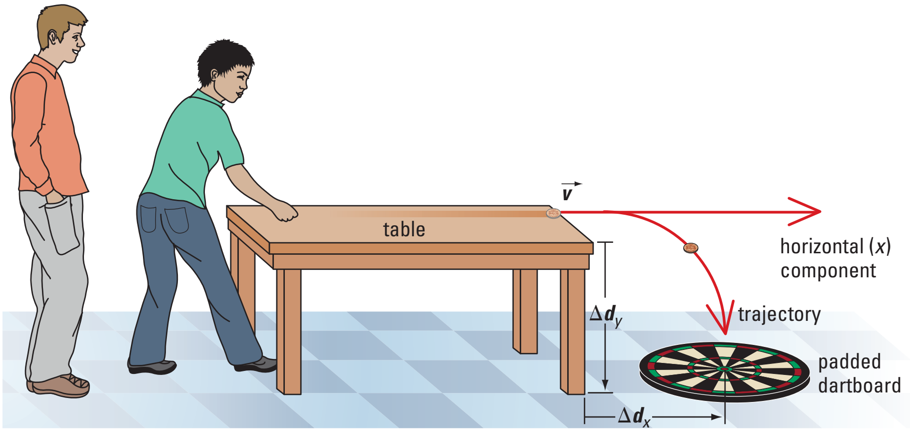
From section 1.6, you know that gravity influences the vertical motion of a projectile by accelerating it downward. From Figure 2.64, note that gravity has no effect on an object's horizontal motion. So, the two components of a projectile's motion can be considered independently. As a result, a projectile experiences both uniform motion and uniformly accelerated motion at the same time! The *horizontal motion* of a projectile is an example of uniform motion; the projectile's horizontal velocity component is constant. The *vertical motion* of a projectile is an example of uniformly accelerated motion. The object's acceleration is the constant acceleration due to gravity or 9.81 m/s^2 [down] (neglecting friction).

Concept Check

In a table, classify the horizontal and vertical components of position, velocity, and acceleration of a horizontally launched projectile as uniform or non-uniform motion.

Objects Launched Horizontally

Suppose you made a new game based on a combination of shuffleboard and darts. The goal is to flick a penny off a flat, horizontal surface, such as a tabletop, and make it land on a target similar to a dartboard beyond the table. The closer your penny lands to the bull's eye, the more points you score (Figure 2.65).



▲ **Figure 2.65** An object launched horizontally experiences uniform horizontal motion and uniformly accelerated vertical motion.

In the game, once the penny leaves the tabletop, it becomes a projectile and travels in a parabolic path toward the ground. In section 1.6, you studied motion that was caused by acceleration due to gravity. The velocity of an object falling straight down has no horizontal velocity component. In this game, the penny moves both horizontally and vertically, like the ball on the right in Figure 2.64. In this type of projectile motion, the object's initial vertical velocity is zero.

Because the projectile has a horizontal velocity component, it travels a horizontal distance along the ground from its initial launch point. This distance is called the projectile's **range** (Figure 2.66). The velocity component in the y direction increases because of the acceleration due to gravity while the x component remains the same. The combined horizontal and vertical motions produce the parabolic path of the projectile.

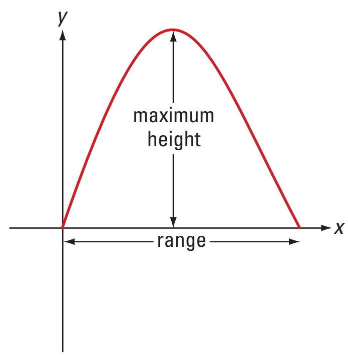
PHYSICS INSIGHT

For a projectile to have a non-zero velocity component in the vertical direction, the object must be thrown up, down, or at an angle relative to the horizontal, rather than sideways.

range: the distance a projectile travels horizontally over level ground

Concept Check

(a) What factors affecting projectile motion in the horizontal direction are being neglected?
(b) What causes the projectile to finally stop?
(c) If the projectile's initial velocity had a vertical component, would the projectile's path still be parabolic? Give reasons for your answer.

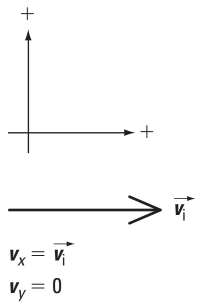


▲ **Figure 2.66** The range of a projectile is its horizontal distance travelled.

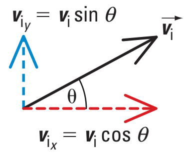
Solving Projectile Motion Problems

In this chapter, you have been working with components, so you know how to solve motion problems by breaking the motion down into its horizontal (x) and vertical (y) components.

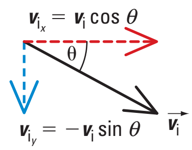
Before you solve a projectile motion problem, review what you already know (Figure 2.67).



▲ **Figure 2.67(a)**
The projectile is given an initial horizontal velocity.



▲ **Figure 2.67(b)**
The projectile is given an initial horizontal velocity and an upward vertical velocity.



▲ **Figure 2.67(c)**
The projectile is given an initial horizontal velocity and a downward vertical velocity.

e MATH

To explore and graph the relationship between the velocity and position of an object thrown vertically into the air, visit www.pearsoned.ca/school/physicssource.

<u>x direction</u>

- There is no acceleration in this direction, so $a_x = 0$. In this text, a_x will always be zero. The projectile undergoes uniform motion in the x direction.
- The general equation for the initial x component of the velocity can be determined using trigonometry, e.g., $v_{i_x} = v_i \cos \theta$.
- The range is Δd_x.
- Because the projectile is moving in both the horizontal and vertical directions at the same time, Δt is a common variable.

<u>y direction</u>

- If up is positive, the acceleration due to gravity is down or negative, so $a_y = -9.81 \text{ m/s}^2$.
- The y component of the initial velocity can be determined using trigonometry, e.g., $v_{i_y} = v_i \sin \theta$.
- The displacement in the y direction is Δd_y.
- Time (Δt) is the same in both the x and y directions.

▼ **Table 2.4** Projectile Problem Setup

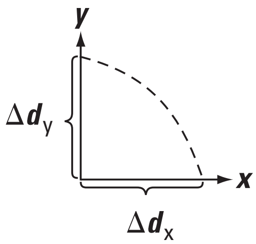
***x* direction**	***y* direction**
$a_x = 0$	$a_y = -9.81 \text{ m/s}^2$
$v_{i_x} = v_i \cos \theta$	$v_{i_y} = v_i \sin \theta$
	v_{i_y} can be positive or negative depending on the direction of $\vec{v}_i$.
$\Delta d_x = v_x \Delta t$	$\Delta d_y = v_{i_y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$

If you check the variables, you can see that they are v_i, Δt, Δd, and a, all of which are present in the equation $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} (\Delta t)^2$. In the horizontal direction, the acceleration is zero, so this equation simplifies to $\Delta \vec{d} = \vec{v}_i \Delta t$. The next example shows you how to apply these equations.

Example 2.10

Head-Smashed-In Buffalo Jump, near Fort Macleod, Alberta, is a UNESCO heritage site (Figure 2.68). Over 6000 years ago, the Blackfoot people of the Plains hunted the North American bison by gathering herds and directing them over cliffs 20.0 m tall. Assuming the plain was flat so that the bison ran horizontally off the cliff, and the bison were moving at their maximum speed of 18.0 m/s at the time of the fall, determine how far from the base of the cliff the bison landed.

▲ **Figure 2.68**



◄ **Figure 2.69**

Practice Problems

1. A coin rolls off a table with an initial horizontal speed of 30 cm/s. How far will the coin land from the base of the table if the table's height is 1.25 m?
2. An arrow is fired horizontally with a speed of 25.0 m/s from the top of a 150.0-m-tall cliff. Assuming no air resistance, determine the distance the arrow will drop in 2.50 s.
3. What is the horizontal speed of an object if it lands 40.0 m away from the base of a 100-m-tall cliff?

Answers

1. 15 cm
2. 30.7 m
3. 8.86 m/s

Given

For convenience, choose forward and down to be positive because the motion is forward and down (Figure 2.69).

x direction	y direction
$v_{i_x} = 18.0$ m/s	$a_y = 9.81\text{ m/s}^2\text{ [down]} = +9.81\text{ m/s}^2$
	$\Delta d_y = 20.0$ m

Required

distance from the base of the cliff (Δd_x)

Analysis and Solution

Since there is no vertical component to the initial velocity of the bison, $v_{i_y} = 0$ m/s. Therefore, the bison experience uniformly accelerated motion due to gravity in the vertical direction but uniform motion in the horizontal direction resulting from the run.

From the given values, note that, in the y direction, you have all the variables except for time. So, you can solve for time in the y direction, which is the time taken to fall.

y direction:

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$= 0 + \frac{1}{2}a_y\Delta t^2$$

PHYSICS INSIGHT

For projectile motion in two dimensions, the time taken to travel horizontally equals the time taken to travel vertically.

eSIM

Analyze balls undergoing projectile motion. Follow the eSim links at www.pearsoned.ca/school/physicssource.

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{2(20.0 \text{ m})}{9.81 \text{ m/s}^2}}$$

$$= 2.019 \text{ s}$$

x direction:
The time taken for the bison to fall vertically equals the time they travel horizontally. Substitute the value for time you found in the *y* direction to find the range. Since the bison had a uniform horizontal speed of 18.0 m/s, use the equation $\Delta d_x = v_{i_x}\Delta t$.

$$\Delta d_x = (18.0 \text{ m/s})(2.019 \text{ s})$$

$$= 36.3 \text{ m}$$

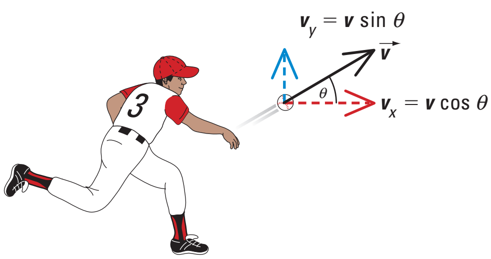
Paraphrase
The bison would land 36.3 m from the base of the cliff.

Objects Launched at an Angle

▼ **Figure 2.70** Baseball is all about projectile motion.

Baseball is a projectile game (Figure 2.70). The pitcher throws a ball at the batter, who hits it to an open area in the field. The outfielder catches the ball and throws it to second base. The runner is out. All aspects of this sequence involve projectile motion. Each sequence requires a different angle on the throw and a different speed. If the player miscalculates one of these variables, the action fails: Pitchers throw wild pitches, batters strike out, and outfielders overthrow the bases. Winning the game depends on accurately predicting the components of the initial velocity!

For objects launched at an angle, such as a baseball, the velocity of the object has both a horizontal and a vertical component. Any vector quantity can be resolved into *x* and *y* components using the trigonometric ratios $R_x = R \cos \theta$ and $R_y = R \sin \theta$, when θ is measured relative to the *x*-axis. To determine the horizontal and vertical components of velocity, this relationship becomes $v_x = v \cos \theta$ and $v_y = v \sin \theta$, as shown in Figure 2.71.



▲ **Figure 2.71** The horizontal and vertical components of velocity

Solving problems involving objects launched at an angle is similar to solving problems involving objects launched horizontally. The object experiences uniform motion in the horizontal direction, so use the equation $\Delta d_x = v_{i_x}\Delta t$. In the vertical direction, the object experiences uniformly accelerated motion. The general equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ still applies, but in this case, v_{i_y} is not zero. The next example shows you how to apply these equations to objects launched at an angle.

Example 2.11

Baseball players often practise their swing in a batting cage, in which a pitching machine delivers the ball (Figure 2.72). If the baseball is launched with an initial velocity of 22.0 m/s [30.0°] and the player hits it at the same height from which it was launched, for how long is the baseball in the air on its way to the batter?

▲ **Figure 2.72**

Given

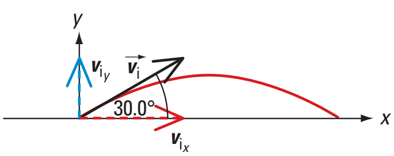
$\vec{v}_i = 22.0$ m/s [30.0°]

Required

time (Δt)

Analysis and Solution

Choose forward and up to be positive (Figure 2.73). First find the components of the baseball's initial velocity.



◀ **Figure 2.73**

x direction

$$\begin{aligned} v_{i_x} &= v_i \cos\theta \\ &= (22.0\ \text{m/s})(\cos 30.0°) \\ &= 19.05\ \text{m/s} \end{aligned}$$

y direction

$$\begin{aligned} v_{i_y} &= v_i \sin\theta \\ &= (22.0\ \text{m/s})(\sin 30.0°) \\ &= 11.00\ \text{m/s} \end{aligned}$$

Since the ball returns to the same height from which it was launched, $\Delta d_y = 0$. With this extra known quantity, you now have enough information in the *y* direction to find the time the ball spent in the air.

Practice Problems

1. A ball thrown horizontally at 10.0 m/s travels for 3.0 s before it strikes the ground. Find
   (a) the distance it travels horizontally.
   (b) the height from which it was thrown.
2. A ball is thrown with a velocity of 20.0 m/s [30°] and travels for 3.0 s before it strikes the ground. Find
   (a) the distance it travels horizontally.
   (b) the height from which it was thrown.
   (c) the maximum height of the ball.

Answers

1. (a) 30 m
   (b) 44 m
2. (a) 52 m
   (b) 14 m
   (c) 19 m

PHYSICS INSIGHT

Be careful to follow the sign convention you chose. If you chose up as positive, a_y becomes −9.81 m/s².

info BIT

The world's fastest bird is the peregrine falcon, with a top vertical speed of 321 km/h and a top horizontal speed of 96 km/h.

e WEB

The fastest speed for a projectile in any ball game is approximately 302 km/h in jai-alai. To learn more about jai-alai, follow the links at www.pearsoned.ca/school/physicssource.

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$0 = (11.00 \text{ m/s})\Delta t + \frac{1}{2}(-9.81 \text{ m/s}^2)(\Delta t)^2$$

Isolate Δt and solve.

$$(4.905 \text{ m/s}^2)(\Delta t)^{\cancel{2}} = (11.00 \text{ m/s})(\cancel{\Delta t})$$

$$\Delta t = \frac{11.00 \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}}{4.905 \dfrac{\text{m}}{\text{s}^2}}$$

$$= 2.24 \text{ s}$$

Paraphrase

The baseball is in the air for 2.24 s.

PHYSICS INSIGHT

Since the vertical velocity of the ball at maximum height is zero, you can also calculate the time taken to go up and multiply the answer by two. If down is positive,

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{0 \text{ m/s} - (-11.00 \text{ m/s})}{9.81 \text{ m/s}^2}$$

$$= \frac{11.00 \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}}{9.81 \dfrac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$= 1.121 \text{ s}$$

The total time the baseball is in the air is $2 \times 1.121 \text{ s} = 2.24 \text{ s}$.

info BIT

The longest speedboat jump was 36.5 m in the 1973 James Bond movie *Live and Let Die*. The boat practically flew over a road.

How far would the baseball in Example 2.11 travel horizontally if the batter missed and the baseball landed at the same height from which it was launched? Since horizontal velocity is constant,

$$\Delta d_x = v_{i_x}\Delta t$$

$$= (19.05 \text{ m/s})(2.24 \text{ s})$$

$$= 42.7 \text{ m}$$

The baseball would travel a horizontal distance of 42.7 m.

In the next example, you are given the time and are asked to solve for one of the other variables. However, the style of solving the problem remains the same. In any problem that you will be asked to solve in this course, you will always be able to solve for one quantity in either the x or y direction, and then you can substitute your answer to solve for the remaining variable(s).

Example 2.12

A paintball directed at a target is shot at an angle of 25.0°. If paint splats on its intended target at the same height from which it was launched, 3.00 s later, find the distance from the shooter to the target.

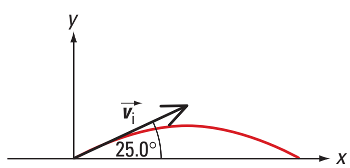
Given

Choose down and right to be positive.

$\vec{a} = a_y = 9.81 \text{ m/s}^2 \text{ [down]} = +9.81 \text{ m/s}^2$

$\theta = 25.0°$

$\Delta t = 3.00 \text{ s}$



▲ **Figure 2.74**

Required

range (Δd_x)

Analysis and Solution

Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$. Since the height of landing is the same as the launch height, $\Delta d_y = 0$.

y direction:

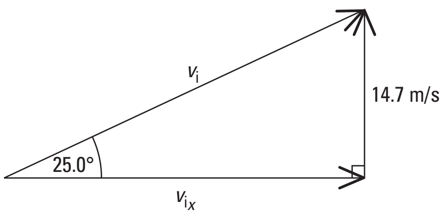
$$\begin{aligned}\Delta d_y &= v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2\\ 0 &= v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2\\ v_{i_y}\Delta t &= -\frac{1}{2}a_y(\Delta t)^2\\ v_{i_y} &= -\frac{1}{2}a_y\Delta t\\ &= -\frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(3.00\ \cancel{\text{s}})\\ &= -14.7\ \text{m/s}\end{aligned}$$

Since down is positive, the negative sign means that the direction of the vertical component of initial velocity is up.

x direction:

Find the initial horizontal speed using the tangent function. Because there is no acceleration in the x direction, the ball's horizontal speed remains the same during its flight: $a_x = 0$.

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ \text{adjacent} &= \frac{\text{opposite}}{\tan\theta}\\ &= \frac{14.7\ \text{m/s}}{\tan 25.0°}\\ &= 31.56\ \text{m/s}\end{aligned}$$



▲ **Figure 2.75**

From Figure 2.75, the adjacent side is v_{i_x} and it points to the right, so $v_{i_x} = 31.56$ m/s.

Now find the horizontal distance travelled.

$$\begin{aligned}\Delta d_x &= v_{i_x}\Delta t\\ &= (31.56\ \text{m/s})(3.00\ \text{s})\\ &= 94.7\ \text{m}\end{aligned}$$

Paraphrase

The distance that separates the target from the shooter is 94.7 m.

PHYSICS INSIGHT

Alternatively, the time taken to reach maximum height is the same time taken to fall back down to the same height. So, the paintball is at its maximum height at 1.50 s. The speed at maximum height is zero. If up is positive,

$$\begin{aligned}\vec{a} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t}\\ \vec{v}_i &= \vec{v}_f - \vec{a}\Delta t\\ &= 0\ \text{m/s} - \vec{a}\Delta t\\ &= -(-9.81\ \text{m/s}^2)(\Delta t)\\ &= +(9.81\ \text{m/s}^2)(1.50\ \text{s})\\ &= +14.7\ \text{m/s}\end{aligned}$$

The sign is positive, so the direction is up.

Practice Problems

1. Determine the height reached by a baseball if it is released with a velocity of 17.0 m/s [20°].
2. A German U2 rocket from the Second World War had a range of 300 km, reaching a maximum height of 100 km. Determine the rocket's maximum initial velocity.

Answers

1. 1.72 m
2. 1.75×10^3 m/s [53.1°]

The points below summarize what you have learned in this section.

- To solve problems involving projectiles, first resolve the motion into its components using the trigonometric functions, then apply the kinematics equations.
- Perpendicular components of motion are independent of one another.
- Horizontal motion is considered uniform and is described by the equation $\Delta\vec{d} = \vec{v}\Delta t$, whereas vertical motion is a special case of uniformly accelerated motion, where the acceleration is the acceleration due to gravity or 9.81 m/s^2 [down].
- A projectile's path is a parabola.
- In the vertical direction, a projectile's velocity is greatest at the instant of launch and just before impact, whereas at maximum height, vertical velocity is zero.

2.4 Check and Reflect

Knowledge

1. Platform divers receive lower marks if they enter the water a distance away from the platform, whereas speed swimmers dive as far out into the pool as they can. Compare and contrast the horizontal and vertical components of each type of athlete's motion.
2. For a fixed speed, how does the range depend on the angle, θ?
3. (a) For a projectile, is there a location on its trajectory where the acceleration and velocity vectors are perpendicular? Explain.
   (b) For a projectile, is there a location on its trajectory where the acceleration and velocity vectors are parallel? Explain.
4. Water safety instructors tell novice swimmers to put their toes over the edge and jump out into the pool. Explain why, using concepts from kinematics and projectile motion.

Applications

5. Participants in a road race take water from a refreshment station and throw their empty cups away farther down the course. If a runner has a forward speed of 6.20 m/s, how far in advance of a garbage pail should he release his water cup if the vertical distance between the lid of the garbage can and the runner's point of release is 0.50 m?
6. A baseball is thrown with a velocity of 27.0 m/s [35°]. What are the components of the ball's initial velocity? How high and how far will it travel?
7. A football is thrown to a moving receiver. The football leaves the quarterback's hands 1.75 m above the ground with a velocity of 17.0 m/s [25°]. If the receiver starts 12.0 m away from the quarterback along the line of flight of the ball when it is thrown, what constant velocity must she have to get to the ball at the instant it is 1.75 m above the ground?
8. At the 2004 Olympic Games in Athens, Dwight Phillips won the gold medal in men's long jump with a jump of 8.59 m. If the angle of his jump was 23°, what was his takeoff speed?
9. A projectile is fired with an initial speed of 120 m/s at an angle of 55.0° above the horizontal from the top of a cliff 50.0 m high. Find
   (a) the time taken to reach maximum height
   (b) the maximum height with respect to the ground next to the cliff
   (c) the total time in the air
   (d) the range
   (e) the components of the final velocity just before the projectile hits the ground

Extension

10. Design a spreadsheet to determine the maximum height and range of a projectile with a launch angle that increases from 0° to 90° and whose initial speed is 20.0 m/s.

eTEST

To check your understanding of projectile motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

Key Terms and Concepts

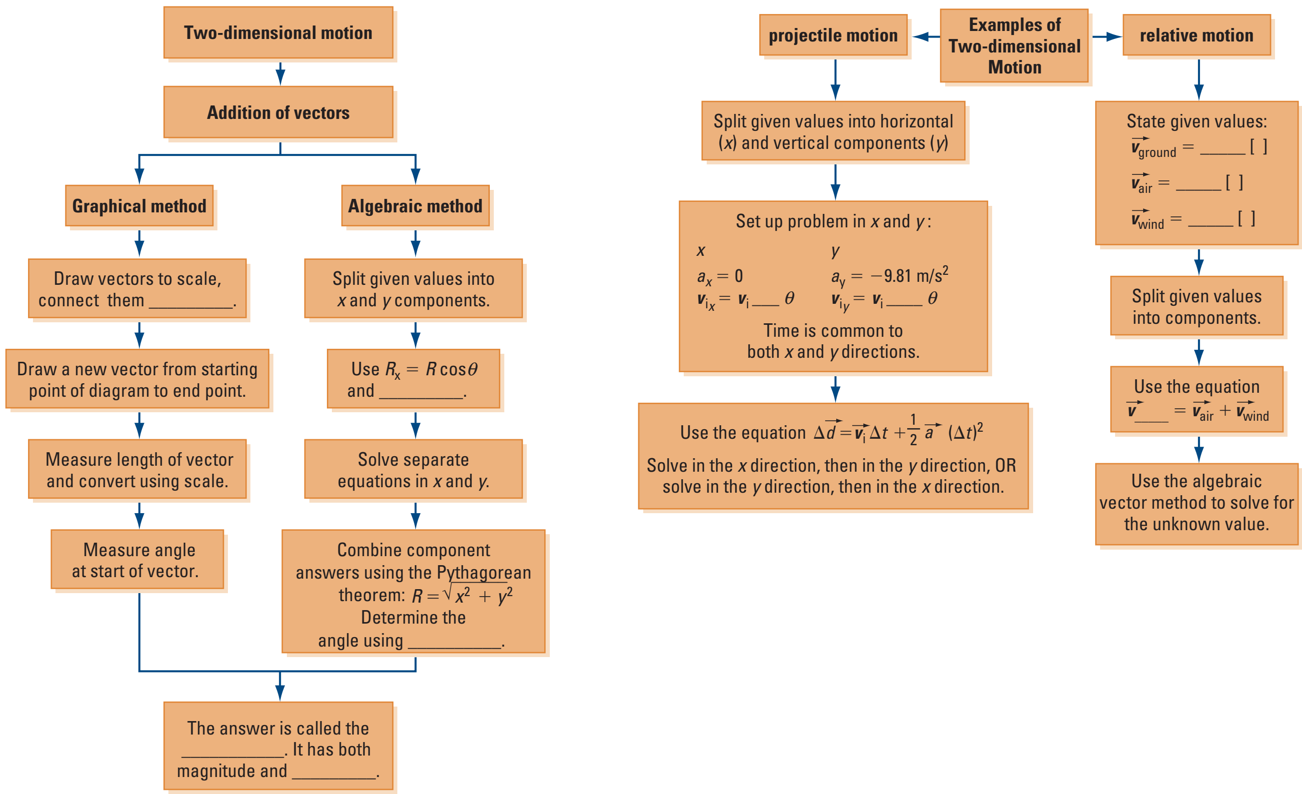
collinear
resultant vector
components
polar coordinates method
navigator method
non-collinear
relative motion
ground velocity
air velocity
wind velocity
trajectory
range

Key Equations

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2 \qquad \Delta\vec{d} = \vec{v}\Delta t$$

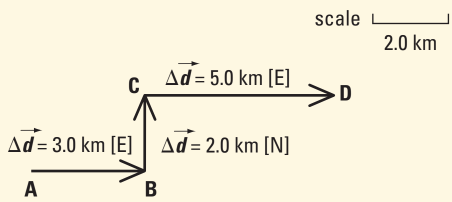
Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.



Knowledge

1. (2.2) During the Terry Fox Run, a participant travelled from A to D, passing through B and C. Copy and complete the table using the information in the diagram, a ruler calibrated in millimetres, and a protractor. In your notebook, draw and label the displacement vectors AB, BC, and CD and the position vectors AB, AC, and AD. Assume the participant's reference point is A.



	Distance Δd (km)	Final position $\vec{d}$ (km) [direction] reference point	Displacement $\Delta\vec{d}$ (km) [direction]
AB			
BC			
CD			
AC			
AD			

2. (2.2) Determine the *x* and *y* components of the displacement vector 55 m [222°].

3. (2.4) What is the vertical component for velocity at the maximum height of a projectile's trajectory?

4. (2.4) During a field goal kick, as the football rises, what is the effect on the vertical component of its velocity?

5. (2.1) Fort McMurray is approximately 500 km [N] of Edmonton. Using a scale of 1.0 cm : 50.0 km, draw a displacement vector representing this distance.

6. (2.1) Give one reason why vector diagrams must be drawn to scale.

7. (2.2) Using an appropriate scale and reference coordinates, graphically solve each of the following:

(a) 5.0 m [S] and 10.0 m [N]

(b) 65.0 cm [E] and 75.0 cm [E]

(c) 1.0 km [forward] and 3.5 km [backward]

(d) 35.0 km [right] −45.0 km [left]

8. (2.4) For an object thrown vertically upward, what is the object's initial horizontal velocity?

Applications

9. The air medivac, King Air 200, flying at 250 knots (1 knot = 1.853 km/h), makes the trip between Edmonton and Grande Prairie in 50 min. What distance does the plane travel during this time?

10. A golf ball is hit with an initial velocity of 30.0 m/s [55°]. What are the ball's range and maximum height?

11. Off the tee box, a professional golfer can drive a ball with a velocity of 80.0 m/s [10°]. How far will the ball travel horizontally before it hits the ground and for how long is the ball airborne?

12. A canoeist capable of paddling north at a speed of 4.0 m/s in still water wishes to cross a river 120 m wide. The river is flowing at 5.0 m/s [E]. Find

(a) her velocity relative to the ground

(b) the time it takes her to cross

13. An object is thrown horizontally off a cliff with an initial speed of 7.50 m/s. The object strikes the ground 3.0 s later. Find

(a) the object's vertical velocity component when it reaches the ground

(b) the distance between the base of the cliff and the object when it strikes the ground

(c) the horizontal velocity of the object 1.50 s after its release

14. If a high jumper reaches her maximum height as she travels across the bar, determine the initial velocity she must have to clear a bar set at 2.0 m if her range during the jump is 2.0 m. What assumptions did you make to complete the calculations?

15. An alligator wishes to swim north, directly across a channel 500 m wide. There is a current of 2.0 m/s flowing east. The alligator is capable of swimming at 4.0 m/s. Find

(a) the angle at which the alligator must point its body in order to swim directly across the channel

(b) its velocity relative to the ground

(c) the time it takes to cross the channel

16. A baseball player throws a ball horizontally at 45.0 m/s. How far will the ball drop before reaching first base 27.4 m away?

17. How much time can you save travelling diagonally instead of walking 750 m [N] and then 350 m [E] if your walking speed is 7.0 m/s?

18. How long will an arrow be in flight if it is shot at an angle of 25° and hits a target 50.0 m away, at the same elevation?

19. A pilot of a small plane wishes to fly west. The plane has an airspeed of 100 km/h. If there is a 30-km/h wind blowing north, find
    (a) the plane's heading
    (b) the plane's ground speed

20. At what angle was an object thrown if its initial launch speed is 15.7 m/s, it remains airborne for 2.15 s, and travels 25.0 m horizontally?

21. A coin rolls off a 25.0° incline on top of a 2.5-m-high bookcase with a speed of 30 m/s. How far from the base of the bookcase will the coin land?

22. Starting from the left end of the hockey rink, the goal line is 3.96 m to the right of the boards, the blue line is 18.29 m to the right of the goal line, the next blue line is 16.46 m to the right of the first blue line, the goal line is 18.29 m right, and the right board is 3.96 m right of the goal line. How long is a standard NHL hockey rink?

23. A plane with a ground speed of 151 km/h is moving 11° south of east. There is a wind blowing at 40 km/h, 45° south of east. Find
    (a) the plane's airspeed
    (b) the plane's heading, to the nearest degree

24. How long will a soccer ball remain in flight if it is kicked with an initial velocity of 25.0 m/s [35.0°]? How far down the field will the ball travel before it hits the ground and what will be its maximum height?

25. At what angle is an object launched if its initial vertical speed is 3.75 m/s and its initial horizontal speed is 4.50 m/s?

Extensions

26. During the Apollo 14 mission, Alan Shepard was the first person to hit a golf ball on the Moon. If a golf ball was launched from the Moon's surface with a velocity of 50 m/s [35°] and the acceleration due to gravity on the Moon is $-1.61\ \text{m/s}^2$,
    (a) how long was the golf ball in the air?
    (b) what was the golf ball's range?

27. An airplane is approaching a runway for landing. The plane's air velocity is 645 km/h [forward], moving through a headwind of 32.2 km/h. The altimeter indicates that the plane is dropping at a constant velocity of 3.0 m/s [down]. If the plane is at a height of 914.4 m and the range from the plane to the start of the runway is 45.0 km, does the pilot need to make any adjustments to her descent in order to land the plane at the start of the runway?

Consolidate Your Understanding

Create your own summary of kinematics by answering the questions below. If you want to use a graphic organizer, refer to Student References 4: Using Graphic Organizers on pp. 869–871. Use the Key Terms and Concepts listed on page 113 and the Learning Outcomes on page 68.

1. Create a flowchart to describe the different components required to analyze motion in a horizontal plane and in a vertical plane.

2. Write a paragraph describing the similarities and differences between motion in a horizontal plane and motion in a vertical plane. Share your thoughts with another classmate.

Think About It

Review your answers to the Think About It questions on page 69. How would you answer each question now?

eTEST

To check your understanding of two-dimensional motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

Are Amber Traffic Lights Timed Correctly?

Scenario

The Traffic Safety Act allows law enforcement agencies in Alberta to issue fines for violations using evidence provided by red light cameras at intersections. The cameras photograph vehicles that enter an intersection *after* the traffic lights have turned red. They record the time, date, location, violation number, and time elapsed since the light turned red. The use of red light cameras and other technology reduces the amount of speeding, running of red lights, and collisions at some intersections.

The length of time a traffic light must remain amber depends on three factors: perception time, reaction time, and braking time. The sum of perception time and reaction time is the time elapsed between the driver seeing the amber light and applying the brakes. The Ministry of Infrastructure and Transportation's (MIT) *Basic Licence Driver's Handbook* allows for a perception time of 0.75 s and a reaction time of 0.75 s. The braking time is the time it takes the vehicle to come to a full stop once the brakes are applied. Braking time depends on the vehicle's initial speed and negative acceleration. The MIT's predicted braking times are based on the assumption that vehicles travel at the posted speed limit and have a uniform acceleration of -3.0 m/s^2. Other factors that affect acceleration are road conditions, vehicle and tire performance, weather conditions, and whether the vehicle was travelling up or down hill.

If drivers decide to go through an intersection safely (go distance) after a light has turned amber, they must be able to travel not only to the intersection but across it before the light turns red. The go distance depends on the speed of the vehicle, the length of the intersection, and the amount of time the light remains amber. If the driver decides to stop (stop distance), the vehicle can safely do so only if the distance from the intersection is farther than the distance travelled during perception time, reaction time, and braking time.

As part of a committee reporting to the Ministry of Infrastructure and Transportation, you must respond to concerns that drivers are being improperly fined for red light violations because of improper amber light timing. You are to decide how well the amber light time matches the posted speed limit and intersection length. Assume throughout your analysis that drivers travel at the posted speed limits.

Planning

Research or derive equations to determine

(a) a car's displacement during reaction time

(b) stop distance

(c) go distance

(d) amber light time

(e) displacement after brakes are applied

(f) amount of time elapsed after the brakes are applied

Assessing Results

After completing the project, assess its success based on a rubric* designed in class that considers

- research strategies
- experiment techniques
- clarity and thoroughness of the written report
- effectiveness of the team's presentation

Materials

- measuring tape, stopwatch

Procedure

1. Design a survey to measure the amber light times at 10 different intersections near your school. For each intersection, record its length. ***Use caution around intersections due to traffic! You may wish to estimate the length of the intersection by counting the number of steps it takes you to cross and measuring the length of your stride.***
2. Apply suitable equations to determine appropriate amber light times for the 10 different intersections.
3. Calculate stop distances and go distances for a range ($\pm$10 km/h) of posted speed limits for each intersection and plot graphs of stop distance and go distance against posted speed.

Thinking Further

1. Research the effectiveness of red light cameras in reducing accidents, speeding, and red light violations. Using your research, recommend a course of action to increase vehicle-rail safety at light-controlled railway crossings.
2. Based on your surveys and investigation, recommend whether existing amber light times should be increased, decreased, or left alone. Consider posted speeds against actual speeds and wet against dry surface conditions.
3. Prepare a presentation to the other members of your committee. Include graphs and diagrams.

*Note: Your instructor will assess the project using a similar assessment rubric.

UNIT I SUMMARY

Unit Concepts and Skills: Quick Reference

Concepts	Summary	Resources and Skill Building
Chapter 1	**Graphs and equations describe motion in one dimension.**	
	1.1 The Language of Motion	
Scalar and vector quantities	A scalar quantity consists of a number and a unit.	Section 1.1
	A vector quantity consists of a number, a unit, and a direction.	Section 1.1
Distance, position, and displacement	Distance is the length of the path taken to travel from one position to another. Position is the straight-line distance from the origin to the object's location. Displacement is the change in position.	Figure 1.5 Figures 1.3, 1.4, 1.5, Example 1.1 Figures 1.3, 1.4, 1.5, Example 1.1
	1.2 Position-time Graphs and Uniform Motion	
Slope of a position-time graph	A position-time graph for an object at rest is a straight line with zero slope. A position-time graph for an object moving at a constant velocity is a straight line with non-zero slope. The greater the slope of a position-time graph, the faster the object is moving.	Figure 1.14 Figures 1.12, 1.15(b), 1-3 Inquiry Lab Examples 1.2, 1.3, 1-3 Inquiry Lab
	1.3 Velocity-time Graphs: Uniform and Non-uniform Motion	
Slope of a velocity-time graph	A velocity-time graph for an object experiencing uniform motion is a horizontal line. The slope of a velocity-time graph represents acceleration.	Figure 1.24 Figures 1.24, 1.30, Example 1.5
Position-time, velocity-time, acceleration–time graphs representing accelerated motion	The position-time graph for an object undergoing uniformly accelerated motion is a curve. The corresponding velocity-time graph is a straight line with non-zero slope. The corresponding acceleration-time graph is a horizontal line.	Figures 1.28–1.31, Example 1.5
Instantaneous velocity	The slope of the tangent on a position-time curve gives instantaneous velocity.	Figure 1.29, Example 1.5
	1.4 Analyzing Velocity-time Graphs	
Area under and slope of a velocity-time graph	The area under a velocity-time graph represents displacement; slope represents acceleration.	Figure 1.41, Examples 1.6, 1.8, 1.9
Average velocity	Average velocity represents total displacement divided by time elapsed.	Figure 1.45, Examples 1.7, 1.9
Velocity-time graphs	You can draw acceleration-time and position-time graphs by calculating and plotting slope and area, respectively, of a velocity-time graph.	Examples 1.10, 1.11
	1.5 The Kinematics Equations	
Kinematics equations	When solving problems in kinematics, choose the equation that contains all the given variables in the problem as well as the unknown variable.	Figures 1.53–1.55 Examples 1.12–1.16
	1.6 Acceleration Due to Gravity	
Projectile motion straight up and down	Gravity causes objects to accelerate downward.	1-6 QuickLab, 1-7 Inquiry Lab, 1-8 QuickLab, Examples 1.17–1.19
Maximum height	At maximum height, a projectile's vertical velocity is zero. The time taken to reach maximum height equals the time taken to fall back down to the original height.	Figures 1.63–1.65 Examples 1.18, 1.19
Chapter 2	**Vector components describe motion in two dimensions.**	
	2.1 Vector Methods in One Dimension	
Adding and subtracting vectors	Add vectors by connecting them tip to tail. Subtract vectors by connecting them tail to tail.	Examples 2.1, 2.2
	2.2 Motion in Two Dimensions	
Components	To add vectors in two dimensions, draw a scale diagram, or resolve them into their components and use trigonometry to find the resultant.	Examples 2.3–2.5, Figures 2.31, 2.33–2.37
	2.3 Relative Motion	
Relative motion	To solve relative motion problems, use trigonometry, with ground velocity as the resultant. If the vectors are not perpendicular, resolve them into their components first.	Examples 2.6–2.9
	2.4 Projectile Motion	
Projectile motion in two dimensions	The shape of a projectile's trajectory is a parabola. Horizontal and vertical components of projectile motion are independent. To solve projectile problems in two dimensions, resolve them into their horizontal and vertical components. Then use the kinematics equations. The time taken to travel horizontally equals the time taken to travel vertically.	2-4 QuickLab, Figure 2.62 2-5 QuickLab, Figure 2.64 Examples 2.10–2.12, Figures 2.67, 2.71

UNIT I REVIEW

Vocabulary

1. Using your own words, define these terms:
   acceleration
   acceleration due to gravity
   air velocity
   at rest
   collinear
   components
   displacement
   distance
   ground velocity
   instantaneous velocity
   kinematics
   navigator method
   non-collinear
   non-uniform motion
   origin
   polar coordinates method
   position
   projectile
   projectile motion
   range
   relative motion
   resultant vector
   scalar quantity
   tangent
   trajectory
   uniform motion
   uniformly accelerated motion
   vector quantity
   velocity
   wind velocity

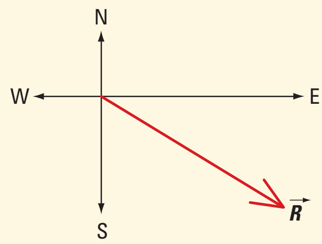
Knowledge

CHAPTER 1

2. Describe how scalar quantities differ from vector quantities.

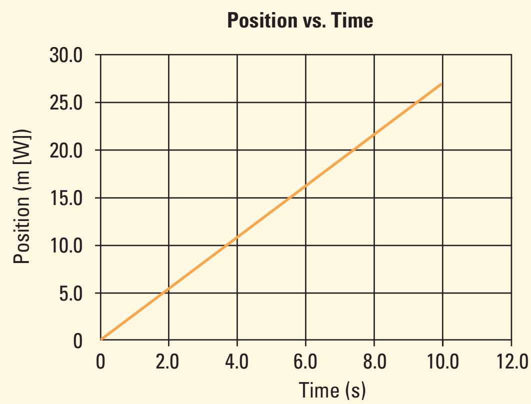
CHAPTER 2

3. Resolve the following vectors into their components:
   (a) 5.0 m [90°]
   (b) 16.0 m/s [20° S of W]
4. Using an appropriate scale and reference coordinates, draw the following vectors:
   (a) 5.0 m/s [0°]
   (b) 25.0 m/s² [60° N of E]
   (c) 1.50 km [120°]
5. Using a scale of 1.0 cm : 3.5 km, determine the magnitude and direction of the vector below.



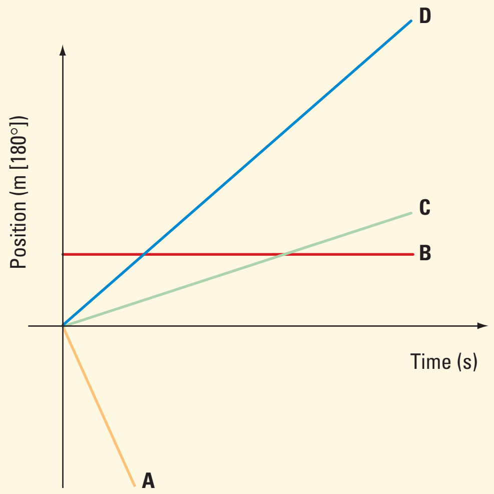
Applications

6. A wildlife biologist records a moose's position as it swims away from her. Using the graph below, determine the moose's velocity.



7. Sketch a position-time graph for each statement below. Assume that right is positive.
   (a) object accelerating to the right
   (b) object accelerating to the left
   (c) object travelling at a constant velocity left
   (d) object at rest
   (e) object travelling with constant velocity right
8. Hockey pucks can be shot at speeds of 107 km/h. If a puck is shot at an angle of 30°, determine how long the puck is in the air, how far it will travel, and how high it will be at the peak of its trajectory.
9. Sketch two different position-time graphs for objects with a negative velocity.
10. Sketch two different velocity-time graphs for objects with a negative acceleration.

11. From the position-time graph below, determine which object has the greatest velocity.



12. Solve each of the following equations for initial velocity, $\vec{v}_i$, algebraically.

(a) $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

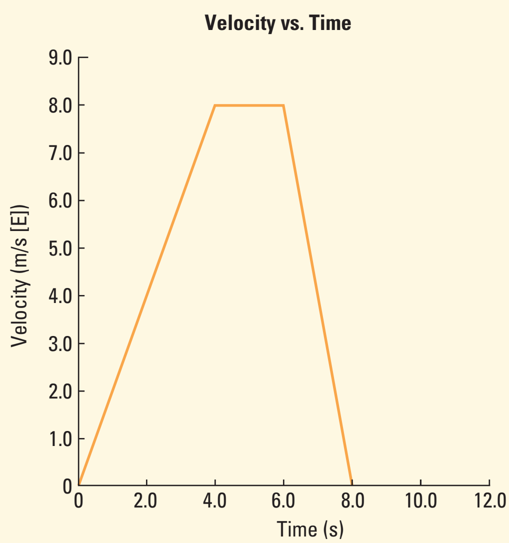
(b) $\Delta\vec{d} = \vec{v}_i \Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$

(c) $\Delta\vec{d} = \dfrac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$

13. The longest kickoff in CFL history is 83.2 m. If the ball remains in the air for 5.0 s, determine its initial speed.

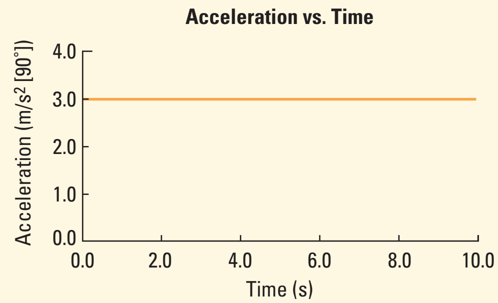
14. Determine the speed of a raven that travels 48 km in 90 min.

15. Describe the motion of the object illustrated in the graph below.



16. (a) What is the change in velocity in 10.0 s, as illustrated in the acceleration-time graph below?

(b) If the object had an initial velocity of 10 m/s [90°], what is its final velocity after 10.0 s?



17. How far will a crow fly at 13.4 m/s for 15.0 min?

18. How long will it take a car to travel from Valleyview to Grande Prairie if its speed is 100 km/h? The map's scale is 1 cm : 118 km.

19. A baseball player hits a baseball with a velocity of 30 m/s [25°]. If an outfielder is 85.0 m from the ball when it is hit, how fast will she have to run to catch the ball before it hits the ground?

20. Determine the magnitude of the acceleration of a Jeep Grand Cherokee if its stopping distance is 51.51 m when travelling at 113 km/h.

21. What is the velocity of an aircraft with respect to the ground if its air velocity is 785 km/h [S] and the wind is blowing 55 km/h [22° S of W]?

22. An object undergoing uniformly accelerated motion has an initial speed of 11.0 m/s and travels 350 m in 3.00 s. Determine the magnitude of its acceleration.

23. Improperly installed air conditioners can occasionally fall from apartment windows down onto the road below. How long does a pedestrian have to get out of the way of an air conditioner falling eight stories (24 m)?

24. An object is launched from the top of a building with an initial velocity of 15 m/s [32°]. If the building is 65.0 m high, how far from the base of the building will the object land?

25. Two friends walk at the same speed of 4.0 km/h. One friend steps onto a travelator moving at 3.0 km/h. If he maintains the same initial walking speed,
   (a) how long will it take him to reach the end of the 100-m-long travelator?
   (b) what must be the magnitude of the acceleration of the other friend to arrive at the end of the travelator at the same time?

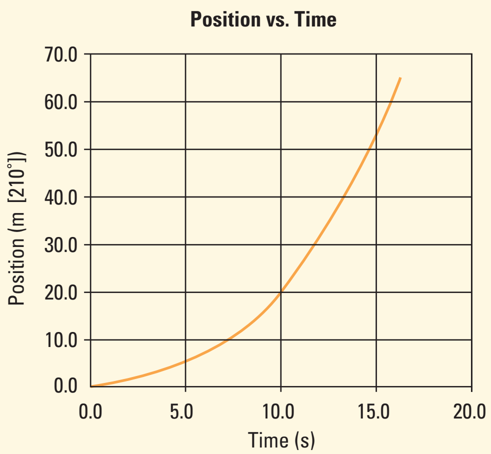
26. How far will a vehicle travel if it accelerates uniformly at 2.00 m/s² [forward] from 2.50 m/s to 7.75 m/s?

27. An object is thrown into the air with a speed of 25.0 m/s at an angle of 42°. Determine how far it will travel horizontally before hitting the ground.

28. Determine the average velocity of a truck that travels west from Lloydminster to Edmonton at 110 km/h for 1.0 h and 20 min and then 90 km/h for 100 min.

29. What distance will a vehicle travel if it accelerates uniformly from 15.0 m/s [S] to 35.0 m/s [S] in 6.0 s?

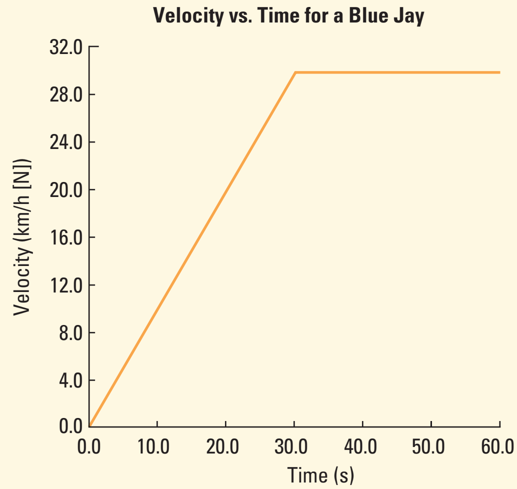
30. From the graph below, determine the instantaneous velocity of the object at 5.0 s, 10.0 s, and 15.0 s.



31. A speedboat's engine can move the boat at a velocity of 215 km/h [N]. What is the velocity of the current if the boat's displacement is 877 km [25° E of N] 3.5 h later?

32. An object starts from rest and travels 50.0 m along a frictionless, level surface in 2.75 s. What is the magnitude of its acceleration?

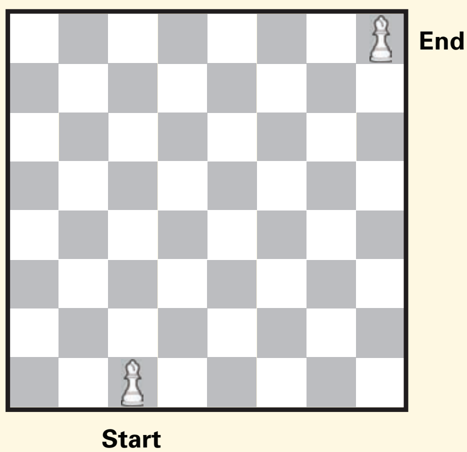
33. Determine the displacement of the blue jay from the velocity-time graph below.



34. Sketch a position-time graph for an object that travels at a constant velocity of 5.0 m/s for 10 s, stops for 10 s, then travels with a velocity of −2.0 m/s for 20 s.

35. Determine the height reached by a projectile if it is released with a velocity of 18.0 m/s [20°].

36. The bishop is a chess piece that moves diagonally along one colour of square. Assuming the first move is toward the left of the board, determine
   (a) the minimum number of squares the bishop covers in getting to the top right square.
   (b) the bishop's displacement from the start if the side length of each square is taken as 1 unit and each move is from the centre of a square to the centre of another square.



37. A wildlife biologist notes that she is 350 m [N] from the park ranger station at 8:15 a.m. when she spots a polar bear. At 8:30 a.m., she is 1.75 km [N] of the ranger station. Determine the biologist's average velocity.

38. A bus travels 500 m [N], 200 m [E], and then 750 m [S]. Determine its displacement from its initial position.

39. Match the motion with the correct position-time graph given below. Identify the motion as at rest, uniform motion, or uniformly accelerated motion.

(a) an airplane taking off

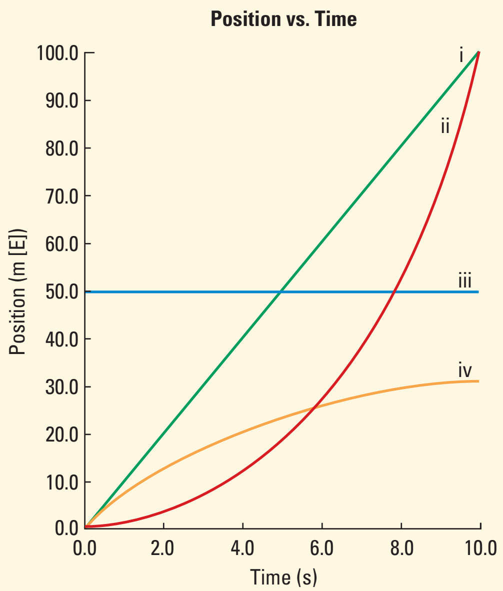
(b) an airplane landing

(c) passing a car on the highway

(d) waiting at the red line at Canada Customs

(e) standing watching a parade

(f) travelling along the highway on cruise control



40. Determine the magnitude of the acceleration of a Jeep Grand Cherokee that can reach 26.9 m/s from rest in 4.50 s.

Extensions

41. A penny is released from the top of a wishing well and hits the water's surface 1.47 s later. Calculate

(a) the velocity of the penny just before it hits the water's surface

(b) the distance from the top of the well to the water's surface

42. A balloonist drops a sandbag from a balloon that is rising at a constant velocity of 3.25 m/s [up]. It takes 8.75 s for the sandbag to reach the ground. Determine

(a) the height of the balloon when the sandbag is dropped

(b) the height of the balloon when the sandbag reaches the ground

(c) the velocity with which the sandbag hits the ground

43. A motorcycle stunt rider wants to jump a 20.0-m-wide row of cars. The launch ramp is angled at 30° and is 9.0 m high. The landing ramp is also angled at 30° and is 6.0 m high. Find the minimum launch velocity required for the stunt rider to reach the landing ramp.

Skills Practice

44. Draw a Venn diagram to compare and contrast vector and scalar quantities.

45. Draw a Venn diagram to illustrate the concepts of graphical analysis.

46. A swimmer wants to cross from the east to the west bank of the Athabasca River in Fort McMurray. The swimmer's speed in still water is 3.0 m/s and the current's velocity is 4.05 m/s [N]. He heads west and ends up downstream on the west bank. Draw a vector diagram for this problem.

47. For an experiment to measure the velocity of an object, you have a radar gun, probeware, and motion sensors. Explain to a classmate how you would decide which instrument to use.

48. Design an experiment to determine the acceleration of an object rolling down an inclined plane.

49. Construct a concept map for solving a two-dimensional motion problem involving a projectile thrown at an angle.

50. Explain how you can use velocity-time graphs to describe the motion of an object.

Self-assessment

51. Describe to a classmate which kinematics concepts and laws you found most interesting when studying this unit. Give reasons for your choices.

52. Identify one issue pertaining to motion studied in this unit that you would like to investigate in greater detail.

53. What concept in this unit did you find most difficult? What steps could you take to improve your understanding?

54. As a future voter, what legislation would you support to improve vehicular and road safety?

55. Assess how well you are able to graph the motion of an object. Explain how you determine a reference point.

***e*TEST**

To check your understanding of kinematics, follow the eTest links at www.pearsoned.ca/school/physicssource.